The 'Celtic Case': Guarantees, transparency and dual debt crises

Preliminary Version

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Abstract

Bank liability guarantee schemes have traditionally been viewed as costless measures to shore up investor confidence and stave off bank runs. However, as the experience of some European countries, most notably Ireland, has demonstrated, the credibility and effectiveness of these guarantees is crucially intertwined with the sovereign's funding risks. Employing methods from the literature on global games, we develop a simple model to explore the functional co-dependence between the rollover risks of a bank and a government, which are connected through the government's guarantee of bank liabilities. We show the existence and uniqueness of the joint equilibrium and derive its comparative static properties. In solving for the optimal guarantee, we further show that its credibility may be improved through policies that promote balance sheet transparency.

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1. Introduction

Motivated by the multitude of bank debt guarantee programs in many countries that were issued mainly in the aftermath of the Lehman default in 2008, this paper asks under which conditions such guarantee schemes can be successfully implemented. We tackle this question by breaking it down into several smaller, more specific questions. Firstly, when a government is itself exposed to funding risks, how does the issuance of a banking sector liability guarantee scheme affect the behavior of sovereign and bank creditors? Secondly, how does the guarantee impact on the ex ante probabilities of banking and sovereign default, as well as on the likelihood of a systemic crisis? Thirdly, is there a guarantee that optimally trades off the risk of sovereign and bank default? Finally, how does the effectiveness of the (optimal) guarantee depend on policies that influence balance sheet transparency and the liquidity of banks and sovereigns alike?

The global financial crisis was marked by a severe loss of confidence by investors in financial markets the world over. The triggers were revelations of losses on United

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States sub–prime mortgages and other toxic financial assets by banks. An immediate consequence was a freeze in interbank money markets, as banks ceased lending to each other.[1] Figure 1 illustrates this development. It shows the EURIBOR-OIS spread, a measure for interbank market tensions in the euro area, which sharply and abruptly increased by a factor of 3 following the collapse of Lehman Brothers in September 2008. Similarly, Figure 2 shows the change in banking sector and sovereign credit default swap (CDS) spreads between January 2007 and late September 2008 (shortly after the default of Lehman). Approximating the change in default probabilities by the change in CDS–spreads, one sees a marked increase in the default probabilities banking sectors in almost all countries.

In light of such deteriorating conditions, governments the world over sought to introduce measures to mitigate systemic risks and shore up confidence in their domestic financial sectors. An opening salvo for many was to introduce contingent guarantee schemes for retail and wholesale deposits in banks. These schemes were viewed as cost effective measures to stave off bank runs, whereby governments lend their own creditworthiness to the financial sector.[2] Table 1 provides a brief overview of schemes introduced in several countries. Figure 3 compares the size of these schemes in percent of the respective country's GDP. The schemes in Italy and Spain amounted to about 3% and 9% of GDP, respectively, while in Austria and the Netherlands they totaled at, roughly, 30% of GDP. All these were, however, dwarfed by the measures introduced in Ireland, which guaranteed all bank liabilities for a period of two years with no monetary cap. The broad mandate of the Irish scheme, which amounted to roughly 244% of GDP followed from the consensus that, as Patrick Honohan (2010), governor of the Central Bank of Ireland, noted, *"No Irish bank should be allowed to fail"*.

In general, the guarantee schemes were largely successful in alleviating banking sector default risk, yet, at the same time, they led to an increase in sovereign default risk. This can be seen from Figure 4 which compares the change in sovereign CDS–spreads with the change in banking sector CDSs. Based on this measure, it appears that the increase in the sovereigns' default probabilities was of much smaller magnitude than the reduction in the respective banking sector default probability. This phenomenon indicates that the guarantees not only led to a re–allocation of risks between banks and governments, but they may have also reduced economy–wide risks. We will come back to this phenomenon in Section 6 below.

Again, the case of Ireland requires particular attention as it can be considered exemplary for the dramatic consequences that may follow from tying the government's funding situation to that of its banking sector by means of debt guarantees. Before the crisis, Ireland was a 'sound' country with low government debt and deficit, enjoying decent growth prospects and facing low sovereign funding costs (see Figure 5). Against this background, Ireland issued its first bank liability guarantee program in October 2008. The guarantee had the immediate effect of driving down CDS–spreads for the banking sector. However, questions pertaining to the guarantee's credibility and the Irish government's ability to pay out the guarantee were it to fall due, came to the fore and sovereign funding costs and default risk began to increase. Moreover, the guarantee obviously failed to prevent large withdrawals away from Irish banks to the financial sectors in countries like Germany, the Netherlands and Luxemburg. Figure 6 illustrates this development by showing the

[1]See Taylor and Williams (2008) or Holthausen and Pill (2010) for a detailed investigation of interbank money markets during the 2007–08 financial crisis.

[2]See Schich and Kim (2011) for an overview of banking sector safety nets.

net TARGET2 liabilities of the Irish Central Bank, which serves as a proxy for the cumulative net outflows of euro denominated liquidity.[3] These events culminated in the nationalization of Anglo-Irish Bank in January 2009, and the Irish government seeking a bail-out on 21 November, 2010, jointly from the European Union's European Financial Stability Facility and the International Monetary Fund.

