Revisiting the inflation – output gap relationship for France using a wavelet transform approach

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Abstract

The purpose of the paper is to revisit the inflation – outputgap relationship using a new approach known as the wavelet transform. This approach combines the classical time series analysis with frequency domain analysis and presents the advantages of assessing the co-movement of the two series in the context of both time and frequencies. Using discrete and continuous wavelet methodologies for study of the inflation – output gap nexus in the case of France, we determine that the output gap is able to predict the inflation dynamics in the short- and medium-runs, and these results have important implications to the Phillips curve theory. More precisely, we discovered that in a discrete wavelet framework, the short- and medium-term fluctuations of both variables are more closely correlated, whereas the continuous wavelet analysis states that the output gap leads inflation in short-run.

Keywords: Phillips curve; Output gap – inflation co-movement; Wavelet framework; France.

JEL code: C22, E31, E32, E52, E60.

1. Introduction

Analysis of the relationship between inflation and the output gap (or economic activity) has been a constant topic in macroeconomics for half a century. Indeed, this relationship plays a fundamental role in the policy-making process, thereby justifying the major interest of academics and policy-makers in assessing the mechanisms of interdependence between these two macroeconomic variables. Despite the large body of literature on this topic, no unitary vision exists to describe the exact structure of this complex

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relationship. The explanations of this bilateral relationship have evolved over time according to different theoretical and methodological backgrounds, as well as the cyclical evolution of macroeconomic variables.

The first formal analysis of this relationship was developed in 1958 by Philips, who fit a statistical equation for the change in nominal wages and the unemployment rate in the UK and found a stable negative short-run relationship between these variables. To render this technique more useful to policymakers, the original version of the Phillips curve was transformed from a wage-change equation to a price-change equation (Claus, 2000), and the unemployment gap was adopted to reflect the view that economic fluctuations are the result of both demand and supply shocks. However, instead of the unemployment gap, the output gap is often included as a measure of excess demand in Phillips curve analysis (Claus, 2000).
From an empirical point of view, the original Philips curve, which provides a short-run trade-off between inflation and output, was invalidated in 1970 per the stagflation phenomenon that followed the oil crisis. From a theoretical point of view, this idea was first challenged by Phelps (1967) and Friedman (1968), who argued using adaptive expectations that the apparent trade-off between inflation and output would tend to be a temporary phenomenon.
The development of rational expectations theory has represented an important step in analysing the inflation-output gap relationship. Indeed, according to the Lucas critique, the agents are able to build correct expectations of future inflation, and consequently, there is no trade-off between inflation and output gap even in the short-run.

Structural models were subsequently developed, and the New Keynesian Phillips Curve (NKPC) became the main foundation for inflation – output gap analysis. These models are micro-founded and consider sticky prices and purely forward-looking inflation expectations. Variants of the NKPC depend on the choice of price-setting models and on the measure of real marginal costs (see Montoya and Döhring, 2011). For example, Roberts (1995) considered that the aggregate real marginal cost is proportional to the output gap measured using de-trending techniques.

However, despite the well-defined theoretical framework of the NKPC, empirical evidence has not supported the presumptions of the model with respect to the role of the output gap and expectations in explaining inflation dynamics (Abbas and Sgro, 2011). One shortcoming of the NKPC was associated with an immediate response of inflation to monetary policy shocks. By considering selected backward-looking behaviour, Galí and Gertler (1999) developed the so-called hybrid NKPC. Thus, hybrid models emerged that nest the traditional Phillips curve (backward-looking) and the NKPC (forward-looking)[1]. In addition, the real marginal cost is the theoretically appropriate measure of real sector inflationary pressures as opposed to the cyclical measures used in traditional Phillips curve analysis, such as de-trended output or unemployment (Galí et al., 2001).

Nevertheless, the role of the output gap in explaining inflation dynamics cannot be neglected in the NKPC (Paul, 2009; Zhang and Murasawa, 2011; Montoya and Döhring, 2011). Consequently, a series of studies focused on the changes in the output gap rather than the level of the output gap, or the so-called “speed limit effect”. A speed limit effect exists when the change in the output gap causes inflation to increase even if the level of the output gap is negative. When a negative output gap closes and the change in the output gap is positive as growth increases, upward pressure on inflation may occur.

Empirical evidence of speed limit effects is mixed. For example, Lown and Rich (1997) found a statistically significant rate of change effect for the output gap in the estimation of a price-inflation Phillips curve for the US over the period 1965-1996, whereas Dwyer et al. (2010) found rather limited evidence of the speed limit effect for the UK over the period 1980-2010.

Another development in this area considers the nonlinearities of the inflation – output gap nexus. The majority of analyses using Phillips curves are based on the assumption that the trade-off between inflation and activity is linear and symmetric. However, it is possible that the magnitude of effects from the level and the change of the output gap depend on the sign of the output gap such that the relationship is asymmetric.

The concept underlying the output gap’s role in explaining inflation dynamics is that, due to the presence of short-term price rigidities, demand shocks provoke a supply reaction that causes the actual and potential outputs to differ. However, these differences (i.e., an output gap different from zero) cannot last in the long term and will trigger a price adjustment process to restore equilibrium (Bolt and van Els, 2000). Several studies in this direction include those of Laxton et al. (1999), which imposed a mode convexity for estimating the Phillips curve, Céspedes et al. (2005), which analysed structural breaks in NKPC, and Baghli et al. (2007), which studied the asymmetries with respect to the output – inflation trade-off in

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[1] In the opinion of Russell and Banerjee (2008), the Friedman-Phelps Phillips curve and New Keynesian Phillips curve models are special cases of the hybrid Phillips curve. However, even with partial progress on this topic, the hybrid form of the Phillips curve was in its turn has been criticized. For example, Estrella and Fuhrer (2002) show that the correlations between inflation, future inflation and real marginal costs are not reflected by the data.

the Euro area. In the same line and more recently, Komlan (2013) tested for the presence of nonlinearities with respect to the output gap in the central bank policy reaction function.

The potential output is sometimes also defined as the output level in a situation of stable inflation, and thus, an output gap may also be viewed as a tension variable that leads inflation. Consequently, another body of literature has focused on output gap estimation (i.e., Gerlach and Peng, 2006). The continuously growing literature on this subject relates to the difficulty of deriving an output gap because potential output is an unobserved variable[2].

Nevertheless, regardless of the method of assessing the output gap, the empirical results do not always validate the theoretical background of the inflation – output gap relationship[3]. This effect is sometimes due to the empirical approaches, such the frequently used GMM estimator[4]. The GMM is a well-known approach used to study the Philips Curve, but at the same time, it is associated with small-sample problems and the choice of appropriate instruments (Dees et al., 2008; Tillmann, 2009). Moreover, the frequently used econometric approaches ignore the non-stationary characteristic of the variables[5].

To overcome the limitations of non-stationary data, Ashley and Verbrugge (2006), Assenmacher-Wescheand Gerlach (2008a; 2008b) and more recently, Haug and Dewald (2012) propose a frequency-domain analysis that combines the Phillips curve and the macroeconomic variable co-movement theories to test the relationship between money growth, output gap and inflation. The frequency-domain methods explore the high- and low-frequency components of the variables and interpret the results in terms of frequency bands rather than time horizons. Similarly, Assenmacher-Wesche and Gerlach (2007) used the band spectral estimator[6] for non-stationary data and proved a one-to-one relationship between

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[2] For a literature review of the different methodologies on output gap estimation see, e.g., Claus (2000), Tillmann (2009), Boug et al. (2010), Zhang and Murasawa (2011) and Bolt and Van Els (2000).

