# Macroeconomic Effects of Bank Recapitalizations<sup>\*</sup> Work-in-Progress

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#### Abstract

We build a dynamic stochastic general equilibrium model where investments by entrepreneurs and banks can be leveraged by external funding but are subject to a dual moral hazard problem. In our model banks' monitoring investments have a variable scale and real opportunity cost. As a result, the monitoring investments vary over the business cycle which implies that not only the aggregate amount bank capital and entrepreneurial wealth but also their composition matters in the propagation of shocks. We show that in equilibrium bank capital is scarce and that it greatly amplifies the investment shocks but dampens many other type of shocks. We also study capital injections from the government to banks. We show that capital injections can be useful as a shock cushion, but they may be counter-productive if the aim is to avoid deleveraging and to boost investments.

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# 1 Introduction

Governments' capital injections to the banking system have been an important tool in attempts to support credit flows during financial crises. In the crisis episodes that took place over the period 1970 to 2007, government recapitalization of banks averaged around eight percent of GDP (Laeven and Valencia, 2008). These resolution measures were present in 33 crisis episodes out of 42. Already in the early stages of the ongoing crisis (up until 2009), government capital injections had exceeded five per cent of GDP in the US and the UK, and 2.5 % in the euro area.<sup>1</sup> Possible new capital injections are a topical policy issue in Europe today.

In this paper we analyse capital injections from the government to the banking sector in a dynamic stochastic general equilibrium (DSGE) model with financial frictions. In our model framework, both banks' balance sheets and the balance sheets of non-financial firms play a role in macro-financial linkages, but in equilibrium bank capital tends to be scarce, compared to firm capital: a given change in bank capital has a larger impact on the macroeconomy than a corresponding change in firm capital. Hence, it is rather natural for the government to target the banks, rather than the nonfinancial sector.

Our framework builds on the Holmström and Tirole (1997) model of financial intermediation.<sup>2</sup> In the DSGE models building on Holmström and Tirole (1997) (see Aikman and Paustian (2006), Faia (2010) and Meh and Moran (2010)<sup>3</sup>) entrepreneurs and banks can leverage their investments by using external funding but this leverage creates moral hazard problems. Hence sufficiently large banks' and entrepreneurs' own stakes in the projects are needed to maintain their incentives, which implies that the aggregate amount of informed capital (=the sum of bank capital and entrepreneurial wealth) in the economy plays a crucial role in the propagation of shocks. In this

<sup>3</sup>Early attempts to introduce a Holmström-Tirole type financial friction in macroeconomic models include Castrén and Takalo (2000) and Chen (2001).

<sup>&</sup>lt;sup>1</sup>See calculations in Alessandri and Haldane (2009). See also European Commission (2011).

<sup>&</sup>lt;sup>2</sup>While earlier models of macro-financial linkages (notable examples include Kiyotaki and Moore 1997, Carlstrom and Fuerst 1997, and Bernanke, Gertler and Gilchrist 1999) typically focused on the balance sheets of non-financial firms and treated financial intermediation as a veil, in recent years an increasing number of macro models with banks has been developed, notable examples include Gertler and Karadi (2010) and Gertler and Kiyotaki (2011). However, many of these new generation macro-banking models abstract from the balance sheets of non-financial firms. The Holmström - Tirole (1997) framework is attractive in the sense that it allows the simultaneous analysis of both banks' balance sheets and the balance sheets of non-financial firms.

framework, however, quantitative implications of bank capital cannot easily be disentangled from those of entrepreneurial wealth. These models also require a bank's asset portfolio to be completely correlated, and make assumptions that render them incomparable with the standard New Keynesian framework.

We extend the DSGE framework building on Holmström and Tirole (1997) to allow for the separate roles of bank capital and entrepreneurial wealth. There are several novel features in our model: First, like in the simultaneously written paper by Christensen, Meh and Moran (2011), we allow monitoring investments to be continuous: the more the banks invest in costly monitoring, the lower the entrepreneurs' private benefits from unproductive projects but the less the banks can lend. Second, we treat monitoring investments truly monetary and private benefits truly private in the sense that the former has opportunity costs but the latter does not have. These features imply that the banks monitoring investments vary over the business cycle and that not only the aggregate amount of informed capital but also its composition matters in the propagation of shocks. Third, we distinguish between bankers and banks. In our model, a bank is a balance sheet entity with a capital structure but only a banker faces an incentive problem. This is not only realistic but also allows us to relax the assumption of a completely correlated investment portfolio of a bank. The distinction between bankers and banks is also instrumental when we introduce an aggregate investment shock, which plays a key role in our model. Finally, we strive to benchmark our model to the standard New Keynesian framework which requires a number of subtle but important changes to the previous macro literature building on Holmström and Tirole (1997).

The key results of the modelling effort are the following: i) In equilibrium bank capital is scarce in the sense that the ratio of bank capital to entrepreneurial wealth is smaller than what would maximize the investments and output. Also, a given change of bank capital affects aggregate investments more than an equal proportional change of entrepreneurial wealth. ii) Bank capital is more vulnerable to aggregate investment shocks than entrepreneurial capital. iii) Given properties i) and ii), bank capital plays a more important role in the propagation of investment shocks, and in macroeconomic dynamics, than entrepreneurial capital.

Given the importance of bank capital in macro-financial linkages, our model forms an attractive framework for studying capital injections by the government. An *ex post* capital injection distorts bankers' monitoring incentives and the banks' involvement becomes more expensive for the entrepreneurs. This arises because the government-owned capital is more expensive than the households' deposits. In such a situation capital injections may accelerate deleveraging and lower aggregate investments. The result is reversed if the conditions of the government-owned capital are more favourable than those of deposits. Capital injections can be done *ex ante*, i.e. before the investment shock arrives. In such a case, they form a pre-emptive 'cushion' and the policy can be productive in mitigating develeraging and stabilizing the economy.

In the next section we describe the basic model. In Section 3 we explain why bank capital is scarce in equilibrium. In Section 4 we introduce an investment shock into the model, and discuss the distinction between bankers and banks. In Section 5 we explain how we calibrate the model and in Section 6 we study the impulse responses of financial and macro variables to a number of shocks. In Section 7 we analyze capital injections from the government to banks. Finally Section 8 concludes.

# 2 The model

We consider a discrete time economy with three classes of agents: households, entrepreneurs, and bankers. There are also three sectors of production: i) competitive firms producing final goods from intermediate goods; ii) monopolistically competitive firms producing the intermediate goods from labour supplied by households and capital supplied by entrepreneurs, and iii) entrepreneurs producing capital goods. Households own the firms producing the final and intermediate goods. The production of capital is subject to a dual moral hazard problem in the sense of Holmström and Tirole (1997): First, entrepreneurs, who may obtain external finance from households and banks, have temptation to choose less productive projects with higher private benefits. Second, bank monitoring mitigates the entrepreneurs' moral hazard temptations but since the banks use deposits from the households to finance the entrepreneurs, there is an incentive to shirk in costly monitoring.

### 2.1 Households

Following Gertler and Kiyotaki (2011) and Gertler and Karadi (2010) we assume that there is a representative household with a continuum of members of measure unity. Within the household there are three types of members: consumer-workers, entrepreneurs and bankers. This assumption, as becomes clear in the next subsection, allows us to introduce a Holmström-Tirole type financial intermediation while maintaining a direct connection to the standard New Keynesian framework. The head of a household decides on behalf of its members how much the household works, consumes, and invests in capital and bonds. The problem of the representative household is thus

$$\max_{\{C_t \ge 0, L_t \ge 0\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1 - \sigma} C_t^{1 - \sigma} - \frac{\xi}{1 + \phi} L_t^{1 + \phi} \right],\tag{1}$$

subject to a budget constraint:

$$B_t + P_t(C_t + q_t I_t + T_t) = W_t L_t + (1 + r_{t-1})B_{t-1} + P_t r_t^K K_{t-1}.$$
 (2)

In (1) and (2)  $\xi > 0$ ,  $\phi > 0$  and  $\sigma \in (0,1)$  are parameters of the household's utility function,  $\beta \in (0,1)$  is the rate of time preference,  $C_t$  denotes the household's consumption in period t,  $L_t$  hours worked, and  $I_t^h$  investments with  $q_t$  being its price.  $B_t$  is the end of period one period nominal bond and  $1 + r_{t-1}$  is the gross nominal interest rate on bond holdings,  $P_t$ price level of consumption basket,  $T_t$  lump-sum transfers (including the dividends from monopolistically competitive firms owned by the household, and accumulated assets from exiting entrepreneurs and bankers (see the next subsection)),  $W_t$  nominal wage,  $r_t^K$  the real rental rate of capital, and, finally,  $K_t$  is the capital stock that accumulates according to the law of motion

$$K_{t+1} = (1-\delta)K_t + p_H R I_t, \tag{3}$$

where  $\delta$  is the rate of depreciation, and  $p_H$  and R are parameters, which are related to the production of capital goods; these parameters are defined more precisely in Section 2.5. Note that we assume, as in Carlstrom and Fuerst (1997) but unlike, e.g., Faia (2010) and Meh and Moran (2010), that bank deposits are intra-period deposits that give zero return. They can, consequently, be excluded from intertemporal budget constraint (2). While being somewhat controversial the assumption facilitates comparison of our model with the standard New Keynesian framework. We later elaborate the implications of this assumption.