The 'Celtic case', as we may call it, is far removed from what governments would hope to achieve by issuing bank debt guarantee schemes. The events that followed were a direct consequence of the false belief that a guarantee will shore up investor confidence, without placing any strain on a government's own funding needs, and hence, on the credibility in keeping its guarantee promises. Or, as one financial market participant bluntly put it to the Wall Street Journal (2011) when asked to comment on the on-going banking sector problems in the euro area, "*How useful would bank guarantees from member states be if these member states are themselves shut out of financial markets?*".

In this paper we present a simple model consisting of a government, one bank and a large pool of bank and sovereign creditors. Bank creditors must decide whether to rollover their loans to the bank or to foreclose on them. Their decisions depend on the bank's recourse to liquidity and the contingent guarantee provided for by the government. Sovereign creditors, in turn, decide on whether to continue lending to the government or to withdraw. The decisions of sovereign creditors depend on the government's available resources and the possible payment of the bank guarantee. Using standard techniques from the literature on global games, we embed our model in an incomplete information setting, where creditors face strategic uncertainty concerning the actions of other creditors, as well as fundamental uncertainty over the bank's and the government's recourse to liquidity. Following well established lines of reasoning, we show that our model exhibits a unique equilibrium in threshold strategies, and that there are no other equilibria in non-threshold strategies. Key to this result is the assumption that the bank's recourse to liquidity and the government's debt are uncorrelated. We justify this assumption on the grounds that large banks can readily tap into global financial markets to shore up their liquidity, while a government's fortunes are more closely tied to its country's productivity.

Our model displays *strategic complementarities* within each group of creditors. That is, the incentives of individual bank (sovereign) creditors to rollover are increasing in the mass of bank (sovereign) creditors who also rollover. Furthermore, bank creditors' incentives to rollover are also increasing in the mass of sovereign creditors who lend to the government. Hence, sovereign creditors' actions are strategic complements for bank creditors. But the converse does not hold. The incentives of a sovereign creditor to lend are decreasing in the mass of bank creditors who rollover. The actions of bank creditors are therefore *strategic substitutes* for sovereign creditors.

To better appreciate this latter result, suppose that, following the introduction of a guarantee, a large fraction of bank creditors' rollover their loans. However, if the bank were to still fail, a large guarantee payout would come due, which would add to the government's debt burden. Anticipating such an outcome, sovereign creditors would become doubtful about the government's solvency and more reluctant to rollover their own claims. This result must be interpreted with caution and against the background of our model.

[3] While the Irish guarantee scheme was introduced in October 2008, the outflows continued until May 2009, when they peaked at approximately €100 billion. While there was a reversal of trends between May and September 2009, the pace of withdrawals accelerated shortly thereafter and continued through 2010, and peaking only in January 2011. See Bindseil and König (2012) for details on the role and mechanics of the TARGET2 system during the financial crisis.

Although the government in the model wishes to avoid a bank default, which would result in real output losses, we abstract away from direct payments being made by the bank to the government. If, for example, the government could collect taxes from the bank, its liquidity situation would be directly intertwined with the bank and the strategic substitutes effect would be less pronounced. However, since such taxes may distort the incentives of the bank to act with prudence and remain solvent, we abstract from their inclusion in order to derive the 'pure' strategic interactions between the different groups of creditors. Finally, using numerical methods we investigate how the optimal guarantee size, and the welfare properties it induces, relate to the underlying model parameters. The optimal guarantee is obtained by minimizing a cost of crisis function, which is a weighted sum of the output losses attributed to individual bank and government defaults, and the dual default event. Increases in the ex ante expected recourse to liquidity of bank and government sustain a maximal guarantee level policy. We also find that policies that promote bank's balance sheet transparency are welfare enhancing. These gains are further improved with added balance sheet transparency of the government.

The paper is structured as follows. We introduce the canonical bank debt rollover model in Section **??**. In Section 4, the guarantee-funding problem of a sovereign guarantor is explicitly introduced. The comparative statics properties of this extended model are provided in Section **??**. Most of the mathematics and all proofs are relegated to the Appendix.

2. Relation to the Literature

The modern theoretical perspective on banks' maturity and liquidity mismatches, and deposit guarantees is based on the seminal model of Diamond and Dybvig (1983) on optimal deposit contracts and bank runs. They show the existence of multiple, self-fulfilling equilibria for a bank with short-term financed illiquid assets. In one equilibrium, the bank is run upon by all depositors and fails as its reserves are not sufficient. In the second equilibrium, only a small amount of withdrawals occurs and the bank's liquidity is sufficient to avoid default. The two equilibria are brought about by a mis-coordination of beliefs. Deposit insurance, which is financed by taxes, helps overcome this multiplicity by increasing depositors' expected payoffs from not prematurely withdrawing. The existence of such a deposit insurance is sufficient to avoid a bank run, without ever having to be paid out.