[3]For example, the ECB (2009) shows that movements in the economic slack have played a fairly modest role in the inflation process in the Euro area.

[4] For literature on the use ofGMM to examine the Philips Curve see, e.g., Jondeau and Le Bihan, (2005), Céspedes et al. (2005), Leith and Malley (2007), Mihailov et al. (2011) and Zhang and Murasawa (2011).

[5]Russell and Banerjee (2008) show that at least two reasons justify the question of inflation stationarity in an inflation-targeting framework. The first reason is the assumption that monetary authorities respond to a series of shocks by making discrete changes in their implicit inflation target. The second reason is the assumption that inflation shocks are quite frequent and that monetary authorities adjust the target rate of inflation at least partially in response to these shocks.

[6] Spectral analysis highlights the cyclical properties of the data (Tiwari, 2012; Shahbaz et al., 2012).

inflation and money growth at low frequencies and between inflation and output gap at high frequencies.

However, in losing the time-information, the frequency-domain methods present important limits. Although it allows quantification of co-movement at the frequency level, such a measure disregards the potential evolution of the co-movement over time (Rua, 2010). A new methodology used in economics known as wavelet analysis combines time and frequency domain analyses and is more appropriate for assessing the output gap role in inflation dynamics (see Rua, 2010 for an exhaustive description of the methodology). As shown by Mitra and al. (2011), an important aspect of wavelets is that they are localised in time and space and can be considered as a refinement of Fourier analysis.

Despite the growing literature using wavelets in economics and finance[7], none of the previous studies have explicitly focused on the output gap – inflation nexus. The aim of our paper is to examine the relationship between inflation and the output gap using wavelet analysis to simultaneously assess how variables are related at different frequencies and how such a relationship has evolved over time by capturing the non-stationary features. Moreover, our study uses both discrete and continuous wavelet methodologies to better understand the structure of the co-movement between these two macroeconomic variables. The use of discrete wavelet analysis allows us to avoid non-stationarity problems, whereas the continuous wavelet approach allows us to estimate the correlation degree of the variables.

Another important contribution of our research is related to the analysis of the French case. With the founding of the European Monetary Union (EMU), many studies have focused on analysing the relationship between inflation and the output gap at the aggregate Euro area level[8], leaving out the case of large European economies, such as France. The few studies that approach the French case are those of Jondeau and Pelgrin (2009) and Imbs et al. (2011), which developed a heterogeneity-correcting estimation technique and applied it to sector data to assess the hybrid NKPC. Along the same lines, Crédit Agricole (2009) discovered a positive role for output gap pressures in influencing the inflation in France.

Study of the French case can be interesting for at least two main reasons. First, France is a large developed country that has maintained a stable economy for more than half a

[7] See e.g.,In and Kim (2006), Naccache (2011), Gallegati et al. (2011), Aguiar-Conraria and Soares (2011a), Jammazi (2012), Benhmad (2012), Dajcman et al. (2012), Tiwari (2013), Tiwari et al. (2013a,b) and Trezzi (2013).

[8] Papers that offer insights on this topic are Bolt and van Els (2000), Baghli et al. (2007), Assenmacher-Wesche and Gerlach (2008a), ECB 2009), Tillmann (2009) and Montoya and Döhring (2011).

century and whose long-run statistical data are available and reliable. Thus, France can serve as a reliable benchmark for implementing new methodological tools to examine the structure of the inflation – output gap relationship as well as its evolution over time.

Second, France has experienced a particular evolution of its inflation phenomenon during the first stage of the actual global crisis. Indeed, for the first time since 1957, the annual inflation rate became negative in 2009. These recent and appealing dynamics of the inflation in France reinforce the interest in a close analysis of the inflation – output gap relationship, which could provide relevant explanations according to a distinction between the long- and short-run dynamics of those macroeconomic variables.

The remainder of the paper is structured as follows. Section two describes the discrete and the continuous wavelet methodology and the data. Section three is dedicated to the data description and to results analysis. The last section provides conclusions.

2. Methodology

As we have shown, the majority of the research on the output gap – inflation relationship has concentrated on the time – domain approach. Only a few analyses of the output gap – inflation nexus that were carried out in the frequency domain can be found in the literature. Therefore, this work represents an important contribution to the literature because the relationship between variables may exist at different frequencies: short, medium and long frequencies. We refer to Fourier analysis, which allows us to study the cyclical nature of a time series in the frequency domain.

In spite of its utility, the time information of a time series is lost under the Fourier transform, making it difficult to discriminate among ephemeral relationships or to identify structural changes that are rather essential for macroeconomic variables. As Rua (2010) showed, a caveat of the windowed Fourier transform is that the window width and thus the time resolution are constant for all frequencies. Another important argument against the application of Fourier transforms is the reliability of the results. According to the assumption of this approach, non-stationary data are not valid for implementation of this approach.

To overcome this obstacle, Gabor (1946) introduced the short-time Fourier transform (STFR) in which the time series are classified into smaller sub-samples and applied this transformation to each sub-sample. The STFR is time- and frequency-localized, but certain issues remain with the frequency-time resolution trade-off. The limitation of this approach is related to its efficiency with respect to the correct choice of the window and its constancy over time because it applies an equal frequency resolution across all dissimilar frequencies (Raihan et al., 2005). This transform does not allow for any time dependence of the signal. Therefore, this approach cannot provide any information on the time evolution of its spectral characteristics.

The wavelet approach was proposed in the 1980s by Grossmann and Morlet (1984) and Goupillaud et al. (1984) to address the limitations of the Fourier transform. This approach uses local base functions that can be stretched and translated with flexible resolution in both frequency and time (Rua, 2010)[9]. More precisely, the wavelet transform routinely allows adjustments in the high or low frequencies, with a short window for high frequencies and a long window for lower frequencies. In this context, time compression or dilatation is applied rather than a variation of frequency in the modulated signal. According to Aguiar-Conraria and Soares (2011b), the major advantage of the wavelet transform is its ability to perform local analysis of a time-series because the length of wavelets varies endogenously.

Mathematically, as Tiwari et al. (2013a) show, the wavelet transform includes two types of wavelet, namely, the father wavelets ϕ, which operate with the low frequency-flattened components of a signal (the trend components),and the mother wavelets ψ, which use the high-frequency details components (all of the deviation from the trend):

Father wavelets $\int \phi(t)dt = 1$, and

Mother wavelets $\int \psi(t)dt = 0$.

Under the wavelet transform, a time-series $f(t)$ can be decomposed as follows:

$$f(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + ... + \sum_k d_{1,k}\psi_{1,k}(t) \quad (1)$$

where J represents the number of multi-resolution levels, and k describes the ranges from 1 to the number of coefficients in each level. The coefficients $s_{J,k}$, $d_{J,k}$,..., $d^{1,k}$ are the wavelet transform coefficients and $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ illustrate the approximated wavelets functions. The wavelet transforms become:

$$s_{J,k} = \int \phi_{J,k}(t) f(t)dt \quad (2)$$

$$d_{j,k} = \int \psi_{j,k}(t) f(t)dt \text{, for j=1,2,.......J} \quad (3)$$

[9] Wavelets offer a better signal representation using multi-resolution analysis, with balanced resolution at any time and frequency compared with the STFR.

where J describes the maximum integer such that 2^J has a value less than the number of observations.