Solving the household's optimization problem yields the following familiar first order conditions for  $B_t$ ,  $L_t$  and  $K_t$ , respectively:

$$1 = \beta E_t \left\{ (1+r_t) \left( \frac{C_t^{\sigma}}{C_{t+1}^{\sigma}} \frac{P_t}{P_{t+1}} \right) \right\},\tag{4}$$

$$C_t^{\sigma} \xi L_t^{\phi} = \frac{W_t}{P_t},\tag{5}$$

$$q_{t} = \beta E_{t} \left\{ \frac{C_{t}^{\sigma}}{C_{t+1}^{\sigma}} \left[ r_{t+1}^{K} + q_{t+1}(1-\delta) \right] \right\}.$$
 (6)

### 2.2 Entrepreneurs and bankers

In every period, household members split into three categories: a part of them become entrepreneurs, another part become bankers, and the rest remain consumer-workers. As will be explained in detail below, each entrepreneur has access to different investment projects with stochastic returns. Each banker manages a financial intermediary (a bank) that obtain deposits from households and finance entrepreneurs. In general, entrepreneurs and banks earn higher return to their risky investments than households earn to their safe deposits. Hence it is optimal for the entrepreneurs and bankers to keep building their assets until exiting their industries (cf. Gertler and Karadi (2010), and Gertler and Kiyotaki (2011).. In each period, exit occurs with probabilities  $1 - \lambda^e$  and  $1 - \lambda^b$  where  $\lambda^e \in (0, 1)$  and  $\lambda^b \in (0, 1)$  denote the entrepreneur's and banker's survival probabilities. In a steady state, the number of household members becoming entrepreneurs and bankers equal the number of exititing entrepreneurs and bankers. The exiting entrepreneurs and bankers give their accumulated assets to the household which in turn provide new entrepreneurs and bankers with some initial investment capital.

Within the household there is a perfect consumption insurance against the risks entrepreneurs and bankers take. Hence entrepreneurs and bankers consume in each period like ordinary consumer-workers, i.e., all household members consume an equal amount in each period.

### 2.3 Final good production

Competitive firms produce the final good by assembling a continuum of intermediate goods, indexed by  $i \in [0,1]$ , by using the standard constantelasticity-of-substitution (Dixit-Stiglitz) aggregator

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}},$$

where  $\epsilon > 0$  is the elasticity of substitution and  $Y_t(i)$  denotes the use of intermediate good *i* in period *t*. The final good producers choose the level of  $Y_t(i) \forall i$  and  $\forall t$  to maximize their profits subject to a zero-profit condition. Solving this maximization problem yields

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\epsilon} Y_t,$$

where  $P_t(i)$  is the price of intermediate good *i* and

$$P_t = \left[\int_0^1 P(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$
(7)

is the aggregate price index.

#### 2.3.1 Intermediate good production

The firms in the intermediate good sector combine capital  $K_t(i)$  and labour  $L_t(i)$  using the Cobb-Douglas production function

$$Y_t(i) = K_t(i)^{\alpha} \left( Z_t L_t(i) \right)^{1-\alpha},$$

where the common labour-augmenting technology is given by  $Z_t$ . Cost minimization results in the following real marginal costs

$$MC_t = \left(\frac{r_t^K}{\alpha}\right)^{\alpha} \left(\frac{W_t/P_t}{Z_t(1-\alpha)}\right)^{1-\alpha}$$
(8)

and conditional factor demands for labour

$$(1-\alpha)\frac{Y_t(i)}{L_t(i)} = \frac{W_t}{P_t} \tag{9}$$

and for capital

$$\alpha \frac{Y_t\left(i\right)}{K_{t-1}} = r_t^K.$$
(10)

Following Calvo (1983) each intermediate good firm may revise price of its product only with the probability  $1 - \theta$ ,  $\theta \in [0, 1]$ , in any given period. Assuming that the probability is independent of the length of the time and the time elapsed since the last adjustment, the fraction  $1 - \theta$  of firms may change their price in each period while the rest, fraction  $\theta$ , of the firms keep their price unchanged. Let  $P_t^*$  denote the price level of the firm that receives price change signal. When making its pricing decision, the firm takes into account that it cannot change the price it chooses now with probability  $\theta$  over each future period. That is, when a firm can change its price, its problem is

$$\max_{P_t^{\star} \ge 0} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left\{ P_t^{\star} Y_{t+k}(P_t^{\star}) - \Psi_{t+k} \left[ Y_{t+k}(P_t^{\star}) \right] \right\},\$$

where  $\Psi_{t+k}[Y_{t+k}(P_t^*)]$  is the firm's total costs that depends on the demand function,  $Y_{t+k}(P_t^*) = (P_t^*/P_{t+k})^{-\epsilon}Y_{t+k}$ , which is due to the households' consumption index, and  $Q_{t,t+k} \equiv \beta^k (C_{t+k}^h/C_t^h)^{-\sigma} (P_t/P_{t+k})$  is the nominal stochastic discount factor that the household uses to price any financial asset. Note that our standard household's optimization problem yields the very same pricing kernel for the one-period bond. The first order condition for the firm's problem is given by

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^{\star}) \left( P_t^{\star} - \mathcal{M}\psi_{t,t+k} \right) = 0, \qquad (11)$$

where  $\psi_{t,t+k} = \Psi'_{t+k} (Y_{t+k}(P_t^*))$  is nominal marginal costs and  $\mathcal{M} \equiv \epsilon / (\epsilon - 1)$  is the frictionless mark-up. Equation (11) gives the optimal price  $P_t^*$  for all the firms that may set their price level. As a result the aggregate price index (7) may be re-expressed as

$$P_t = \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^{\star})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}.$$

#### 2.4 Monetary policy

We assume that monetary policy follows a simple Taylor rule

$$1 + r_t = \frac{\Pi_t^{\phi_\pi}}{\beta}$$

where  $\Pi_t \equiv P_t/P_{t-1}$  is the (gross) inflation rate and  $\phi_{\pi} > 1$ .

### 2.5 Production of Capital

Capital demanded by the firms in the intermediate good sector is produced by entrepreneurs who have access to different investment projects with variable scale and stochastic returns. While the entrepreneurs generally have some initial wealth of their own, they can attempt to leverage their investments by borrowing from banks and households. When this occurs, we can either think that households invests their funds directly in the entrepreneurs' projects along with the capital from banks or that the households first deposit their funds with the banks, which then invest the deposits in the projects, along with their own capital. For clarity of presentation, we work with the latter interpretation.

All successful projects transform *i* units of final goods to Ri (R > 1) units of capital goods while failed projects yield nothing. The projects differ in their probability of success and private benefits associated with them: There is a "good" project that is successful with probability  $p_H$  and involves no private benefits to the entrepreneur. There is also a continuum of bad projects that have common success probability  $p_L$  ( $0 \le p_L < p_H < 1$ ) but differ in the amount of private benefits  $b, b \in (0, \overline{b}]$ , attached to them. Hence, especially when the entrepreneurs are leveraged, they have an incentive to choose a bad project with a high private benefits.

Bankers are endowed with a monitoring technology that reduces the level of private benefits of bad projects. In contrast to much earlier macro literature, we allow for monitoring to have a variable scale. Monitoring at the intensity level c ( $c \ge 0$ ) eliminates all bad projects with  $b \ge b(c)$ , with  $b'(c) \leq 0, b''(c) \geq 0$ , and  $\lim_{c\to\infty} b'(c) = 0$ . Monitoring is costly, requiring investments of real resources: obtaining monitoring intensity c entails the bank paying *ci* units of final goods to consumer-workers. That is, the more a banker invests in monitoring the less his bank can lend to the entrepreneur.<sup>4</sup> Hence the banker must be provided incentives to monitor. Note also that because of the marginal effect of monitoring investments is decreasing, the bankers will never want to eliminate the entrepreneurs' private benefits completely. Hence, despite monitoring the entrepreneurs must be provided incentives to choose the good project. In sum, there are two moral hazard problems: one between bankers and entrepreneurs and another between bankers and households (depositors). The moral hazard problems may be solved by choosing a proper financing contract.