Morris and Shin (2000) and Goldstein and Pauzner (2005) solve the multiple equilibria problem by extending the setup of Diamond and Dybvig to an incomplete information setting where information on fundamentals, i.e., the liquidity of the bank, is not common knowledge. Employing the global games approach of Morris and Shin (1998, 2003), they solve for the unique equilibrium in threshold strategies. If the information received by depositors is sufficiently precise and banks' fundamentals are below a critical threshold, most depositors withdraw, leading to bank failure. If fundamentals are strong, then most depositors stay. Importantly, in equilibrium the amount actually paid out due to the deposit guarantee is low as there are only a few depositors who roll over despite the bank's default. This logic has recently been translated to government guarantee schemes by Kasahara (2009) and Bebchuk and Goldstein (2010). Kasahara considers a standard global game model, where creditors to a firm enjoy the benefit of a government-financed debt guarantee. He shows that the guarantee removes inefficient coordination failures only if the government combines this policy with an information policy where it provides a sufficiently precise public signal about the firm's fundamental. While the guarantee in Kasahara's model is exogenously financed, he also considers potential costs that may arise when the guarantee creates adverse incentives and leads to a moral hazard problem on the side of the firm.

Bebchuk and Goldstein (2010) consider a stylized global game model where the coordination failure occurs among banks who can decide whether to lend to the real economy or not. Among other policy measures, they consider how a guarantee of banks' loans could overcome the no-lending- or 'credit-freeze-equilibrium'. Similar to the effect of a deposit insurance in a bank-run model, they find that when the guarantee is sufficiently high, the risk of coordination failure may be reduced to zero. Goldstein and Bebchuk focus especially on the 'global game solution' of vanishing fundamental uncertainty and they conclude that "*(...) government's guarantees (...) do not lead to any capital being spent (...) this mechanism leads to an improvement in the threshold below which a credit freeze occurs without any actual cost*" (p. 25). The authors nevertheless acknowledge that the validity of a guarantee mechanism crucially "*depends on the credibility of the government in providing the guarantee*" (p. 26). Our model adds to this recent literature by explicitly considering the credibility of the guarantee by adding a refinancing problem for the sovereign guarantor. As will be explained in greater detail below, Goldstein's and Bebchuk's conclusion still hold in our model whenever fundamental uncertainty vanishes. Yet, whenever bank creditors face some fundamental uncertainty, the guarantee induces a higher default risk of the the sovereign.

Cooper (2012) shows a similar result in a multiple equilibrium model of sovereign debt pricing. He studies how a guarantee by a sound country shifts strategic uncertainty towards the guarantor. In the absence of fundamental uncertainty, beliefs of creditors are not affected and the guarantee simply acts as a device that selects the good equilibrium. Yet, when fundamental uncertainty is present, the guarantee may influence the price of the sound country's debt. The guarantee then connects the countries and creates a contagion channel which was not present before.

Acharya et al. (2011) consider the related problem of financial sector bailouts and their impact on sovereign credit risk. Bank bailouts are financed by taxing the non-financial sector of the economy. While the bailout is successful in alleviating problems of the banks, the higher tax burden of the non-financial sector reduces the economy's growth rate. The government's task is thus to optimize the economy's welfare and to set the optimal tax rate. We abstract in this paper from taxation and finely focus on the coordination problem between bank and sovereign creditors. This emphasis on joint coordination failures allows us to address more clearly the issues of the governments' "ability-to-pay" and the credibility of the guarantee. The government in our model then sets the optimal guarantee in order to minimize the expected costs of crises and coordination failures.

Closely related to our model is the 'twin crises' global game of Goldstein (2005), which also includes two groups of agents, currency speculators who attack a pegged exchange rate, and bank creditors who hold foreign currency denominated claims against a domestic bank. The (exogenous) political decision by a government to peg the exchange rate connects the actions of the two groups of agents. The greater the fraction of speculators who attack the currency, the more likely a devaluation of the currency becomes, and hence the more likely is the bank to default due to the currency mis-match on its balance sheet. Conversely, the greater the fraction of bank creditors who withdraw their funds, the larger is the outflow of foreign reserves, and it becomes more likely that the currency peg will break down. The actions of bank creditors and speculators are strategic complements. They reinforce each other which gives rise to a vicious circle. In our model it is also an exogenous political decision, guaranteeing bank debt, that leads to the connection of the actions of sovereign and banking creditors. But unlike Goldstein's twin crisis theory, our model does not universally display strategic complementarities between the actions of the two groups. While foreclosures of sovereign creditors spur withdrawals of bank creditors, since the former raise the likelihood of a sovereign default and therefore decrease the likelihood of a guarantee payment to the latter, the converse does not hold. Moreover, in Goldstein's model, the bank's and the sovereign's financial strength is determined by the same fundamental, whilst the financial strength of the respective institutions in our model is driven by different, independently distributed fundamentals.

Global games with different fundamentals have not yet been studied in the literature to a great extent. Two examples related to our paper are Dasgupta (2004) and Manz (2010). Dasgupta models financial contagion in a global game between two banks in different regions that are exposed to independent regional shocks. Linkages between banks are created by cross-holdings of deposits in the interbank market and regional shocks may, therefore, trigger contagious bank failures in equilibrium. Manz also considers a global game with two independently distributed fundamentals to study information-based contagion between distinct sets of creditors of two firms. Creditors have imperfect information about both, their debtor firm's fundamental and a common hurdle function which a fundamental must pass for the respective firm to become solvent. In contrast to Dasgupta, his model has a sequential structure where creditors to the second firm can observe whether the first firm failed or not. This observation acts like a common signal and provides second firm creditors some information about the hurdle which in turn influences their decision to liquidate their own claim or not. While we also resort to the assumption of independently distributed fundamentals, creditor decisions are taken simultaneously, which implies that informational contagion, based on the observation of a particular outcome in one refinancing game, cannot occur. Rather, the spill-overs between the bank's and the sovereign's refinancing problem are determined by the guarantee.