The coefficients $d_{J,k}$,….., $\mathrm{d}^{1,k}$ reveal an increasingly finer scale deviation from the smooth trend, and $s_{J,k}$ is the smooth coefficient that captures the trend. As a consequence, the initial $f(t)$ series under a wavelet approximation can be expressed as follows:

$$f(t) = S_{J,k}(t) + D_{J,k}(t) + D_{J-1,k}(t) + \ldots.. + D_1(t) \tag{4}$$

where $S_{J,k}$ indicates the smooth signal and $D_{J,k}$, $D_{J-1,k}$, $D_{J-2,k} \ldots D_{1,k}$ indicate the detailed signals.

These smooth and detailed signals can be written as follows:

$$S_{J,k} = \sum_k s_{J,k}\phi_{J,k}(t),\ D_{J,k} = \sum_k d_{J,k}\psi_{J,k}(t) \text{ and } D_{1,k} = \sum_k d_{1,k}\psi_{1,k}(t),\ j = 1,2,\ldots.,J-1 \tag{5}$$

The $S_{J,k}$, $D_{J,k}$, $D_{J-1,k}$, $D_{J-2,k} \ldots D_{1,k}$ are listed in increasing order of the finer scale components.

Wavelet transforms are broadly divided into three classes: discrete, continuous and fast wavelets. In economics and finance, wavelet analyses are mostly limited to the implementation of one or several variants of the discrete wavelet transform (DWT)due to the simplicity of the DWT and the advantages for data decomposition of many variables at the same time.

Most recently, tools associated with the continuous wavelet transform (CWT) have become more widely used. The CWT methodology was not popular in economics because it was based only on two-variable analysis. Currently, new advancements allow the use of more than two variables (known as conditioning variables) and make the CWT a rather attractive approach. The CWT is computationally complex and contains a high amount of redundant information (Gençay et al., 2002), which is also rather finely detailed.

We must state that the discrete variant of the wavelet transform is grounded on the same concepts as the CWT. However, a complementarity exists between the DWT and CWT, which recommends the use of the two approaches to obtain robust results. Each of the two methods presents specific advantages and drawbacks.

For example, the DWT is thought to be more parsimonious because it uses a limited number of translated and dilated versions of the mother wavelet to decompose a given signal (Gençay et al., 2002). Therefore, we obtain no redundant information from this method. Nevertheless, various constraints must be considered if applying discrete wavelet analysis, including the level of decomposition, the type of wavelet transform that must be used and how the bound conditions at the end of the series are to be handled.

In contrast, the CWT returns an array that is one dimension larger than the input data. With the CWT, the variation in the time series data can be obtained more easily. Based on a single diagram, one can immediately conclude the evolution of the variable variances at several time scales. However, because used a non-orthogonal set of wavelets is used, the data are highly correlated, and thus a larger redundancy is found. Moreover, the CWT is an implementation of a wavelet transform that uses arbitrary scales and nearly arbitrary wavelets. All of these observations recommend the use of both approaches due to their complementary natures. The DWT and CWT methodologies are described below.

2.1. The discrete wavelet approach

According to Daubechies (1992), the wavelet filter coefficients are $h_1 = (h_{1,0},...h_{1,L-1},0,...,0)^T$, which compactly supports the Daubechies wavelet unit scale and is zero-padded to length N.

By this definition, we consider $h_{1,j} = 0$ for $l > L$. In addition, the wavelet filter must satisfy three properties:

$$\sum_{i=0}^{L-1} h_{1,l} = 0; \quad \sum_{l=0}^{L-1} h_{1,l}^2 = 1; \quad \sum_{l=0}^{L-1} h_{1,l}h_{1,l+2n} = 0 \quad \text{for all non-zero integers } n \tag{6}$$

Based on these conditions, the wavelet filter must sum to zero (have a zero mean), must have unit energy and must be orthogonal to its even shifts.

Consider $g_1 = (g_{1,0},...g_{1,L-1},0,...,0)^T$ as the zero-padded scaling filter coefficients, which are defined via $g_{1,l} = (-1)^{l+1} h_{1,L-1-1}$, and let $x_0,...,x_{N-1}$ be a time-series. For scales with $N \geq L_j$, where $L_j = (2^j - 1)(L-1) + 1$, to obtain the wavelet coefficients, the time-series can be filtered using h_j:

$$W_{j,t} = 2^{j/2}\tilde{W}_{j,2^j(t+1)+1}, \qquad \left[(L-2)\left(1-\frac{1}{2^j}\right)\right] \leq t \leq \left[\frac{N}{2^j} - 1\right], \tag{7}$$

where $\tilde{W}_{j,t} = \dfrac{1}{2^{j/2}} \displaystyle\sum_{2^{j/2}}^{L_j-1} h_{j,l} x_{t-1}, \qquad t = L_j - 1,......N-1$

The $\tilde{W}_{j,t}$ associated coefficients with changes on a scale of length $\tau_j = 2^{j-1}$ are performed by sub-sampling every $2^j th$ of the $\tilde{W}_{j,t}$ coefficients.

Two main drawbacks characterise the orthogonal discrete wavelet transform: the dyadic length requirement (i.e., a sample size divisible by 2^j) and the wavelet and scaling coefficients, which are not shift-invariant as a result of their sensitivity to circular shifts (involving a decimation operation). To overcome these limitations, the maximal overlap DWT (MODWT) approach is proposed, which represents a type of compromise between the DWT and the CWT.

The MODWT, which is a non-orthogonal variant of the DWT, does not decimate the coefficients, and the number of scaling and wavelet coefficients at every level of transform is the same as the number of sample observations. Even if the MODWT loses orthogonality and efficiency in computation, this approach does not contain limitations for any sample size and is shift-invariant. The wavelet coefficients $\tilde{w}_{j,t}$ and scaling coefficients $\tilde{V}_{j,t}$ at levels $j; j = 1,..., J$, are:

$$\tilde{W}_{j,t} = \sum_{l=0}^{L-1} \tilde{g}_l \tilde{v}_{j-1,t-1 \bmod N} \quad and \quad \tilde{v}_{j,t} = \sum_{l=0}^{L-1} \tilde{h}_l \tilde{v}_{j-1,t-1 \bmod N} \tag{8}$$

The wavelet and scaling filters $\tilde{g}_l, \tilde{h}_l$ are rescaled as $\tilde{g}_j = g_j / 2^{j/2}$, $\tilde{h}_j = h_j / 2^{j/2}$. The differences between the generalised averages of the scale data $\tau = 2^{j-1}$ are non-decimated wavelet coefficients. Moreover, an extra benefit is also generated by the MODWT because it provides for all functions of the DWT (e.g., the MODWT can handle any sample size, is translation-invariant, and can provide an increase in resolution at coarser scales).In the wavelet correlation analysis, we also note that the MODWT offers a larger sample size and produces a more asymptotically efficient wavelet covariance estimator than the DWT.