#### 2.5.1 The Financing Contract

At each period t, there are three contracting parties: entrepreneurs, bankers and households (depositors). Following the standard practice in the literature we assume limited liability and focus on the class of one-period optimal contracts where the entrepreneurs invests all their own wealth  $n_t$  in their projects. The financial contract then stipulates how much the required funding of the project of size  $i_t$  comes from banks  $(a_t)$  and households  $(d_t)$  and how the project's return R in case of success is shared among the entrepreneur  $(R_t^e)$ , her bankers  $(R_t^b)$ , and outside investors / workers  $(R_t^w)$ .

As mentioned, we allow bankers to choose the banks' monitoring intensity  $c_t$  That is, a banker, given her share from the project returns will maximize the bank's profits by choosing monitoring intensity. As we assume a competitive banking sector, we can equivalently proceed as if the entrepreneur would control the monitoring intensity directly. The problem of a representative entrepreneur is then given by

$$\max_{t,a_t,d_t,R_t^e,R_t^b,R_t^w,c_t} q_t p_H R_t^e i_t$$

subject to her and her banker's incentive constraints

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$$q_t p_H R_t^e i_t \ge q_t p_L R_t^e i_t + b\left(c_t\right) i_t,\tag{12}$$

<sup>&</sup>lt;sup>4</sup>Except for this difference, the modelling of dual moral hazard problem follows Section IV.4. of Holmström and Tirole (1997).

$$q_t p_H R_t^b i_t \ge q_t p_L R_t^b i_t + (1 + r_t^d) c_t i_t,$$
(13)

the outside investors' (or workers') and the banker's participation constraints

$$q_t p_H R_t^w i_t \ge \left(1 + r_t^d\right) d_t,\tag{14}$$

$$q_t p_H R_t^b i_t \ge (1 + r_t^a) a_t, \tag{15}$$

and the resource constraints for the investment inputs and outputs

$$a_t + d_t - c_t i_t \ge i_t - n_t, \tag{16}$$

$$R \ge R_t^e + R_t^b + R_t^w. \tag{17}$$

In words, (16) implies that the aggregate supply of investment funds must satisfy their aggregate demand and (17) that the total returns must be enough to cover the total payments.

The constraints (12)-(17) are familiar from the earlier macro literature based on Holmström and Tirole (1997). Besides endogenous monitoring intensity, there are, however, two other differences. First, as shown by (13), we assume that the bank capital invested in monitoring  $c_t i_t$  has an opportunity cost  $1 + r_t^d$ . We think this is quite natural when monitoring investments are assumed to be monetary (see (16)), as in the macro literature. Second, we, following Holmström and Tirole (1997) assume, that the private benefit is truly private in the sense that it cannot invested in the market (see (12)).

It is clear that all constraints bind in equilibrium. It is also clear that the entrepreneur wants to invest as much as possible, i.e., she wants to raise as much funds from outside as possible without breaking the households' and banks' participation and incentive constraints. Using these standard equilibrium properties, we solve the entrepreneur's program in two steps. In the first step we take the intensity of monitoring  $c_t$  and, by implication, the level of private benefits  $b(c_t)$  as given and solve for the maximum size of the investment project  $i_t$  for a given level of entrepreneurial wealth  $n_t$ . As the second step, we solve for the equilibrium levels of monitoring  $c_t$  and private benefits  $b(c_t)$ .

#### 2.5.2 Investment and Leverage at the Firm Level

In this section we use  $b_t \equiv b(c_t)$  to shorten notation and to emphasize the exogenous nature of  $c_t$  and  $b_t$  here. In the Holmström-Tirole framework the maximum investment size depends on how much funds can be raised from households which in turn depends on how much of the project returns can credibly be pledged to the households. From the entrepreneur's and the

banker's incentive constraint (12) and (13) we see that the entrepreneur and the banker must get no less that  $b_t/(q\Delta p)$  and  $(1 + r_t^d) c_t/(q\Delta p)$ , respectively, in case of success, as otherwise these 'insiders' will misbehave. Substitution of  $R_t^b = (1 + r_t^d) c_t/(q_t\Delta p)$  and  $R_t^e = b_t/(q_t\Delta p)$  for the return-sharing constraint (17) shows that the workers can be promised at most

$$R_t^w = R - \frac{\left(1 + r_t^d\right)c_t}{q_t\Delta p} - \frac{b_t}{q_t\Delta p}.$$
(18)

Substituting (18) for the household's participation constraint (14) yields

$$p_H\left(q_t R - \frac{\left(1 + r_t^d\right)c_t}{\Delta p} - \frac{b_t}{\Delta p}\right) = \left(1 + r_t^d\right)\frac{d_t}{i_t}.$$
(19)

Next, we combine the banker's incentive constraint (13) with her participation constraint (15) and (16) to obtain

$$\frac{d_t}{i_t} = 1 + c_t - \frac{p_H}{\Delta p} \frac{1 + r_t^d}{1 + r_t^a} c_t - \frac{n_t}{i_t}$$

which can be then substituted for (19). Solving the resulting equation for  $i_t$  gives

$$i_t = \frac{n_t}{g\left(r_t^a, r_t^d, q_t; c_t, b_t\right)} \tag{20}$$

where

$$g(r_t^a, r_t^d, q_t; c_t, b_t) = \frac{p_H b_t}{\Delta p} + \left[1 + \frac{p_H}{\Delta p} \left(1 - \frac{1 + r_t^d}{1 + r_t^a}\right)\right] \left(1 + r_t^d\right) c_t - \chi_t \quad (21)$$

is the inverse degree of leverage, i.e., the smaller is g(.) the larger is the size of the investment project  $i_t$  for a given level of entrepreneurial wealth  $n_t$ . The term  $\chi_t \equiv p_H q_t R - 1 - r_t^d$  in (21) denotes the net present value of the good investment project, which must be positive in equilibrium.

#### 2.5.3 Monitoring

In this subsection we solve for monitoring investments  $c_t$ . This allows us to derive the rate of return to bank capital  $r_t^a$ , aggregate deposit used in investment projects  $D_t$ , and aggregate investments  $I_t$ . By using (12) and (20) we can rewrite the entrepreneur's expected profits as  $p_H q_t R_t^e i_t =$  $p_H n_t b(c_t) / \left[g\left(r_t^a, r_t^d, q_t, c_t\right) \Delta p\right]$ . Since the entrepreneur chooses  $c_t$  to maximize her expected profits we may re-express the entrepreneur's problem as

$$\min_{c_t \ge 0} \frac{g\left(r_t^a, r_t^d, q_t, c_t\right)}{b\left(c_t\right)},\tag{22}$$

where g() is given by (21). In words the entrepreneur wants to maximize the product of leverage and private benefits as larger private benefits translate in equilibrium to a larger share of the project returns. However, striving to obtain larger leverage requires giving up private benefits (project returns) so as to raise the costly bank capital (see (21)).

To derive an tractable analytic solution to (22), we specify the following functional form for  $b(c_t)$ :

$$b(c_t) = \begin{cases} \Gamma c_t^{-\frac{\gamma}{1-\gamma}} & \text{if } c_t > \underline{c} \\ b_0 & \text{if } c \le \underline{c} \end{cases},$$
(23)

where  $\Gamma > 0$ ,  $b_0 > 0$ ,  $\gamma \in (0, 1)$ , and  $\underline{c} \geq 1$ . The first row of (23) shows how  $b(c_t)$  is strictly convex for  $c_t > \underline{c}$  and that the monitoring technology is the more efficient the larger is  $\gamma$  or the smaller is  $\Gamma$ . The second row implies that there is a minimum efficient scale for monitoring investments or an upper bound for private benefits. This upper bound ensures that the net present value of a bad project is negative even for low levels of  $c_t$ .<sup>5</sup>

Under the minimum scale requirement, the entrepreneur may choose a corner solution with no monitoring  $c_t = 0$ ,  $b(c_t) = b_0$ , or a unique interior solution  $c_t = c_t^*$ . Substituting (21) and (23) for (22) and solving the problem gives the unique interior solution as

$$c_t^* = \frac{\gamma \frac{\Delta p}{p_H} \chi_t}{\left(1 + r_t^d\right) \left(1 + \frac{\Delta p}{p_H} - \frac{1 + r_t^d}{1 + r_t^a}\right)}.$$
(24)

In the appendix we study the conditions under which we can rule out the corner solution. (These conditions are met around the steady state that we study in this paper.)

#### 2.5.4 Investment and Leverage at the Aggregate Level

We proceed under the assumption that all projects will be monitored with the same intensity (24), and all entrepreneurial firms have the same capital structure. That is, for all entrepreneurs the ratios  $a_t/i_t$ ,  $d_t/i_t$ , and  $n_t/i_t$  are the same.<sup>6</sup> Given this symmetry, moving from the firm level to the aggregate level is simple. Clearly

$$\frac{a_t}{i_t} = \frac{A_t}{I_t}, \ \frac{d_t}{i_t} = \frac{D_t}{I_t}, \ \frac{n_t}{i_t} = \frac{N_t}{I_t}.$$
 (25)

<sup>&</sup>lt;sup>5</sup>Naturally, we have experimented with many other functional forms but they result in considerably uglier algebra than (23) without yielding additional insights.