3. Canonical bank debt rollover game

In this section, we describe the canonical rollover game that serves as the workhorse for the remainder of the paper. We introduce an exogenously financed guarantee and discuss the relationship between balance sheet transparency and the costliness of the guarantee.

3.1. Model description

A bank, indexed b, is indebted to risk-neutral creditors $n_b \in [0, N_b]$, where $N_b \in \mathbb{R}_+$ measures the bank's exposure to funding illiquidity. Creditors hold identical claims against the bank with face value of one monetary unit. The bank's recourse to cash is summarized by the random variable $\theta_b \in [-\eta_b, \eta_b + \theta_b^0]$. We may think of θ_b as being comprised of two parts. First, there are the liquid assets (cash) on the bank's balance sheet, which directly contribute to increasing θ_b. Second, the bank can raise cash by entering into secured finance arrangements – for example, repurchase agreements and covered bonds – where it pledges illiquid assets to investors in exchange for cash. These investors, who are not explicitly modeled, include other commercial banks, hedge funds, and also the central bank. The size of θ_b is thus further influenced by haircuts to collateral and fire-sale discounts. In total, the ex-ante mean recourse to liquidity is given by $\theta_b^0/2$.

Creditors simultaneously decide whether to rollover their loans to the bank, or to foreclose on them and walk away with their initial deposits. The set of actions of creditor n_b is given by $\{0,1\}$, where 0 denotes that the creditor *rolls over* his loan, while 1 denotes *withdrawing*. Defining $\lambda_b \in [0,1]$ as the fraction of bank creditors who withdraw, the bank defaults whenever aggregate withdrawals exceed the available liquid resources,

$$\lambda_b N_b \geq \theta_b.$$

We assume that all bank creditors have common payoffs, which are summarized in Table 2.

		Bank	
		Default	Survive
Bank Creditor	Withdraw	C_b	C_b
	Rollover	ℓ	D_b

Table 2: Typical bank creditor's payoffs.

Withdrawal by a creditor may entail additional transaction costs, which are subtracted from the unit claim held against the bank. Thus, the net payoff from withdrawing is $C_b \leq 1$, which is independent of whether the bank defaults or survives. If, however, the creditor rolls over his loan and the bank survives, he is paid back $D_b > 1$, which includes both the original amount lent, plus additional interest payments. Finally, if the bank defaults, then creditors who rolled over their loans receive a fraction ℓ of their unit claim. Hence, $\ell \times 1$ can be interpreted as the payment stemming from a bank liability guarantee scheme of the government. For the moment we assume that ℓ is exogenously financed and that creditors receive ℓ with probability one in case it is due. We further assume that $D_b > C_b \geq \ell \geq 0$, which entails that creditors face a coordination problem.[4]

3.2. Tripartite classification of the fundamental

The bank debt rollover game exhibits a *tripartite classification* of the fundamental θ_b, which is characteristic of such coordination games.[5] For $\theta_b < 0$, the bank always defaults, irrespective of the fraction λ_b of creditors who foreclose. We refer to this as the *fundamental insolvency* case or the efficient default. It is a dominant action for creditors to withdraw in this case. For $\theta_b > N_b$, the bank always survives, even if all creditors were to foreclose their loans. Here it is dominant for all creditors to rollover their loans.

The unique Nash-Equilibrium for $\theta_b < 0$ is all creditors withdrawing and the bank defaulting, whereas the unique Nash-Equilibrium for $\theta_b > N_b$ is all creditors rolling over their loans and the bank surviving. However, under the assumptions of *common knowledge* of θ_b, the game exhibits multiple (pure strategy) equilibria for intermediate values $\theta_b \in [0, N_b]$. The equilibria in this interval are sustained by common self-fulfilling expectations about the behavior of other creditors. In one equilibrium, each creditor expects that all other creditors will withdraw, and hence withdrawing is the best response to this belief. In aggregate, this leads to bank default, which vindicates the initially held beliefs. In the second equilibrium, each creditor expects all other creditors to rollover their loans. This implies that each creditor chooses to rollover as the best response to this belief. The resulting outcome is one where the bank survives, which once again vindicates the beliefs of creditors.[6]

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[4] For simplicity, we deliberately ignore the possibility of default due to insolvency at some later date which may occur even though the rollover has been successfully managed.

[5] See e.g. Diamond and Dybvig (1983), in the context of bank-runs, and Obstfeld (1996) in the context of currency crises.

[6] See Morris and Shin (2003).