2.2. The continuous wavelet approach

Application of the discrete wavelet analysis depends on the level of decomposition. Therefore, the findings of this approach can be difficult to understand and interpret. The continuous transform performs better than the discrete wavelet transform in this situation. Indeed, according to Aguiar-Conraria et al. (2008), the economic applications of the (discrete) wavelet transform have mainly involved use as low and high pass filters. Moreover, with the DWT, it is difficult to simultaneously analyse two (or more) time series variables. Consequently, the CWT represents a reliable analysis of the time frequency dependencies between two time series. In addition, Hudgins et al. (1993) and Torrence and Compo (1998) developed the cross wavelet power, the cross wavelet coherency and the phase difference methodologies[10]. Although the cross wavelet tool improves the interactions between two time series at different frequencies and better explains how they evolve over time[11], the wavelet coherency can be interpreted as a correlation coefficient in the time-frequency space. In addition, the term "phase" implies the position in the pseudo-cycle of the series as a function of frequency (Tiwari, 2013).

2.2.1. The continuous wavelet transform

In both the frequency and time scales, the wavelet is a function with a zero mean. We can characterise a wavelet by how localised it is in time (dt) and frequency ($d\omega$ or the bandwidth). The conventional approach of the Heisenberg uncertainty principle explains that a trade-off always exists between localisation in time and frequency.

We define a limit to the smallness of the uncertainty product $dt \cdot d\omega$. According to the specification of a particular wavelet, the Morlet wavelet is defined as:

$$\psi_0(\alpha) = \pi^{-1/4} e^{i\omega_C \alpha} e^{-\frac{1}{2}\alpha^2} \tag{9}$$

where ω_C a dimensionless frequency and α is a dimensionless time.

When using wavelets for feature extraction purposes, the Morlet wavelet (with ω_C=6) is a good choice because it provides a satisfactory balance between time and frequency localisation. We therefore restrict our further treatment to this wavelet. The idea behind the CWT is to apply the wavelet to the time series as a band pass filter. The wavelet is stretched in time by varying its scale (s) such that $\alpha = s.\ t$ and normalising it to unit energy. For the Morlet wavelet (with ω_C=6), the Fourier period (λ_{wt}) is nearly equal to the scale ($\lambda_{wt} = 1.03$

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[10] An extensive description of these techniques can be found in Grinsted et al. (2004) and Aguiar-Conraria and Soares (2011b). The methodology of continuous wavelet transform, cross wavelet transform and wavelet coherency that we present in this work is heavily drawn from Grinsted et al. (2004).

[11] In brief, the cross wavelet power of two time series illustrates the confined covariance between the time series.

s). The CWT of a time series $a_t, t = 1,.....,N-1, N$ with uniform time steps δt is defined as the convolution of x_n with the scaled and normalised wavelet. We write:

$$W_t^A(s) = \sqrt{\frac{\delta t}{s}} \sum_{t=1}^{N} x_t \psi_0 \left[(t-n)\frac{\delta t}{s} \right] \tag{10}$$

We define the wavelet power as $|W_t^A(s)|^2$.The complex argument of $W_t^A(s)$ could be interpreted as the local phase. The CWT contains edge artefacts because the wavelet is not completely localised in time. It is therefore useful to introduce a cone of influence (COI) in which the edge effects cannot be ignored. In this work, we take the COI as the area in which the wavelet power caused by a discontinuity at the edge has dropped to e^{-2} of the value at the edge. The statistical significance of the wavelet power can be assessed relative to the null hypotheses that the signal is generated by a stationary process with a given background power spectrum (P_k).

Although Torrence and Compo (1998) have shown that the statistical significance of wavelet power can be assessed against the null hypothesis that the data-generating process is given by an AR(0) or AR(1) stationary process with a certain background power spectrum (P_k), for more general processes, one must rely on Monte Carlo simulations. Torrence and Compo (1998) computed the white-noise and red-noise wavelet power spectrum from which they derived the corresponding distribution for the local wavelet power spectrum at each time *n* and scale *s*under the null as follows:

$$D\left(\frac{|W_t^A(s)|^2}{\sigma_A^2} < p \right) = \frac{1}{2} P_k \chi_v^2(p) \tag{11}$$

where v is equal to 1 for real and 2 for complex wavelets.

2.2.2. The cross wavelet transform

The cross wavelet transform (XWT) of two time series a_t and b_t is defined as $W^{AB} = W^A W^{B*}$, where W^A and W^B are the wavelet transforms of x and y, respectively, and * denotes complex conjugation. We further define the cross wavelet power as $|W^{AB}|$. The complex argument $\arg W^{ab}$ can be interpreted as the local relative phase between a_t and b_t in time-frequency space. The theoretical distribution of the cross wavelet power of two time series with background power spectra P_k^A and P_k^B is given in Torrence and Compo (1998) as:

$$D\left(\frac{\left|W_t^A(s)W_t^B*(s)\right|}{\sigma_A\sigma_B}<p\right)=\frac{z_v(p)}{v}\sqrt{P_k^AP_k^B} \qquad (12)$$

where $Z_v(p)$ is the confidence level associated with the probability p for a pdf defined by the square root of the product of two χ^2 distributions.

Although the wavelet power spectrum depicts the variance of a time series with occurrences of large variance indicating large power, the cross wavelet power of two time series depicts the covariance between these time series at each scale or frequency.

2.2.3. The wavelet coherence

According to the Fourier spectral approaches, the wavelet coherency (WTC) can be defined as the ratio of the cross-spectrum to the product of the spectrum of each series and can be treated as the local correlation both in time and frequency between two time series. At the same time, the wavelet coherency can be defined as the ratio of the cross spectrum to the product of the spectrum of each series (Aguiar-Conraria et al., 2008).

Following Torrence and Webster (1999), we define the WTC of two time series as[12]:

$$R_t^2(s)=\frac{\left|S\left(s^{-1}W_t^{AB}(s)\right)\right|^2}{S\left|\left(s^{-1}\left|W_t^A(s)\right|^2\right)\right|.S\left|\left(s^{-1}\left|W_t^B(s)\right|^2\right)\right|} \qquad (13)$$

where S is a smoothing operator.

Based on the work of Aguiar-Conraria and Soares (2011c), we focus on the wavelet coherency instead of the wavelet cross spectrum because the wavelet coherency presents the advantage of normalisation by the power spectrum of the two time series.

2.2.4. The cross wavelet phase angle

Because we are interested in the phase difference between the components of the two time series, we need to estimate the mean and confidence interval of the phase difference. We use the circular mean of the phase over regions with greater than 5% statistical significance

[12] Notice that this definition closely resembles that of a traditional correlation coefficient, and it is useful to think of the wavelet coherence as a localized correlation coefficient in time-frequency space.

(i.e. outside the COI) to quantify the phase relationship. This method is useful and general for calculating the mean phase. The circular mean of a set of angles $(a_t, t = 1,....,n)$ is defined as:

$$a_m = \arg(A,B),\ \ with\ A = \sum_{t=1}^{n} \cos(a_t)\ \ and\ \ B = \sum_{t=1}^{n} \sin(a_t) \qquad (14)$$

It is difficult to calculate the confidence interval of the mean angle reliably because the phase angles are not independent. The number of angles used in the calculation can be set arbitrarily high simply by increasing the scale resolution. However, it is of interest to obtain the scatter of angles around the mean. For this purpose, we define the circular standard deviation as:

$$s = \sqrt{-2\ln(R/t)} \qquad (15)$$

where $R = \sqrt{A^2 + B^2}$.