<sup>&</sup>lt;sup>6</sup>Entrepreneurial firms nonetheless differ in terms of investment scale. If the firm has more entrepreneurial wealth  $n_t$ , the scale of the investment project is larger.

where capital letters stand for aggregate level variables. Then, combining (25), the bank's incentive and participation constraints (13) and (15) yields

$$c_t^* = \frac{\Delta p \left(1 + r_t^a\right) A_t}{p_H \left(1 + r_t^d\right) I_t}.$$
 (26)

Since in equilibrium (26) must be equal to (24), we have

$$1 + r_t^{a*} = \frac{1}{1 + \frac{\Delta p}{p_H}} \left( 1 + r_t^d + \frac{\gamma \chi_t I_t}{A_t} \right).$$
(27)

This shows how return to bank capital tends to be high, when bank capital is relatively scarce, i.e.,  $I_t/A_t$  is large. For (27) to characterize the equilibrium return to bank capital, it must hold that

$$r_t^{a*} > r_t^d. (28)$$

Otherwise,  $r_t^{a*} = r_t^d$ . Again, we proceed under the assumption that (28) holds, and proceed to study aggregate leverage  $1/G_t$  of the economy.

Equations (16) and (25) imply

$$\frac{D_t}{I_t} = 1 + c_t^* - \frac{A_t + N_t}{I_t}.$$
(29)

Substituting (29) for (19) and using (25) yields

$$p_{H}\left(q_{t}R - \frac{\left(1 + r_{t}^{d*}\right)c_{t}^{*} + b\left(c_{t}^{*}\right)}{\Delta p}\right) = \left(1 + r_{t}^{d}\right)\left(1 + c_{t}^{*} - \frac{A_{t} + N_{t}}{I_{t}}\right).$$

Finally, plugging (23), (26), and (27) into the above formula yields after some algebra

$$\left(\frac{A_t}{I_t} + \frac{\gamma\chi_t}{1 + r_t^d}\right)^{\gamma} \left(\frac{N_t}{I_t} + (1 - \gamma)\chi_t\right)^{1 - \gamma} = \left(\frac{p_H}{\Delta p}\right)^{1 - \gamma} \left(1 + \frac{p_H}{\Delta p}\right)^{\gamma} \Gamma^{1 - \gamma}$$
(30)

Equation (30) determines the aggregate investment level  $I_t$  in the economy.

#### 2.6 Accumulation of Capital

After the investment projects are realized, surviving entrepreneurs and bankers receive the proceeds in the form of capital goods so that the amount of capital goods held by entrepreneurs and bankers at the beginning of period t + 1 are  $\lambda^b p_H R_t^b I_t$  and  $\lambda^e p_H R_t^e I_t$ , respectively (recall that  $\lambda^b$  and  $\lambda^e$  are the banker's and entrepreneur's survival probabilities). These capital goods are then rented to producing firms. The value of a unit of capital good at the beginning of period t+1 is thus  $r_{t+1}^{K} + (1-\delta) q_{t+1}$  where the first part is the rental income and the second part the value of undepreciated capital. The total values of banker's and entrepreneur's capital are then given by

$$A_{t+1} = \left( r_{t+1}^{K} + q_{t+1} \left( 1 - \delta \right) \right) \lambda^{b} p_{H} R_{t}^{b} I_{t}$$
(31)

and

$$N_{t+1} = \left(r_{t+1}^{K} + q_{t+1}\left(1 - \delta\right)\right)\lambda^{e} p_{H} R_{t}^{e} I_{t}.$$
(32)

Combining (31) with (15) and (25) yields the following law of motion for bank capital

$$A_{t+1} = \frac{1}{q_t} \left[ A_t \lambda^b \left( 1 + r_t^a \right) \left( r_{t+1}^K + (1 - \delta) \, q_{t+1} \right) \right]. \tag{33}$$

Similarly, we may rewrite (32) as

$$N_{t+1} = \frac{1}{q_t} \left[ N_t \lambda^e \left( 1 + r_t^e \right) \left( r_{t+1}^K + (1 - \delta) \, q_{t+1} \right) \right]$$
(34)

where  $1 + r_t^e \equiv q_t p_H R_t^e I_t / N_t$  denotes the expected rate of return on the entrepreneur's project. This gives the law of motion for entrepreneurial wealth.

Finally, the dynamics of physical capital stock follow the familiar law of motion

$$K_{t+1} = (1 - \delta) K_t + p_H R I_t.$$
(35)

(In our calibration, we set  $p_H * R = 1$ , so that the law of motion of the capital stock reduces to  $K_{t+1} = (1 - \delta) K_t + p_H R I_t$ .)

### 2.7 Equilibrium

Since in our model deposits occur within a period, they carry no interest rate, i.e.,  $r_t^d = 0$ . Given this, (??) and (28), a competitive equilibrium of the economy is a tuple

$$\left\{K_t, C_t, L_t, q_t, MC_t, W_t/P_t, P_t^*, R_t^b, R_t^h, c_t, r_t^a, D_t, I_t, N_{t+1}, A_{t+1}\right\}_{t=0}^{\infty}$$

given by (4), (5), (6), (8), (9), (10), (11), (13), (18), (24), (27), (29), (??), (33), (34) and (35). In what follows, we study the dynamic equilibrium in the neighborhood of the non-stachastic steady state of the model.

# 3 Compositon of informed capital

In this section we demonstrate that bank capital is typically scarce in equilibrium in the sense that a given increase of bank capital raises the aggregate investments more than an equal proportional increase of entrepreneurial wealth.

### 3.1 Investment-maximizing Structure of Informed Capital

Let us seek the ratio of bank to entrepreneurial capital  $\nu_t \equiv A_t/N_t$  that maximizes aggregate leverage  $1/G_t = \frac{I_t}{A_t+N_t}$  in the economy and, by implication, aggregate investments and output for a given level of aggregate informed capital  $A_t + N_t$ . Combining (26) and (27) (recall that in equilibrium  $r_t^d = 0$ ) yields

$$c_t^* = \frac{\frac{\nu_t G_t}{1+\mu_t} + \gamma \chi_t}{1 + \frac{p_H}{\Delta p}}.$$
(36)

Then, substituting (36) for (23) and using (30) gives

$$b_t^* = \frac{\Delta p}{p_H} \left( \frac{G_t}{1 + \nu_t} + (1 - \gamma) \chi_t \right). \tag{37}$$

Next, plugging (36)and (37) into the first row of (23) and totally differentiating gives us the expression  $\frac{dG_t}{d\nu_t}$ , which we then set to zero.<sup>7</sup> The monitoring intensity that maximizes leverage is given by

$$c^{**} = \left(\frac{\gamma}{1-\gamma} \frac{\Gamma}{1+\frac{\Delta p}{p_H}}\right)^{1-\gamma}.$$
(38)

That is, the leverage-maximizing monitoring intensity  $c^{**}$  is constant over time and depends only on the parameters of monitoring technology ( $\gamma$  and  $\Gamma$ ) and projects' success probabilities ( $p_H$  and  $p_L$ ).

Let us use (23) to rewrite (38) as

$$\frac{b^{**}}{c^{**}} = \frac{1-\gamma}{\gamma} \left( 1 + \frac{\Delta p}{p_H} \right). \tag{39}$$

 $\left. \begin{array}{c} \left. 7 \left. \frac{dG_t}{d\nu_t} \right|_{b_t} = -\frac{\frac{\partial b_t}{\partial\nu_t} + \Gamma \frac{\gamma}{1-\gamma} c_t^{-\frac{1}{1-\gamma}} \frac{\partial c_t}{\partial\nu_t}}{\frac{\partial b_t}{\partial G_t} + \Gamma \gamma c_t^{-\frac{1}{1-\gamma}} \frac{\partial c_t}{\partial G_t}} \right. \quad U \text{sing (36) and (37) to obtain the partial derivatives} \\ \left. \begin{array}{c} c & \\ c & \\ \end{array} \right|_{b_t} \left. \frac{\partial dG_t}{\partial G_t} + \Gamma \gamma c_t^{-\frac{1}{1-\gamma}} \frac{\partial c_t}{\partial G_t} \right|_{c_t} \left. \begin{array}{c} C & \\ C & \\ \end{array} \right|_{c_t} \left. \frac{\partial G_t}{\partial G_t} \right|_{c_t} \left. \frac{\partial G_t}{\partial G_t} \right|_{c_t} \left. \begin{array}{c} C & \\ C & \\ \end{array} \right|_{c_t} \left. \frac{\partial G_t}{\partial G_t} \right|_{c_t} \left. \frac{\partial G_t}{\partial G_t} \right|_{c_t} \left. \frac{\partial G_t}{\partial G_t} \right|_{c_t} \left. \begin{array}{c} C & \\ C & \\ \end{array} \right|_{c_t} \left. \frac{\partial G_t}{\partial G_t} \right|_{c_t}$ 

of 
$$c_t$$
 and  $b_t$  gives  $\left. \frac{dG_t}{d\nu_t} \right|_{b_t} = -\frac{\left[ \left( 1 + \nu_t \right) \left( 1 + \frac{\Delta p}{p_H} + \nu_t \Gamma \gamma c_t^{-1} - \gamma \right) \right]}{\left( 1 + \nu_t \right) \left( 1 + \frac{\Delta p}{p_H} + \nu_t \Gamma \gamma c_t^{-1} - \gamma \right)}$ 