3.3. *Information structure and strategies*

To eliminate the multiplicity of equilibria we use the global games approach and relax the assumption of common knowledge about θ_b. This is replaced by a weaker assumption that creditors have heterogeneous and imperfect information concerning the bank's fundamental. Specifically, creditors receive private signals about the fundamental before choosing their action. The signal is modeled as $x_{n_b} = \theta_b + \varepsilon_{n_b}$, where ε_{n_b} is an idiosyncratic i.i.d. noise term that is uniformly distributed over the support $[-\varepsilon_b, \varepsilon_b]$. Following the literature on transparency, as e.g. Heinemann and Illing (2002), Bannier and Heinemann (2005), and Lindner (2006), the dispersion in bank creditors' information ε_b can be interpreted as the *degree of balance sheet transparency* in the banking sector. A higher degree of transparency is therefore associated with a smaller ε_b and a higher precision of private signals which enables creditors to better infer the true fundamental from their observed signal. Creditors use their private signals and the commonly known prior to form individual posteriors $\theta_b|_{x_{n_b}}$ by means of Bayesian updating. Furthermore, to apply global game methods, we need to ensure that the support of the fundamental distribution is sufficiently large to include an upper and a lower dominance region. Given the support of the signal error, a creditor knows for sure that the bank will default whenever he receives a signal $x_{n_b} < -\varepsilon_b$ (even if all other creditors roll over). And similarly, whenever he receives a signal $x_{n_b} > N_b + \varepsilon_b$, he knows for sure that the bank will be able to continue (even when all other creditors withdraw). We assume that the support of θ_b is sufficiently large to include states where all creditors find either rolling over or withdrawing dominant, i.e.

$$[-2\varepsilon_b, N_b + 2\varepsilon_b] \subset [-\eta_b, \theta_b^0 + \eta_b].$$

A strategy for a typical creditor is a complete plan of action that determines for each realization of the signal whether the creditor rolls over or withdraws. Formally, a strategy is a mapping $s_{n_b} : x_{n_b} \mapsto \{0,1\}$. Strategies are *symmetric* if $s_{n_b}(\cdot) = s_b(\cdot)$ for all n_b. A strategy is called a *threshold strategy* if a creditor chooses to withdraw for all x_{n_b} below some critical $\hat{x}_{n_b}$ and rolls over otherwise. Finally, a *symmetric threshold strategy* is a threshold strategy where *all* creditors use the same critical $\hat{x}_b$.

3.4. *Equilibrium*

A symmetric equilibrium of the bank debt rollover game with heterogeneous information for the creditors is given by the strategy $s_b(\cdot)$ and aggregate choice $\lambda(\theta_b)$ such that creditors maximize their expected payoffs and

$$\lambda_b(\theta_b) = \frac{1}{2\varepsilon_b} \int_{\theta_b - \varepsilon_b}^{\theta_b + \varepsilon_b} s_b(x_{n_b}) dx_{n_b}.$$

It is a well established result that coordination games, like our bank debt rollover game, exhibit a unique equilibrium in symmetric threshold strategies.[7] The following proposition re-states this result in terms of our model.

Proposition 1. *The bank debt rollover game has a unique equilibrium summarized by the tuple $(\hat{x}_b, \hat{\theta}_b)$ where*

$$\hat{x}_b = \hat{\theta}_b + 2\varepsilon_b \left(\frac{\hat{\theta}_b}{N_b} - \frac{1}{2} \right) \tag{1}$$

[7] See Morris and Shin (2003). For a general class of distributions of the fundamental, other than the uniform distribution, uniqueness requires that the private signals of creditors are sufficiently precise, i.e. ε_b to be sufficiently small.

and

$$\hat{\theta}_b = \frac{N_b(C_b - \ell)}{D_b - \ell}. \tag{2}$$

All creditors withdraw if $x_{n_b} < \hat{x}_b$ and they rollover if $x_{n_b} > \hat{x}_b$. The bank defaults if and only if $\theta_b < \hat{\theta}_b$.

Proof. See Morris and Shin (2003) for the proof of existence and uniqueness and the Appendix for the calculations of formulae (1) and (2). □

3.5. Changes to the guarantee size

Albeit stylized, we interpret ℓ as the payment from a bank liability guarantee scheme provided by the government. Creditors receive ℓ in the event that they rollover their loans and the bank defaults. If creditors choose to recall their loans, they always receive a payoff of C_b.[8] In absence of the guarantee, i.e. $\ell = 0$, bank creditors will choose to rollover their loans as long as the probability attached to the bank's survival is sufficiently high. In terms of the payoffs, they will rollover as long as the spread between D_b and C_b is large enough to compensate for incurring the risk of ending up with a zero payoff in case of bank default. A positive guarantee $\ell > 0$ reduces the opportunity cost of rolling over (given by $C_b - \ell$) and therefore increases creditors' incentives to rollover. All other things equal, a larger guarantee lowers the critical thresholds $\hat{\theta}_b$ and $\hat{x}_b$, and leads to a higher ex ante survival probability,

$$\frac{\partial \hat{\theta}_b}{\partial \ell} = \frac{N_b(C_b - D_b)}{(D_b - \ell)^2} < 0.$$

3.6. Guarantees and transparency

Such comparative static results and conclusions may have contributed to create the deceptive belief that bank liability guarantee schemes are a costless measure to shore up confidence in financial institutions. And while it is true, that the guarantee serves as a device to change the incentives of creditors to coordinate on the efficient equilibrium, the question remains whether this is indeed a costless policy. To better appreciate the conditions under which this holds true, consider the case where creditors face only