The circular standard deviation is analogous to the linear standard deviation and varies from zero to infinity. A similar result is found if the angles are distributed closely around the mean angle. In certain cases, there might be reasons for calculating the mean phase angle for each scale, and in those cases, the phase angle can be quantified as a number of years.

Monte Carlo simulation methods are used to obtain the statistical significance level of the wavelet coherence. We generate a large ensemble of surrogate data set pairs with the same AR(1) coefficients as the input datasets. We compute the wavelet coherence for each pair and estimate the significance level for each scale using only the values outside the COI.

To conclude, all of these CWT developments supply important information on the common movement of the series. As Grinsted et al. (2004) have shown, the XWT will expose the common power of the series and the relative phase in time-frequency space (we look for the common features of the two series). However, the WTC methods allow us to estimate the presence of a simple cause-effect relationship between the phenomena recorded in the time series. Finally, the COI tests for phase differences between the components of the two time series (i.e., the series are in an anti-phase position or not).

3. Data and empirical results

3.1. Data

For the inflation and output gap, we use monthly data for the period 1957M2-2011M12. The data are extracted from the IFS (International Financial Statistics) CD-ROM of the International Monetary Fund (IMF) (2012).

The inflation is defined as the monthly growth rate (compared with the previous month) of the consumer price index (CPI) expressed in a natural logarithm. We use the industrial production index (IIP) as a proxy for the output growth to estimate the output gap. The output gap is subsequently computed as the difference between the IIP trends obtained based on the HP filter[13]and the observed values of the IIP for France (see Fig. 1).

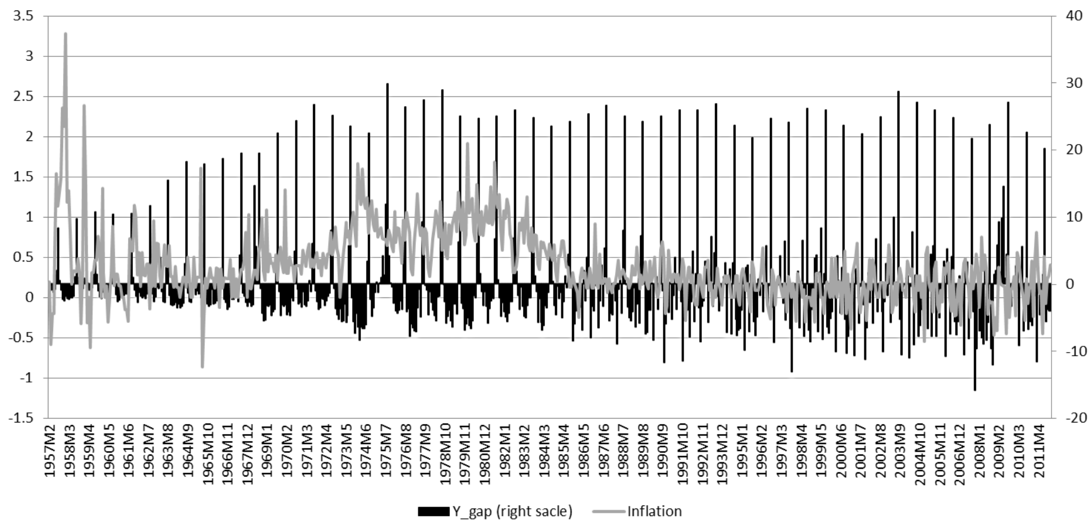


Fig. 1. The trend of the inflation and output gap

The use of IIP as a proxy for the output gap has become popular in economics. As Mitra et al. (2011) show, there is a strong justification for using IIP as a proxy for GDP: "*It follows from the fact that IIP series reflects the efficiency at which the level of technology, the abundance and quality of productive resources and labour force an economy is utilising,*

[13] The Hodrick-Prescott filter is a classic mathematical tool used to filter macroeconomic data series, especially in real business cycle theory. In practice, the stochastic trends (e.g., in the case of output) are often approximated by statistical methods such as the HP filter (see Dees et al., 2008).

which, in turn, reflect the industrial performance of the economy". Consequently, from a global perspective, the IIP supplies additional information on the economy.

The descriptive statistics of the inflation and output gap (Y_gap) are presented in Table 1.

Table 1 Descriptive statistics of inflation and the output gap

	Inflation	Y_gap
Mean	0.388745	0.000581
Median	0.305319	-1.897357
Maximum	3.279010	29.81634
Minimum	-0.860305	-15.93978
Std. Dev.	0.430929	7.999102
Skewness	1.262271	2.014159
Kurtosis	7.404194	7.042514
Jarque-Bera	707.6077	894.2978
Probability	0.000000	0.000000
Observations	659	659

The measure of skewness indicates that both inflation and the output gap are positively skewed. Similarly, both series demonstrate excess kurtosis, i.e., both series are leptokurtic. This type of distribution is quite often in financial and economic variables. The Jarque–Bera normality test rejects the null hypothesis of normality of the series. The data reported in Table 1 show a positive mean for inflation and a nearly zero mean for the output gap. At the same time, the median is positive for inflation and negative for the output gap, which also has a high volatility.

3.2. Results

3.2.1. Results obtained based on the discrete wavelet approach

We decomposed the two series based on the methodology described in the previous section and using s8 filters[14]. Next, we performed an analysis of the relative importance of the short-, medium- and long-term dynamics. For this purpose, we used the energy of the wavelet decomposition[15] of both variables, i.e., the energy of each scale (or frequency), to measure the relative importance of the short-, medium- and long-runs.

An analogy exists between the energy and the variance of each detail level described as the percentage of the overall energy. Hence, the percentage of the variance explained by each

[14] See Appendix 1 for the MODWT decomposition of inflation and the output gap.

[15] The energy represents the percentage of the total variance explained by the different scales.

scale is measured. Percival and Walden (2000) argued the fact that the DWT has the ability to decompose the energy in a time series across scales, and Percival and Mofjeld (1997) proved that the MODWT is also an energy-preserving transform (i.e., the variance of the time-series is preserved in the variance of the coefficients from the MODWT). Consequently, a time-series x(t) with wavelet coefficients for scale j, $\tilde{w}_{j,t}$ and scaling coefficients $\tilde{V}_{j,t}$, from a MODWT has the following energy decomposition:

$$\sum_{t=1}^{N} x^2(t) = \sum_{j=1}^{J}\sum_{t=1}^{N} \tilde{w}_{j,t}^2 + \sum_{t=1}^{N} \tilde{V}_{j,t}^2 \qquad (16)$$

Where N is the number of observations used in the calculation.[16] This method allows separation of the contributions of energy in the time series due to changes at a given scale.

Table 2 presents the energy of each scale (as a percentage of the overall energy) for the two variables under consideration, namely, inflation and output gap. Toobtain an unbiased estimator, the coefficients affected by the boundaries were not included in Table 2. Notice that only six scales were used (the seventh scale is included in the smoothness)[17]. The Daubechies least asymmetric wavelet filter (LA) was used for Table 2 because it is less affected by the boundaries.