Next, plugging (36) and (37) into (39) yields

which simplifies to

$$\frac{\frac{\nu_t}{1+\nu_t}G_t + \gamma\chi_t}{\frac{G_t}{1+\nu_t} + (1-\gamma)\chi_t} = \frac{\gamma}{1-\gamma}$$

$$\frac{A^{**}}{N^{**}} = \nu^{**} = \frac{\gamma}{1-\gamma}.$$
(40)

This is the structure of informed capital that maximizes leverage and investments. In words, to attain the maximum investment scale, the ratio of bank capital to entrepreneurial capital should be equal to the elasticity of monitoring technology.

### 3.2 Steady-state structure of informed capital

It can be shown that in a steady state

$$\frac{A^{SS}}{N^{SS}} \equiv \nu^{SS} = \frac{\gamma}{1-\gamma} \frac{\lambda^b}{\lambda^e} \frac{1-\frac{\lambda^e}{\beta}}{1-\frac{\lambda^b}{\beta}+\frac{\Delta p}{p_H}},$$

so that  $\nu^{SS} < \nu^{**} = \frac{\gamma}{1-\gamma}$  as long as  $\lambda^b < \lambda^e (1 + \Delta p/p_H)$ . In particular, when  $\lambda^b = \lambda^e, \nu^{SS} < \nu^{**}$ . In words, bank capital is generally *scarce* in a steady state compared to the level that would be desirable to maximize aggregate investments and output. Only if the bankers' survival probability is clearly higher to that of the entrepreneurs, they can accumulate enough capital to maximize investment level in the economy

Similarly, it can be shown that near the steady state

$$\frac{d\ln I_t}{d\ln A_t} > \frac{d\ln I_t}{d\ln N_t}$$

as long as  $\lambda^b < \lambda^e (1 + \Delta p/p_H)$ .<sup>8</sup> That is, because bank capital is scarce, increasing it boosts the aggregate investments more than increasing entrepreneurial wealth by an equal amount.

The scarcity of bank capital arises essentially from the real resources consumed by monitoring: the more intensive the monitoring the less resources can be invested in the projects. To maximize investments, this should be taken into account in the bankers' accumulation of capital. However, entrepreneurs and bankers accumulation of capital in equilibrium only reflects their revenue shares, without taking into consideration the resources needed for monitoring.

$${}^{8}\frac{d\ln I_{t}}{d\ln A_{t}} = \frac{dI_{t}}{dA_{t}}\frac{A_{t}}{I_{t}} = \frac{1}{\Omega}\left(1 + \frac{\Delta p}{p_{H}} - \frac{\lambda^{b}}{\beta}\right) \quad \text{and} \quad \frac{d\ln I_{t}}{d\ln N_{t}} = \frac{dI_{t}}{dA_{t}}\frac{N_{t}}{I_{t}} = \frac{1}{\Omega}\left(1 + \frac{\Delta p}{p_{H}}\right)\left(1 - \frac{\lambda^{e}}{\beta}\right)$$
where  $\Omega \equiv 2\left(1 + \frac{\Delta p}{p_{H}}\right) - \frac{\lambda^{e}}{\beta}\left(1 + \frac{\Delta p}{p_{H}}\right) - \frac{\lambda^{b}}{\beta}.$ 

#### 3.3 Interpretation

To further understand why bank capital is scarce in a steady state equilibrium, let us for brevity focus on the case where  $\lambda^b = \lambda^e$ . Let us have a second look at the investment-maximizing composition of informed capital. To maximize the leverage over informed capital, the costs of entrepreneurs' and bankers' involvement should be minimized. The cost of entrepreneurs' involvement is just their expected compensation  $p_H q_t R_t^e$  whereas the cost of bankers' involvement consist of their expected compensation  $p_H q_t R_t^e$  and monitoring costs  $c_t$ .

Essentially, (39) - or (40) - is the solution to the minimization problem

$$\min_{\substack{c_t,b_t\\s.t.}} p_H q_t R_t^e + p_H q_t R_t^b + c_t$$

$$R_t^e = \frac{b_t}{q_t \Delta p}, \quad R_t^b = \frac{c_t}{q_t \Delta p}, \quad b_t = \Gamma c^{-\frac{\gamma}{1-\gamma}}$$
(41)

The first-order conditions of (41) boil down to

$$\frac{p_H q_t R_t^o + c_t}{p_H q_t R_t^e} = \frac{\gamma}{1 - \gamma} \tag{42}$$

It is easy to see that (42) is equivalent to (39) and (40) (since  $R_t^e = \frac{b_t}{q_t \Delta p}$  and  $R_t^b = \frac{c_t}{q_t \Delta p}$ ). Combining (40) and (42) yields

$$\frac{A^{**}}{N^{**}} = \frac{p_H q_t R_t^b + c_t}{p_H q_t R_t^e}$$

Therefore, to maintain the leverage-maximizing structure  $\frac{A^{**}}{N^{**}}$ , bankers and entrepreneurs should accumulate capital in relation to  $(p_H q_t R_t^b + c_t)/p_H q_t R_t^e$ . However, in equilibrium bankers and entrepreneurs accumulate capital in relation to their retained earnings,  $R_t^b/R_t^e$ .

To see the problem from a slightly differently angle, notice that in order to maximize leverage, there should be relatively intensive monitoring by banks  $(c^{**})$ . To achieve this, bank capital should be abundant, making the yield  $r^a$  moderate and bank involvement attractive to entrepreneurs. However, the problem is that a moderate yield (needed to support intense monitoring in equilibrium) cannot sustain a large stock of bank capital.

### 4 Investment shocks

Given the scarcity property established in the previous section, a shock that erodes bank capital is especially detrimental to aggregate investments. Here we introduce an investment shock that plays such a role. We also show that these investment shocks affect entrepreneurial capital to a much smaller extent.

Until now we have assumed that investment projects only involve idiosyncratic shocks: while individual investment projects may succeed or fail, in equilibrium a constant share  $p_H$  of the projects succeed, and  $p_H R * I_t$  units of new capital is produced. In this section we introduce an aggregate investment shock. To be more specific, we assume that in period t success rate of good projects,  $\tilde{p}_{Ht}$ , is

$$\widetilde{p}_{Ht} = p_H (1 + \varepsilon_t^I)$$

and the probability of success of bad projects is

$$\widetilde{p}_{Lt} = p_L(1 + \varepsilon_t^I)$$

where  $\varepsilon_t^I$  is an investment shock. The investment shock affects the accumulation of physical capital (as well and aggregate bank capital and entrepreneurial capital).

$$K_{t+1} = (1 - \delta) K_t + p_H R I_t \left( 1 + \varepsilon_t^I \right)$$

Hence, the investment shock can be introduced into the standard New Keynesian model (with capital) as well.

We have modelled the shock in such a way that we the structure of Holmström-Tirole the financial contract remains essentially intact. (See the appendix for details.) In particular, the investment shock is realized at the end of the period, after the projects have matured and also after capital markets have closed. Thus the investment shock has no effect on variables that are publicly observable in period t, and the financial contracts cannot be made contingent on the realization of the shock.

In particular, we make the following assumptions concerning capital markets. Capital goods from  $\overline{p}_H > \widetilde{p}_H$  (and also  $\overline{p}_H > p_H$ ) projects are sold in the capital markets to capital rental firms, at price  $q_t$ . Then after the capital markets have closed, it turns out that some of these capital goods are of inappropriate quality. Payments are only made for capital goods of appropriate quality, ( $\widetilde{p}_H$  projects).

### 4.1 Bankers and banks

In order to introduce investment shocks, we need to make a distinction between bankers and banks. There is a large number of bankers in each bank. Each banker monitors a single investment project. If the project succeeds, the *banker* herself takes a certain share of the proceeds, and also transfers a certain share of the proceeds to the common pot of the *bank*. Depositors are paid from this common pot. If the project, fails the banker gets nothing (and there is nothing to be transferred to the common pot). The Holmström-Tirole incentive structure applied to the *bankers*. Investment shocks have a levered impact on bankers' capital.