[8]The fact that creditors *always* receive C_b when they choose to foreclose deserves some comment. The interpretation of θ_b as available liquid resources implies that the bank is unable to pay one unit per claimant for $\theta_b < \hat{\theta}_b$. A more plausible setup would then be to impose a 'sequential service constraint' and assume that creditors receive only a fraction of the available resources in the case of bank default, which may be determined by θ_b, the fraction λ_b and possible transaction costs. The resulting payoff from withdrawing would inherit a negative dependency on λ_b. However, the realism added by modeling the problem in this way has to be traded off against technical difficulties that arise due to the resulting *partial strategic complementarities*. The proof of equilibrium employed above relies on the existence of *global strategic complementarities*, i.e. creditors' actions strictly decrease in λ_b. But with the more realistic assumption of a 'sequential service constraint', the expected payoff differential (rolling over vs withdrawing) becomes increasing in λ_b over a certain range. However, as Goldstein and Pauzner (2005) show, under the alternative assumption of the payoff differential obeying a single-crossing property, the nature of the equilibrium remains unaltered. There is still a unique symmetric threshold equilibrium. Under the further restriction to uniform distributions, there are also no other non-threshold equilibria. However, this proof is more involved, leading to more complicated comparative statics calculations that continue to remain qualitatively the same. Thus, to keep the model tractable, we stick to the less realistic assumption that the payoff from withdrawing is fully safe which guarantees the global strategic complementarity property. This is also in line with standard practice in the literature, e.g. Chui et al. (2002) or Morris and Shin (2006). Rochet and Vives (2004) further motivate this approach by appealing to institutional managers who seek to make the right decision, while their payoffs do not depend directly on the face value of their claims.

strategic uncertainty about the behavior of other creditors and no *fundamental uncertainty* about the true realization of θ_b. This corresponds to a high degree of balance sheet transparency with $\varepsilon_b \to 0$, which implies that $\hat{x}_b \to \hat{\theta}_b$ (see equation (1)). All creditors now receive almost the *same* signal and as they all use the same threshold strategy around $\hat{x}_b$, in equilibrium, either everyone rolls over and the bank survives or everyone forecloses and the bank defaults. The payoffs to the creditors are then either D_b, in case they all roll over, or C_b if they all withdraw. While the guarantee payment ℓ raises the creditors' incentives to rollover, it is never paid out. A policymaker could therefore issue an arbitrarily large guarantee and effectively control the likelihood of default without ever having to follow up on its promises. In particular, by setting $\ell = C_b$, the bank's failure threshold converges to $\hat{\theta}_b = 0$ such that only a fundamentally insolvent bank defaults. By this choice of guarantee policy, a policy maker may deliberately avoid inefficient bank runs due to coordination failures.[9]

The result, that guarantees are costless, changes, however, with a lower degree of balance sheet transparency and creditors facing fundamental uncertainty, i.e. $\varepsilon_b > 0$. In this case, some creditors may decide to rollover their loans due to 'misleading' signals $x_{n_b} > \hat{x}_b$, even though the true θ_b is below $\hat{\theta}_b$ and the bank defaults. These creditors become benefactors of the guarantee scheme and receive ℓ. Let γ_b denote the fraction of agents who receive the guarantee payment. By the law of large numbers, the fraction of agents who receive signals above $\hat{x}_b$ is given by the probability that a single signal is above $\hat{x}_b$. So we can write

$$\gamma(\theta_b, \hat{x}_b, \hat{\theta}_b) = \begin{cases} 0 & \text{if } \theta_b > \hat{\theta}_b \\ \frac{\theta_b - \hat{x}_b + \varepsilon_b}{2\varepsilon_b} & \text{if } \theta_b - \hat{x}_b < \varepsilon_b < \hat{x}_b - \theta_b \\ 0 & \text{if } \varepsilon_b < \hat{x}_b - \theta_b \end{cases} \tag{3}$$

Figure 6 plots λ_b and γ_b against the fundamental θ_b for the case with the highest degree of balance sheet transparency (dashed lines) and the case with lower transparency (solid lines). In the former case, λ_b is a step function with a jump discontinuity at $\hat{\theta}_b$. In the latter case, λ_b decreases linearly from 1 to 0 over the range $[\hat{x}_b - \varepsilon_b, \hat{x}_b + \varepsilon_b]$. γ_b is always 0 in the former case, but it increases linearly in θ_b from 0 to $(\hat{\theta}_b - \hat{x}_b + \varepsilon_b)/2\varepsilon_b$ over the range $[\hat{x}_b - \varepsilon_b, \hat{\theta}_b]$ in the latter case. This illustrates how balance sheet transparency influences the possible costliness of the guarantee. The fraction of agents who benefit from the guarantee, and hence the costs created by the guarantee promise, are decreasing in the degree of balance sheet transparency. When balance sheet transparency is rather low, creditors' information is widely dispersed and many creditors may erroneously believe that the bank may not default even if, in fact, it does. These creditors, in turn, become eligible for the guarantee payment ℓ.

Several vital questions arise from these considerations. To which extent do possible costs due to the guarantee pose a threat to the guarantor's own solvency or liquidity position? Are guarantees still effective in reducing the likelihood of bank default whenever one takes the funding risk of the guarantor into account? What are the effects of variations in bank and guarantor liquidity parameters on the behavior of creditors? In what follows, we answer these questions by endogenizing the guarantor's, i.e. the government's, funding risk in the model.

[9]Such a policy has its counterpart in the lender-of-last-resort policy of many major central banks that follow *Bagehot's rule* and grant liquidity and emergency assistance only against eligible collateral to banks that are considered as "sound" by the supervising regulatory authorities.