Table 2 Energy decomposition for inflation and the output gap

Wavelet scales	**Inflation**	**Y_gap**
D1 (2-4 Month Cycles)	10.18%	42.96%
D2 (4-8 Month Cycles)	6.86%	30.56%
D3 (8-16 Month Cycles)	5.95%	21.36%
D4 (16-32 Month Cycles)	4.62%	2.79%
D5 (32-64 Month Cycles)	3.10%	1.93%
D6 (64-128 Month Cycles)	3.21%	0.35%
S6 (Above 128 Month Cycles)	66.04%	0.01%

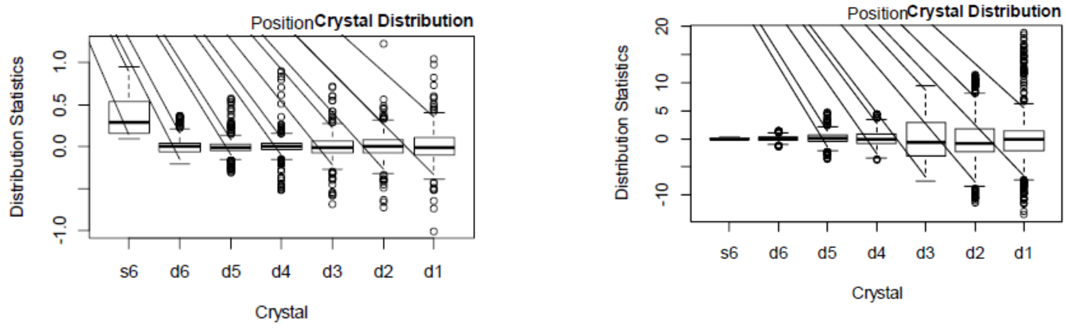
The wavelet scales are represented on the first column of Table 2. The second and third columns respectively present the energy distribution of the inflation and output gap corresponding to the wavelet scales. We discuss energy distribution in four major periods, namely, the short-run (D1+D2), medium-run (D3+D4), long-run (D5+D6) and very long-run (s6). For the output gap, if the short-run dominates all other periods/frequencies and explains

[16] N is not always equal to T because an unbiased estimator of the energy is computed with the coefficients unaffected by the boundary. In this case, N depends on the basis and the number of scales used.

[17] This is applied out to disregard as few of the boundary observations as possible to avoid losing too much information.

most of the variance (73.52%), in the case of inflation, the very long run explains 66.04% of the variance.

Fig.2 presents a box plot for each of the series to illustrate the crystal (scale) energy distribution as presented in Table 2.



Inflation Y_gap

Fig.2. Crystal energy distribution for inflation and the output gap

In the following, we present the analysis of the association between these two series using wavelet covariance and correlation (Fig. 3). The MODWT base wavelet covariance of the inflation and output gap analysis is based on a separation of the effects across timescales and frequency bands. It shows how the two series are associated with one another. Our results state that, the wavelet covariance slowly fluctuates in the analysed period with a flattening tendency for the long-run interval. It is also evident that covariance is negative for all decomposition levels, indicating a trade-off situation.

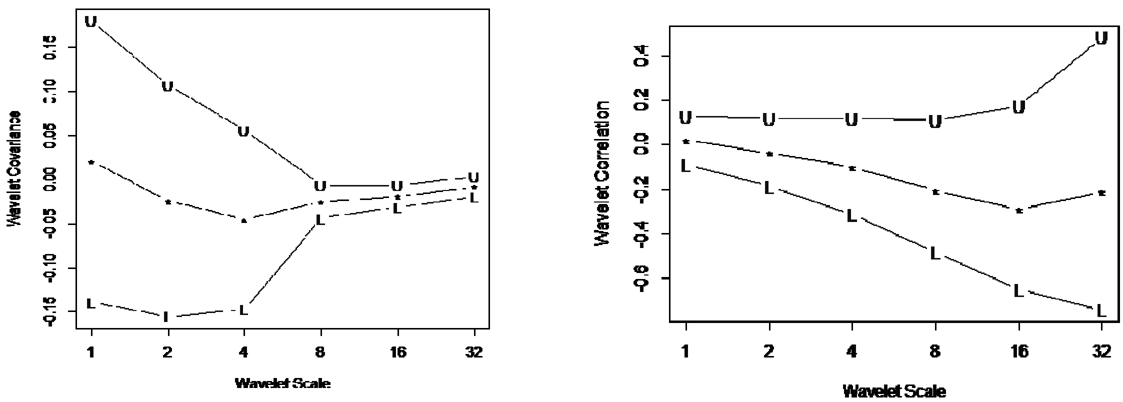


Fig.3.Wavelet covariance and correlation between inflation and the output gap

As shown, there is an increasing association between inflation and the output gap. However, it is difficult to compare the wavelet scales due to the different variability they exhibit. In this case, it is recommended to divide each series by its variance to standardise the covariance, thereby overcoming this influence and making it possible to compare the magnitude of the association across scales. Therefore, in the same Fig. 3, we report the wavelet correlation to examine the magnitude of the association of each series. The figure shows no differences among the short-,medium- and long-runs. In all cases, we observe a negative correlation between inflation and the output gap. However, there is a general tendency in the correlation coefficients to move downwards with the scale, except for the very long-run.

Fig. 4 shows that an approximate linear relationship exists between the wavelet variance and the wavelet scale. The variances of both the inflation and output gap decrease as the wavelet scale increases, and this decline is relatively steeper for the output gap vis-à-vis inflation. More specifically, a wavelet variance in a particular time scale indicates the contribution to the sample variance, and this contribution decreases in the long-run, especially in the case of the output gap.

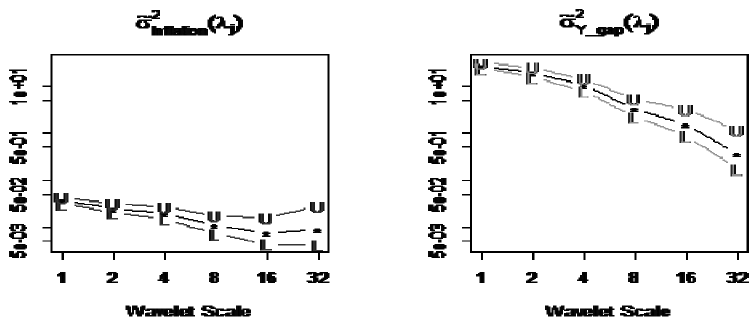


Fig.4. Wavelet variance

Furthermore, we use the wavelet cross-correlation to test the causal relationship between inflation and the output gap in France. We are particularly interested in the one-way causality between the output gap and inflation. The fact that the empirical results show a bi-directional causality is not surprising. As we have shown, certain methodologies related to the output gap computation consider the inflation as a potential determining factor. Fig. 5 illustrates the wavelet cross-correlation between inflation at time t and the output gap at time (t–k) at six levels of decomposition.