The *bank* always pays the depositors the face value of the deposits (plus possible interest payments). Payments to depositors cannot be made contingent on the investment shock, since the depositors do not observe the shock. We assume that the size of the investment shock is not too big, so that the *banks* can always pay the depositors. In other words, the banks do not default.

### 4.2 Investment shocks, bankers' capital and entrepreneurial capital

Thus if there is a negative investment shock, and the bank gets less revenues from the investment projects, bankers' capital absorbs these shocks: Bankers who have monitored successful investment projects get less; if there is a positive investment shock, they get more. A key assumption here is that a *bank* has a large number of investment projects in its portfolio, and the realization of the investment shock is observable at the bank level. Thus the pay-offs of the bankers within in the bank can be made contingent on the realization of the shock. The structure of banker pay-offs is explained in more detail in the appendix.

As a result, a negative investment shock has a levered effect on bankers' aggregate capital: Not only fewer bankers see their project succeed, but also the successful bankers get a smaller share of the pie (since the depositors have to be paid in full). The same logic applies in the opposite direction, if there is a positive investment shock.

More formally, the impact of an investment shock on aggregate bankerowned capital depends on bank leverage

$$LEVB_t = \frac{D_t}{A_t}$$

where  $D_t$  is deposits and  $A_t$  is banker-owned capital. The law of motion of banker-owned capital is

$$A_{t+1} = A_t \lambda^b \left( \frac{r_{t+1}^K + (1-\delta) q_{t+1}}{q_t} \right) \left[ (1+r_t^a) \left( 1 + \varepsilon_t^I \right) + LEVB_t \varepsilon_t^I \right]$$

On the other hand, investment shocks do not have a levered impact on entrepreneurs' capital. Essentially, limited liability is a back-stop to an individual entrepreneur's losses. If the investment project fails, the entrepreneur loses his own capital  $n_t$ . However, successful entrepreneurs cannot be made responsible for the losses incurred by the unsuccessful entrepreneurs.

In the financial contract, the entrepreneur's share of proceeds cannot be made contingent on the investment shock. Given our assumptions, the investment shock does not affect publicly observable (macro) variables in period t. Thus the only effect on aggregate entrepreneurial wealth derives from the fact that fewer investment projects succeed, if there is a negative investment shock. Similarly, a positive investment shock has no levered effect on aggregate entrepreneurial wealth. The law of motion of entrepreneurial wealth is

$$N_{t+1} = N_t \lambda^e \left( \frac{r_{t+1}^K + (1-\delta) q_{t+1}}{q_t} \right) \left( 1 + r_t^e \right) \left( 1 + \varepsilon_t^I \right)$$

where

$$1 + r_t^e = p_H q_t R_t^e I_t / N_t$$

is the expected return to entrepreneurial wealth in the investment projects.

Finally, the introduction of aggregate uncertainty to the investment process, also affects the level of aggregate investments, with given bank capital  $A_t$ , entrepreneurial capital  $N_t$  and price of physical capital  $q_t$ . Essentially, the key aggregate investment equation now takes the form.

$$\begin{pmatrix} (1-\gamma)\,\widehat{\chi}_t + \frac{N_t}{I_t} \end{pmatrix}^{1-\gamma} \left(\gamma\widehat{\chi}_t + \frac{A_t}{I_t}\right)^{\gamma} \\ = \Gamma^{1-\gamma} \left(\frac{p_H}{\Delta p}\eta_t^e\right)^{1-\gamma} \left(1 + \frac{p_H}{\Delta p}\right)$$

where

$$\widehat{\chi}_t = \frac{p_H q_t R}{\eta_t^b} - 1$$

is the risk-adjusted expected net present value of the investment project,  $\eta_t^b = \frac{E[\tilde{v}_t^b]}{E[\tilde{v}_t^b(1+\varepsilon_t^I)]}$ ,  $\eta_t^e = \frac{E[\tilde{v}_t^e]}{E[\tilde{v}_t^e(1+\varepsilon_t^I)]}$ ,  $\tilde{v}_t^b$  is marginal value of bank capital after the realization of the investment shock, and  $\tilde{v}_t^e$  marginal value of entrepreneurial capital after the realization of the shock.

# 5 Calibration

In calibrating the model, we follow the standard New-Keynesian calibration where ever possible. In addition to the standard technology shock, we have aggregate investment shock. Due to this reason, we calibrate the variance of the technology shock smaller than in the RBC literature to Match the variance of output. The calibration of the financial block boils down matching excess return to banks' and entrepreneurial firms' capital, their capital ratios and monitoring costs. In addition we use the loan spreads to calibrate the volatility of investment shocks.

The household utility function is calibrated to imply relatively modest risk aversion ( $\sigma = 2$ ). The labour supply is dampened by the choice of  $\phi = 3$  and  $\xi = 0.5$ . The capital factor share is the usual  $\alpha = 1/3$ . We work with quarterly data, so that  $\beta = 0.995$  and the annualized real interest rate is 2%. The quarterly depreciation rate is  $\delta = 0.025$  matching the (annual) investment to capital ratio of 0.07. To keep the model as close as possible to the basic 'text-book' New Keynesian framework, we adopt the normalization  $p_H R = 1$ . This results in the law of motion of the physical capital stock (35) as  $K_{t+1} = (1 - \delta) K_t + I_t$ . The persistence of the technology shock is  $\rho = 0.979$  and its' standard deviation  $\sigma_{\varepsilon} = 0.0072$ .

The steady-state mark-up is calibrated to 10 percent ( $\epsilon = 10$ ), commonly used in the business cycle literature. The Calvo parameter  $\theta = 0.7$  implies average time between price changes is 3.3 quarters (see Galí *et al* 2001 and Sbordone 2002). Monetary policy follows the strict inflation targeting responding only ( $\phi_{\pi} = 1.5$ ) to the inflation deviating from the target of zero percent.

We construct the steady-state such that there is a fixed subsidy to compensate the imperfect competition. Similarly, we assume investment subsidy to redress the moral hazard in investments. These results efficient steadystate that corresponds that of the standard real business cycle model. Investments' output share is 20 %, and that of the consumption is 80 %.

The key data moments in the financial block are the following:

- Albertazzi and Gambacorta (2009) estimate the return on equity to various countries and country blocks. The average return on equity<sup>9</sup> in 1999–2003 varies from 15 % in the UK and 14 % in the USA to 7 % in the euro area. Haldane and Alessandri (2009) estimate the post 1960's pretax return on equity in the UK to be around 20 % on average. The excess rate of return on bank capital,  $\bar{r}^a$ , is calibrated to be on the upper tail of these values since the bank capital in our theoretical model corresponds the "core", informed capital of banks.
- In calculating the measure of the excess rate of return on entrepreneurial capital,  $\bar{r}^e$ , we use the standard RBC measure 6.5 % for

<sup>&</sup>lt;sup>9</sup>The return on equity is given by profit after tax as a percentage of capital and reserves.

average return to capital in the economy and substract the riskless rate of 2 %.

- Non-financial firms' capital ratio, N/I, seems to be hard to pin down. Graham and Leary (2011) and de Jong, Kabir and Ngyen (2008) report substantial temporal and cross-section/country variation. We choose the magnitude 0.45 that is within the ranges that Rajan and Zingales (1995) estimate for the US.
- In computing the parameter values of the financial block, we correct *banks' capital ratio* by subtracting monitoring costs from the banks' assets. We calibrate this measure to be 8 per cent. It implies that the conventional capital ratio is marginally above eight per cent.
- The estimates of banks' operating expenses are around 1–4 per cent (see, for example, Albertazzi and Gambacorta (2009)). The literature, however, seems to be mute on the magnitude of monitoring activities relative the total operation costs. We approximate the monitoring activities by the ratio of banks' depositary institution loans to total credit market instruments<sup>10</sup>. According to the flow-of-funds statistics, this ratio has averaged in 20 per cent in 1990–2011. This motivates our calibration of the monitoring costs to asset ratio of 0.6 percent (per annum).

The resulted parameter values are reported in the lower panel of table 1.

We use the VIX-index to estimate the persistence and variance of the volatility shock. The sample is limited to the pre-crisis period 1990–2007. The logarithm of the VIX index is normalized to unity. The estimate for the persistence of the volatility shock is modest,  $\hat{\rho}_{\varepsilon^I} = 0.61$  but the estimate of the standard deviation is fairly large  $\hat{\sigma}_{\varepsilon^I} = 0.279$ .

Finally, the standard deviation of the investment shock innovation is calibrated to be 0.01. The low value guarantees that — despite the technical assumption of the Gaussian shock innovations — the probability of  $p^H(1-\varepsilon_t^I)$ exceeding unity is very low.