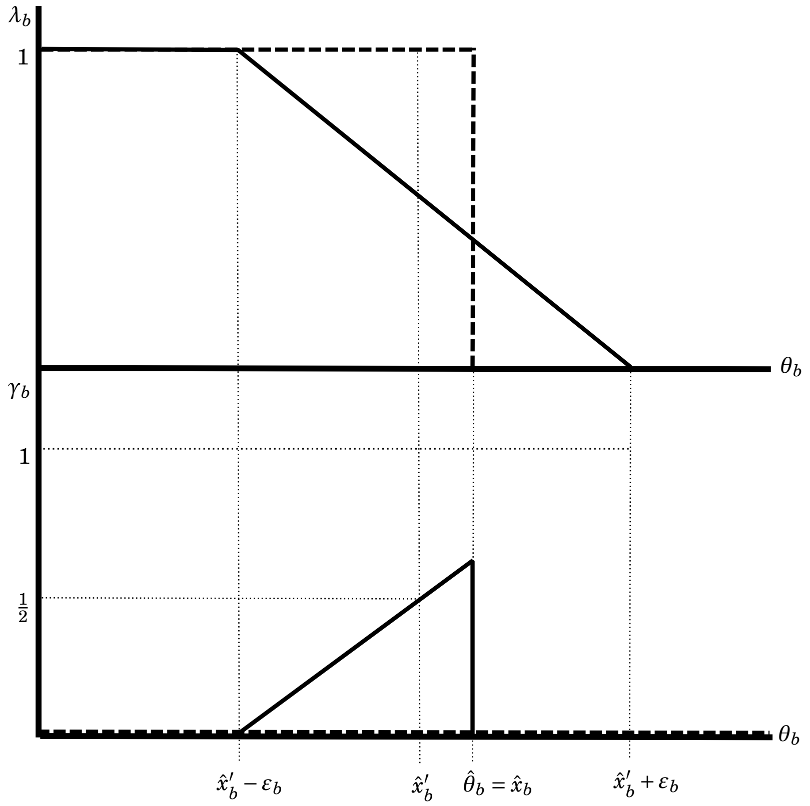


Figure 6: Upper panel: Fraction of bank creditors who withdraw, λ_b. Lower panel: Fraction of bank creditors who receive guarantee payment, γ_b. The case $\varepsilon_b = 0$ is represented by the dotted lines, whereas the case $\varepsilon_b > 0$ is represented by solid lines. An increase in ε_b does not affect $\hat{\theta}_b$, but it changes $\hat{x}_b$ to $\hat{x}'_b$. The diagram is drawn under the assumption that $\frac{C_b-\ell}{D_b-\ell} < \frac{1}{2}$ so that $\hat{x}'_b < \hat{\theta}_b$ if $\varepsilon_b > 0$.

4. Bank debt rollover in the face of sovereign funding risk

4.1. Model description

Building on the canonical debt rollover model we now explicitly introduce the refinancing problem of the government that issued the guarantee. In case of bank default, the government pays out ℓ to those bank creditors that rolled over their loans despite the default. Yet, the government itself faces a refinancing game played by a set of sovereign creditors $n_g \in [0, N_g]$ who are all different from the bank's creditors. We normalize the mass of sovereign creditors to unity, $N_g \equiv 1$. Each sovereign creditor holds a claim with a face value of one monetary unit against the government. Sovereign creditors decide simultaneously whether to continue lending to the government, or to withdraw. The government defaults whenever its liquid resources are insufficient to service debt foreclosures and guarantee payments. We represent the government's liquidity by the random variable θ_g, which is uniformly distributed over $[-\eta_g, \theta_g^0 + \eta_g]$, where $\theta_g^0/2$ is the ex-ante mean recourse to liquidity. We further make the following

Assumption 1: *The government's liquidity θ_g and the bank's liquidity θ_b are independently distributed.*

Sovereign creditors receive a noisy signal $x_{n_g} = \theta_g + \varepsilon_{n_g}$ about the government's liquidity θ_g, with ε_{n_g} being i.i.d. according to a uniform distribution over $[-\varepsilon_g, \varepsilon_g]$. As in the banking game, reduced information dispersion, i.e. a lower ε_g is associated with a higher degree of transparency of the government. Assumption 1 then implies that the signals of bank and sovereign creditors are completely uninformative about the fundamental of the respective other entity.

The payoffs to sovereign creditors are given in Table 4.

		Government	
		Default	Survive
Sovereign Creditor	Withdraw	C_g	C_g
	Rollover	0	D_g

Table 3: Typical sovereign creditor's payoffs.

As is the case for bank creditors, a sovereign creditor who prematurely withdraws receives $C_g < 1$ which is the unit claims less potential transaction costs. If the government manages to survive, the creditor who rolls over receives the full claim D_g. If the government fails, the sovereign creditors who rolled over end up with a zero payoff as there is no guarantee in place for them.

The bank's creditors, however, continue to enjoy the benefit of a guarantee in case the bank defaults and the government survives. The payoffs for a typical bank creditor are shown in Table 4 where we have normalized $C_b = 1$ in order to reflect the relatively small transaction costs in bank funding markets.

		Bank Default		Bank Survive
		Govt Survive	Govt Default	
Bank Creditor	Withdraw	$C_b = 1$	$C_b = 1$	$C_b = 1$
	Rollover	ℓ	0	D_b

Table 4: Updated bank creditor's payoffs.