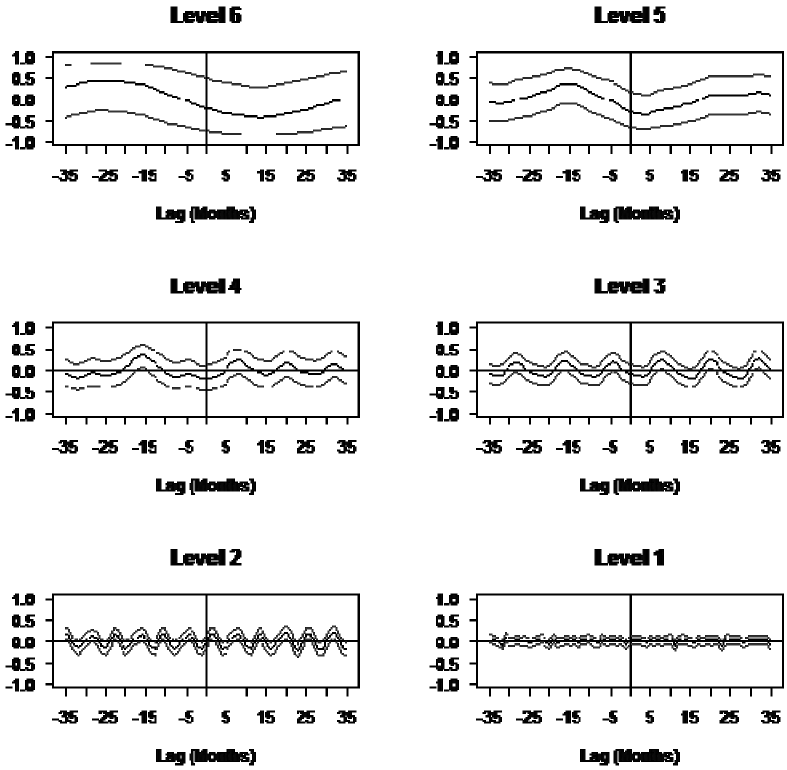


Fig. 5. Wavelet cross-correlation between inflation and output gap
Note: The first variable is the inflation and the second is the output gap

As observed, the short- and medium-term fluctuations of both variables are more closely correlated than those over the long-term with a 35-month lead and lag, and therefore, the magnitude of the cross-correlation becomes smaller by increasing the frequency band (no correlation can be observed for the very-long-run interval). At the sixth level of decomposition, we find that for a 10 to 35 month lag, the cross-correlation is positive, and for a 0 to 35 month lead, the cross-correlation is negative. To restate, we find that at a lag and lead of 35 months, a bidirectional causal relationship exists between the output and inflation at the sixth level of decomposition.

3.2.2. Results obtained based on the continuous wavelet approach

The CWT allows a better interpretation of the results for the evolution of the variables' variances at different time scales[18]. In the case of CWT, the level of decomposition and the type of wavelet transform do not represent a challenge, thus simplifying the identification of common features in the variable characteristics.

[18] We define the short scale as up to 1 year, the medium scaleas 1 to 8 years and the long-run scale as 8 years and beyond.

In Fig. 6, we describe the wavelet power spectrum of inflation and the output gap. The co-movement is presented in a contour plot with three dimensions: time, frequency and colour code. Therefore, to assess whether the series move together and if the strength of the co-movement changes across frequencies and over time, we look to the contour plot. The thick black contour represents the 5% significance level against the red noise. The colour code for power ranges from black (low power) to white (high power).

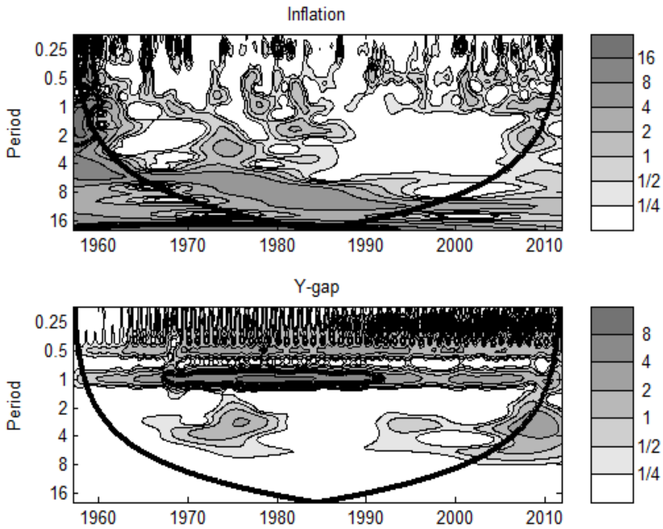
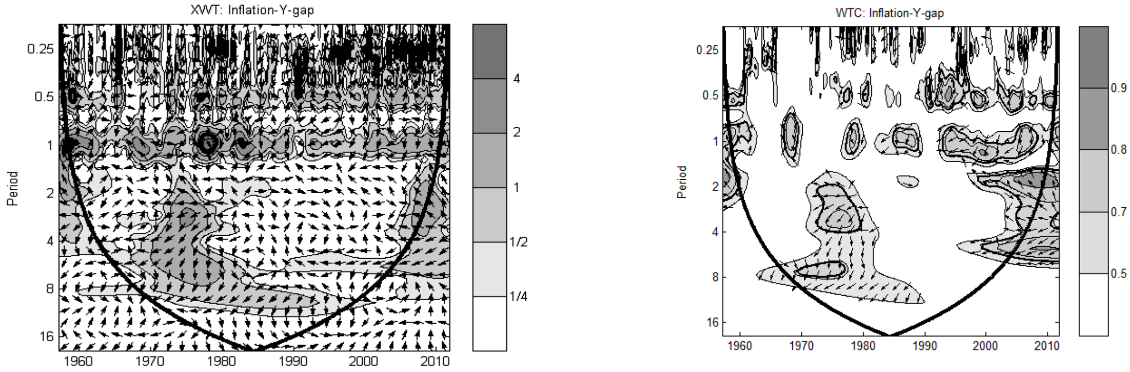


Fig. 6.Continuous wavelet power spectra of inflation and the output gap

Fig. 6 clearly shows the common significant features (at a 5% significance level) in the wavelet power of the two time series, such as the 0.25 to 1year scale that corresponds to the 1993s. Both series also show high power in the 0.5 to 1.25 year scale that corresponds to the period 1996-1997, and the 0.25 to 0.5 year scale that corresponds to the post-2000 period. However, the similarities between the portrayed patterns in this period are low, and it is therefore difficult to determine if this is merely a coincidence. The cross wavelet transform provides clarification in this case (see Fig. 7).



Note: The thick black contour designates the 5% significance level estimated from the Monte Carlo simulations using the phase randomised surrogate series. The cone of influence, which indicates the region affected by edge effects, is also shown with a light black line. The phase differences between the two series are indicated by arrows. Arrows pointing to the right mean that the variables are in phase,to the right and up mean that the Y-gap is leading, andto the right and down mean thattheY-gap is lagging. Arrows pointing to the left mean that the variables are out of phase,to the left and up mean that the Y-gap is lagging and to the left and down mean thatthe Y-gap is leading. Thein-phase condition indicates that variables will have a cyclical effect on each other, and the out-of-phase or anti-phase condition shows that the variables will have an anti-cyclical effect on each other.The Y-axis measures the frequencies and the X-axis represents the time period studied.