# 6 Impulse responses

Figures 1-4 show the impulse responses of a number of key macro variables and financial variables to (i) an investment shock, (ii) a technology shocks,

<sup>&</sup>lt;sup>10</sup>These numbers are from the items FL703068005 and FL704004005 in sector "Private Depository Institutions" in the US flow-of-funds statistics.

Parameter	value	note
Parameters of the New-Keynesian block		
$\beta$	0.9951	discount factor
$\alpha$	0.33	capital share
$\delta$	0.025	rate of decay of capital
ξ	2	parameter of the disutility of labor
$\phi$	0.5	$1/\phi$ Frish elasticity of labor supply
ho	0.979	persistence of technology shock
$\sigma_{arepsilon}$	0.0072	standard deviation of the technology shock innovation
$\sigma$	2	$1/\sigma$ elasticity of intertemporal substitution
$\theta$	0.7	Calvo parameter
$\epsilon$	11	10 % mark-up
$\phi_{\pi}$	1.5	Taylor rule
		Parameters of the financial block
$\lambda^e$	0.9842	survival rate of entrepreneurs
$\lambda^b$	0.9507	survival rate of bankers
$\gamma$	0.3662	$\gamma/(1-\gamma)$ elasticity of monitoring function
Γ	0.0035	parameter of monitoring function
$p_H$	0.95	success probability of a good inv. project
$\frac{\Delta p}{p_H}$	0.0179	$\Delta p \equiv p_H - p_L = 0.017$
$\rho_{\varepsilon^I}$	0.61	persistence of the volatility shock
$\sigma_{\varepsilon^{I}}$	0.279	standard deviation of the volatility shock innovation
$\sigma_I$	0.01	standard deviation of the investment shock innovation

Table 1: Calibrated parameter values

(iii) a demand (or preference) shock and (iv) a mark-up shock. As a benchmark, we also show the impulse responses of the macro variables in the corresponding standard New Keynesian model, with capital but without financial frictions / financial intermediation.

The main finding that emerges from the impulse responses is that financial frictions greatly amplify the effects of investment shocks (Figure 1). The mechanism behind this amplification is twofold.

First, as explained in Section 4, investment shocks have a very strong effect on bank capital: Banks tend to be highly leveraged, with most of their funding consisting of deposits. Even if the investment projects are (as a whole) less successful than expected (there is a negative investment shock), the banks still have to pay the depositors the full a face value of the deposits. Then bank capital serves as a shock buffer and absorbs most of the losses. Likewise, a positive investment shock has a levered effect on bank capital.

By contrast, aggregate entrepreneurial wealth is much less affected by investment shocks. Basically, the investment shock only hits those entrepreneurs whose projects fail, and limited liability is a back-stop to the size of losses.

Second, as explained in Section 3, in equilibrium bank capital tends to be scarce, relative to entrepreneurial wealth. Then a change in bank capital has a very pronounced effect on aggregate investment. This strong effect on investment then also translates into a sizeable effect on real output, employment and other key macro variables. By contrast, an equal (proportional) change in entrepreneurial wealth has a smaller effect on the macro variables.

Figures 2-4 then indicate that the financial frictions introduced in this paper tend to dampen the effects of the more standard shocks (technology shock, demand shock, mark-up shock) on impact. However, the impulse responses of the macro variables tend to be more prolonged than in the standard New Keynesian model. These characteristics essentially derive from the gradual dynamics of bank capital and entrepreneurial wealth in the model with financial frictions.

# 7 Equity injections

In this section we analyze capital injections from the government to the banks. The analysis of capital injections is motivated by two observations, mentioned in earlier in this paper. First, in equilibrium bank capital is scarce, compared to entrepreneurial capital, and changes in bank capital have significant effects on investments. Second, bank capital is vulnerable to investment shocks. Capital injections may have at least two different objective. One objective is be to provide banks a cushion against future negative (investment) shocks. Another possible objective is to avert deleveraging by banks, and to boost aggregate investments. In our framework, capital injections achieve the first objective (to provide a cushion against shocks). However the second objective (to boost investments) may not be attained.

### 7.1 Capital injections and deleveraging

In fact, if policy makers have the second objective in mind, recapitalizing banks may prove to be counterproductive: capital injections may actually accelerate deleveraging and lower aggregate investments. The key assumptions and intuition behind this perhaps somewhat surprising, and provocative, result are as follows. Essentially, capital injections from the government have the wrong effects on the bankers' incentives to monitor: bankers' do not care about what happens to government-owned capital. To put it somewhat differently, bank involvement, i.e. monitoring by bankers, becomes more costly to the entrepreneurs. The entrepreneurs have to give the banker a certain share of proceeds to provide monitoring incentives. But on top of that, also the government-owned capital has to be paid.

Yet another way to express the key intuition is to note that from the point of view of the 'insiders' (bankers and entrepreneurs) government-owned capital is essentially comparable to 'outsider capital', i.e. capital from workers. Moreover, government-owned capital is typically expensive, compared to capital from households. One plausible assumption is that the government requires the same rate of return  $1 + r_t^a$  as bankers. In equilibrium this is higher than the deposit rate.

Then government-owned capital crowds out (more than one-to-one) resources from outsiders (households). Thus in equilibrium there will be less monitoring, and the investment projects will be smaller. The end result is that the aggregate scale of investments falls.

On top of the essentially static effects outlined above, capital injections have dynamic effects as well. While government-owned capital makes bank involvement costlier and less attractive to entrepreneurs (who have to pay both banker-owned and government owned bank capital), the rate of return to banker-owned capital falls. Then a capital injection actually slows down the recovery of banker-owned capital after, say, a negative investment shock. This then further lowers aggregate investments in subsequent periods.

The counterproductive effects of capital injections are illustrated in Figure 5, which shows the impulse responses of output, investments, banker-owned capital and entrepreneurial capital after a negative investment shock. In

the exercise, capital injection takes place after the negative investment shock has realized. The blue line is the impulse response without capital injections, while the green line is the impulse response with capital injections.

The counterproductivity result rests on two assumptions: i) Bankers do not care about government-owned capital; to bankers government-owned capital is essentially like outside funding. ii) Government-owned capital is more expensive than funding received from outsiders (households). There are several ways to get around these assumptions. Then capital injections can *increase* aggregate investments.

First the government can provide capital to banks under favorable terms. Above we assumed that the rate of return to government-owned capital is the same as the rate of return to banker-owned capital,  $1 + r_t^a$ . However, the government can content with a more modest rate of return. Indeed, if the rate of return to government-owned capital is lower than the deposit rate  $1 + r_t^d$  (assumed to be 1 in our model), government owned-capital is actually less expensive than the funds provided by outside investors. This cheap source of funding increases the size of the investment projects, and boosts aggregate investments.

Alternative, the government could also donate the capital to the bankers rather than take an ownership share in the banks. In that case, the capital that is injected to banks provides the bankers the right incentives to monitor, and aggregate investments are boosted. Obviously, however, this kind of arrangement would be subject to serious moral hazard problems.

Finally, when deriving the counterproductivity result, we assumed that government-owned capital is essentially like outside funding for the bankers' point of view. However, there could be incentives in place (not modelled here) so that the bankers would also care about government-owned capital. If this were the case, capital injections would again boost aggregate investments.

#### 7.2 Capital injections as a shock cushion

As mentioned above, in our framework, capital injections do provide a shock cushion. If there is a negative investment shock, government-owned capital takes a part of the hit, and banker-owned capital is less badly affected. The effect of an investment shock to banker-owned bank capital is proportional to bank leverage

$$LEVB_t = \frac{D_t}{A_t + A_t^g}$$

where  $A_t^g$  is government-owned bank capital.

Actually, government-owned capital lowers leverage, and protects the bank in two ways. First, total capital  $A_t^{tot} = A_t + A_t^g$  goes up. Second,

deposits  $D_t$  go down. As discussed above, government-owned capital crowds out funds from outsiders (households) in the financing of investment projects.

The role of government-owned bank capital is illustrated in Figure 6, which shows the recovery of the economy from a negative investment shock with (green line) and without (blue line) capital injections. In the exercise, the government-owned bank capital is already in place, when the negative investment shock hits.

The government-owned bank capital dampens the effect of a negative investment shock on impact. However, also in this setting capital injection slows down the recovery of the economy in the later periods.

# 8 Concluding remarks

In this paper we developed a macro-finance model, where both banks' and firms' balance sheets matter. We showed that in equilibrium, bank capital tends to be scarce, compared to firm capital. Then, a given change in bank capital has a larger impact on aggregate investments than a corresponding change in firm capital. Also, due to bank leverage, bank capital is vulnerable to (negative) investment shocks. For these reasons, bank capital may play a more crucial role in macro-financial linkages, and macro dynamics, than firm capital.