Fig. 7.Crosswavelet power spectrum and coherency of the inflation and output gap

On the left side of Fig.7, we present the results obtained using the XWT. It is very clear from the XWT results that the cross wavelet power spectrum increased after 1990 for the 0.25 to 0.5 year scale. We note one significant area in the one-year scale of 1980. However, although periods and frequencies with significant relationships exist, it is not clear which variables among them are leading or lagging. Overall, taking into account the arrow directions and the high-power regions, we consider that a link exists between inflation and the output gap series, as implied by the cross wavelet power.

Furthermore, it is worth mentioning that the wavelet cross spectrum (i.e., cross wavelet) describes the common power of two processes without normalisation to a single wavelet power spectrum. This method can produce misleading results because one essentially multiplies the continuous wavelet transform of two time series. For example, if one of the spectra is local and the other exhibits strong peaks, those peaks in the cross spectrum can be produced even if they have no association with any relationship between the two series. This observation leads us to conclude that the wavelet cross spectrum is not suitable for testing the significance of relationships between these two time series. Therefore, our conclusion relies on the wavelet coherency (because it is able to detect a significant relationship between two time series; for details, see Section 2.2). However, we can still use the wavelet cross-spectrum to estimate the phase spectrum. The wavelet coherency is used to identify both the frequency bands and the time intervals within which the pairs of indices show co-variance. Finally, we present the results of the cross wavelet coherency on the right side of Fig. 7.

The results from the WTC show that during 1980-1990, the arrows point mostly left-down for up to 0.25 years of scale indicating an anti-phase relationship with the Y-gap leading, whereas during 1995-2012, the arrows point left-up for up to 0.25 years of cycle indicating that the variables are out of phase but Y-gap is lagging. These results mean that for the first period, the output gap causes the inflation, whereas for the second period (1995-2012), the inflation predicts the output gap.

During 1990-2012, for 0.25 to 0.5 years of cycle, the direction of the arrows is not clear, whereas for 0.75 to 1.75 years of cycles during 1982-2008, the arrows point left-down, indicating an anti-phase or anti-cyclical relationship and Y-gap leads. Similar results are obtained in the 1980s for 2 to 8 years of scales, but the opposite case is observed in which the arrows point left-up. The arrows point left-up during 1998-2007 for 2 to 5 years of scale, which indicates an anti-phase relationship between inflation and the Y-gap and lagging by the Y-gap. Consequently, in most of the cases, we observe an anti-phase relationship between the Y-gap and inflation. Thus, in the short-run, the output gap causes inflation, but for the medium-run scale, a bidirectional influence can be observed.

To resume our findings, we conclude that the output gap has an impact on inflation in France, particularly in the short-run. However, it is important to note how the research presented above ties in with the existing empirical findings in the literature. We observe that our results are in agreement with those of Assenmacher-Wesche and Gerlach (2008a), who found that output gap influences inflation only at the high frequency bands (i.e., in the short-run[19]) in a frequency-domain analysis for the Euro area. With respect to the French case, the reported results are in line with those presented by Crédit Agricole (2009) and state that the output gap represents a significant determinant of inflation.

These results have important policy implications for the role of the output gap in explaining inflation. Because France (in addition to Germany) is one of the largest EU economies, the ECB must focus on the output gap in these countries to find an explanation for the inflation dynamics in the Euro area. At the same time, due to the production influence on the price level in France, several conclusions can be drawn for the private sector with respect to the inflationary and/or deflationary signals.

After the crisis, the growth potential appears softer, with a reduced impact on inflation. However, during past years, the output gap acted as a good predictor for the

[19]Montoya and Döhring (2011) found that the output gap has a small impact on inflation in the Euro area.

inflation dynamics in France. Using wavelet analysis, we have shown both the co-movement (wavelet coherency results) and the causality between variables (phase relationship results).

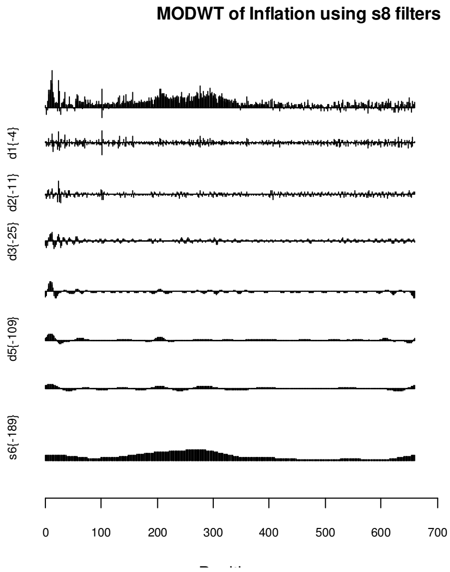
4. Conclusions

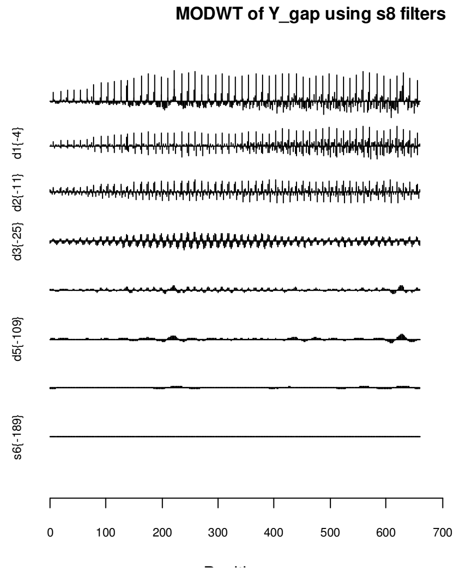
This paper aims to examine the utility of the output gap for explaining inflation dynamics in France using a new approach, namely, the wavelet transform. For a long time, this relationship has served as the workhorse of studies with the Phillips curve, but the empirical results often failed to validate the theoretical assumptions. Although most of the empirical work has been devoted to estimating the output gap and constructing the NKPC in a GMM framework, few researchers have paid attention to the problems related to the non-stationarity of the statistical series. A possible solution advanced in the literature is frequency domain analysis, which ignores the time features of the data. Nevertheless, a more appropriate and better-suited approach would combine the time and frequency domain analyses.

Thus, our study enriches the empirical literature on the Phillips curve in several ways. First, the paper tests the role of the output gap in explaining the inflation dynamics in France, which was a case study that was neglected after the construction of the Euro area. This work extends the work of Crédit Agricole (2009) for French data in the time series by applying the wavelet approach. Second, the paper uses both discrete and continuous wavelets in complementary methodologies and demonstrates that the output gap represents a good predictor of the inflation in the short-run and in the medium-run. In summary, our results suggest that the output gap must be considered as a necessary element for inclusion in the NKPC analysis, and the results support the conclusion that the output gap has important implications for the ECB's monetary policy.

Moreover, our analysis also highlights selected areas for further research. Several extensions of the analysis presented above appear to be warranted. In particular, it would be desirable to perform the same analysis at the Euro area level. However, an intermediary step will be the analysis of the German case, which could prove the robustness of our findings. If the output gap represents a determinant of the inflation in this case as well, it will provide strong evidence for the manner in which the ECB must consider the output gap in its policy decisions.

Appendix 1.MODWT decomposition of inflation and the output gap





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