We also studied capital injections from the government to banks. We showed that capital injections can be useful as a shock cushion, but they may be counter-productive if the aim is to avoid deleveraging and to boost investments.

The model can be extended in various directions: we are working with an extension that aims in analysing the investment shocks in "normal" and turbulent times separately by modifying the model to incorporate risk-averse bankers. Equity injections could be more productive in turbulent times. The model may also be extended to allow for government-owned banks.

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Figure 1: Impulse responses to an investment shock



Figure 2: Impulse responses to a technology shock



Figure 3: Impulse responses to a demand shock



Figure 4: Impulse responses to a mark-up shock



Figure 5: Impulse responses to a negative investment shock. Ex post capital injection



Figure 6: Impulse responses to a negative investment shock. Ex ante capital injection.

### A Ruling out the corner solution

In this appendix we study the conditions under which the corner solution  $(c_t = 0, b(c_t) = b_0)$  can be ruled out. Assume that a firm chooses to be monitored  $(c_t = 0)$ . Then its (maximum) leverage is  $i_t/n_t = \frac{1}{g(r_t^a, r_t^d, q_t; c_t = 0, b_t = b_0)}$ , and by (21)

$$g(r_t^a, r_t^d, q_t; c_t = 0, b_t = b_0) = \frac{p_H}{\Delta p} b_0 - \chi_t$$

Then the expected rate of return to entrepreneurial capital is

$$\widehat{r}_t^e = \frac{\frac{p_H}{\Delta p} b_0}{g\left(r_t^a, r_t^d, q_t; 0, b_0\right)} = \frac{\chi_t}{\frac{p_H}{\Delta p} b_0 - \chi_t}$$

To rule out the corner solution, we must have

$$\widehat{r}_t^e < r_t^e \tag{43}$$

where  $r_t^e$  is the expected rate of return to entrepreneurial capital, if the entrepreneur chooses the interior solution  $c_t = c_t^*$ . In particular, the condition (43) should apply in the steady state, so that we get the condition

$$b_0 \geq \frac{\Delta p}{p_H} \frac{1+\overline{r}^e}{\overline{r}^e} \overline{\chi}$$

One can show that in steady state the rate of return corresponding to the interior solution  $\overline{r}^e = \frac{\beta}{\lambda^e} - 1$ , while the net present value of the investment project

$$\overline{\chi} = \frac{p_H}{\Delta p} \frac{\Gamma^{1-\gamma}}{1-\gamma} \left(1 - \frac{\lambda^e}{\beta}\right) \widehat{\nu}^{-\gamma}$$

where

$$\widehat{\nu} \equiv \frac{\lambda^e}{\lambda^b} \frac{\overline{A}}{\overline{N}} = \frac{\gamma}{1 - \gamma} \frac{1 - \frac{\lambda^e}{\beta}}{1 - \frac{\lambda^b}{\beta} + \frac{\Delta p}{p_H}}$$

so that the condition takes the form

$$b_0 \ge \frac{\Gamma^{1-\gamma}}{1-\gamma} \hat{\nu}^{-\gamma} \tag{44}$$

On the other hand, we must also guarantee that it is optimal to choose the "good" project and the level of monitoring  $c_t^*$ , rather than the "bad" project, maximum level of private payoffs  $b_0$  and no monitoring. For this condition to hold in the steady state, we must have

$$p_H R - \overline{c}^* \geq p_L R + b_0 \Leftrightarrow$$
  
$$b_0 \leq \frac{\Delta p}{p_H} p_H R - \overline{c}^*$$
(45)

One can show that in the steady state

$$\overline{c}^* = (\Gamma \widehat{\nu})^{1-\gamma}$$

Now, to rule out a corner solution, we must find a value of  $b_0$  that satisfies both (44) and (45). Such a value  $b_0$  exists if and only if

$$\frac{\Gamma^{1-\gamma}}{1-\gamma}\widehat{\nu}^{-\gamma} < \frac{\Delta p}{p_H}p_H R - (\Gamma\widehat{\nu})^{1-\gamma} \Leftrightarrow$$
$$(\Gamma\widehat{\nu})^{1-\gamma} \left(\frac{1}{1-\gamma} + \frac{1}{\widehat{\nu}}\right) < \frac{\Delta p}{p_H}p_H R$$
(46)

With our calibration, the condition (46) is satisfied.

### **B** Introducing an investment shock

### **B.1** Timing of events

The timing of events in the investment stage is the following:

- 1. Contracts are designed and signed
- 2. The banks decide how much to monitor, the entrepreneurs choose the project (in equilibrium they always choose the good project)
- 3. The projects are carried out
- 4. The projects are completed, and the capital goods are sold (to capital rental firms) at price  $q_t$
- 5. The proceeds are divided between the entrepreneur, the bank and the outside investors (depositors)
- 6. **Investment shock:** The quality of some of the capital goods is not appropriate. The capital rental firms (that have bought the defective capital goods) are reimbursed by the entrepreneurs and the bankers (but not by the depositors/outside investors).

#### B.2 More detailed structre of stages 4-6

4) The projects are completed, and trade in the capital markets takes place. The market price  $q_t$  is determined.

- At this point it is commonly known that the fraction  $\hat{p}_H$  (<  $p_H$ ) of the projects have succeeded (the capital goods are of the appropriate quality).
- On the other hand, there is also a (small) fraction  $\overline{p}_H$  of projects, whose success is uncertain at this point. We assume that on an average, or as an expectation value, one half of these projects succeed. Then the expected success rate of projects is

$$\widehat{p}_H + \frac{1}{2}\overline{p}_H = p_H$$

- Since trading in capital markets takes place in step 4) the price of capital  $q_t$  can only depend on the expected value  $p_H$ .
- The capital rental firms pay for the fraction  $\hat{p}_H$  of capital goods (which are known to be of good quality).
- Payments for the remaining projects (fraction  $\overline{p}_H$ ) will take place later on, in stage 6.

5) The proceeds are divided between the entrepreneur(s), the banker(s) and the outside investors (depositors)

- The entrepreneurs get  $\hat{p}_H * R_t^e$ , where  $R_t^e$  is the entrepreneur's share of proceeds, as stipulated by the contract
- The banks collect the remaining share  $\hat{p}_H * R^B$ , where  $R^B = R R^e$ .
- Notice: The way the bank's share  $R^B$  is divided between the bankers  $(\widetilde{R}_t^b)$  and the depositors/households/outside investors  $(\widetilde{R}_t^h)$  depends on the realization of the investment shock (thus the tilde)
- The banks pay the depositors  $(1 + r_t^d) * D_t$ , where  $r_t^d$  is the interest rate on deposits (following Calstrom and Fuerst 1997, we assume for simplicity that  $r_t^d = 0$ ), and  $D_t$  is aggregate deposits
- Notice: All deposits  $D_t$  ( + plus possible interests  $r_t^d D_t$ ) are paid at this point.

- Important: the payments to the depositors / outside investors do not depend on the realization of the investment shock (in stage 6)
- Motivation: The payments to depositors can only depend on commonly observed (macro) variables. The price of capital  $q_t$  (determined in stage 4) does not depend on the realization of the investment shock.
- Since the fraction of project that are known to have succeeded  $(\hat{p}_H)$  is large, while the fraction of projects that are still pending  $(\bar{p}_H)$  is small, the banks can always pay the depositors with the income stream  $\hat{p}_H R^B q_t I_t$  they receive in stage 4.

6) It becomes known what share of the remaining (pending) projects has succeeded. The capital goods (of appropriate quality) are delivered to the capital rental firms, as agreed in stage 4, at price  $q_t$  per unit of capital. (The capital goods of inappropriate quality are not delivered and there are no payments for these goods.)

- The entrepreneurs get their share  $R_t^e$  of the proceeds.
- The banks collect the remaining share  $R_t^B = R R^e$ . Since the depositors have already been paid the full amount, in stage 5, the bankers can keep all this money.

#### **B.3** Investment shocks: summary

• In sum, the overall success rate of projects in period t,  $\tilde{p}_{Ht}$ , can be expressed as follows

$$\widetilde{p}_{Ht} = p_H (1 + \varepsilon_t^I)$$

where  $\varepsilon_t^I$  is an investment shock.

• To keep the analysis simple, we also make assumptions guaranteeing that the ratio  $\sim$ 

$$\frac{\Delta \widetilde{p}}{\widetilde{p}_H} = \frac{\Delta p}{p_H} = \text{constant}$$

• This then means that

$$\Delta \widetilde{p} = \widetilde{p}_H \frac{\Delta p}{p_H}$$

and

$$\widetilde{p}_L = \widetilde{p}_H - \Delta \widetilde{p} = \widetilde{p}_H \frac{p_L}{p_H} = p_L (1 + \varepsilon_t^I)$$