Since Assumption 1 appears restrictive, some comments are in order. Firstly, since we interpret θ_b and θ_g as recourses to liquidity for the bank and government, respectively, it may be argued that the correlation between the liquidity available to an internationally active and diversified bank and the government of its jurisdiction is low. Indeed, the liquidity of the government is essentially determined by its revenues from taxes, public dues and tariffs. In contrast, internationally active banks may tap domestic as well as international markets and can issue a greater variety of financial instruments. Moreover, if the bank has branches in other countries, there may be intra-banking group liquidity transactions, so that the bank's liquidity depends on the economic fundamentals in those countries as well. Consequentially, the liquidity situation of the bank need not be strongly correlated with the liquidity situation of its resident government.[10]

Secondly, the assumption should be judged against the clear but narrow objective of our paper, namely that we want to demonstrate how, and to what extent, the introduction of a guarantee induces a dependence between a sovereign debt and bank run crisis. The simplest setting for this analysis is one where, absent the guarantee, there are no dependencies between the two coordination games.

Finally, on purely technical grounds, Assumption 1 allows us to devise a simple proof for the existence of a unique equilibrium in threshold strategies and the non-existence of equilibria in other strategies. The intuition behind this result is straightforward. From

[10]On this point, see also Shin (2012).

Assumption 1 follows that a bank (sovereign) creditor's signal is only informative about the liquidity situation of the bank (sovereign), but completely uninformative about the liquidity of the sovereign (bank). This implies that we can treat the behavior of sovereign creditors in the bank rollover game as exogenously given, and vice versa. Hence, given any arbitrary strategy used by sovereign creditors, we show that the bank creditors' rollover game has a unique equilibrium in threshold strategies and that there are no other equilibria in non-threshold strategies. The following Proposition summarizes this result.

Proposition 2. *There exists a unique equilibrium where sovereign and bank creditors use threshold strategies. There are no other equilibria in non-threshold strategies.*

Proof. See Appendix. □

As a consequence of Proposition 2 we restrict our attention to threshold strategies for sovereign and bank creditors. Absent a guarantee, $\ell = 0$, the two rollover problems are completely independent of each other and the critical thresholds for the government and the bank can be calculated from the respective formulae in Proposition 1. However, when the government issues a guarantee of amount $\ell > 0$, its refinancing problem becomes tied to the bank's rollover problem. For states of the world where the bank defaults, the government faces additional costs due to the guarantee payout. This alters the critical threshold for sovereign creditors, which in turn changes the government's default point *in all states of the world*, even in those where the bank survives. Moreover, the possibility that the government may default changes the critical threshold of bank creditors and thus the bank's default point. We now turn to an explicit derivation of the threshold equilibrium.

4.2. Bank and government default conditions

The possibility of government default does not alter the form of the bank's failure condition, which remains $\lambda_b N_b > \theta_b$. Suppose that bank creditors use a threshold strategy around $\hat{x}_b$. From Equation (1), we obtain that the bank's default point $\hat{\theta}_b$ can be written as a function of the critical threshold signal $\hat{x}_b$ as

$$\hat{\theta}_b(\hat{x}_b) = \frac{\hat{x}_b + \varepsilon_b}{1 + 2\varepsilon_b N_b^{-1}}. \tag{4}$$

Thus, the bank fails if and only if $\theta_b < \hat{\theta}_b(\hat{x}_b)$.

In calculating the government's failure point we must distinguish between two cases. Firstly, if $\theta_b > \hat{\theta}_b$, the bank survives, and the government does not have to make any guarantee payout. Assuming that government creditors use a symmetric threshold strategy around $\hat{x}_g$, the government defaults whenever $\lambda_g > \theta_g$, where λ_g is the fraction of sovereign creditors with signals below $\hat{x}_g$. The government's failure point is calculated as the solution to

$$\begin{aligned} \hat{\theta}_g &= \lambda_g(\hat{\theta}_g) \\ &\overset{LLN}{=} \mathbf{Pr}(x_{n_g} < \hat{x}_g \,|\, \hat{\theta}_g) = \frac{1}{2\varepsilon_g} \int_{-\varepsilon_g}^{\hat{x}_g - \hat{\theta}_g} \mathrm{d}u, \end{aligned}$$

yielding

$$\hat{\theta}_g = \frac{\hat{x}_g + \varepsilon_g}{1 + 2\varepsilon_g}.$$

Secondly, if $\theta_b < \hat{\theta}_b$ and the bank defaults, the government is obliged to pay ℓ to each bank creditor who rolled over his loan. Since bank creditors use the threshold strategy around $\hat{x}_b$, we can use equation (3) to calculate the total guarantee payments, conditional on the realized θ_b, as

$$N_b\,\ell \times \gamma\left(\theta_b, \hat{x}_b, \hat{\theta}_b \middle| \theta_b < \hat{\theta}_b\right) = \frac{\ell N_b}{2\varepsilon_b}\int_{\hat{x}_b}^{\theta_b+\varepsilon_b} \mathrm{d}u.$$

The government's failure point in case of a bank default then follows by solving

$$\hat{\theta}_g - \frac{\ell N_b}{2\varepsilon_b}\int_{\hat{x}_b}^{\theta_b+\varepsilon_b} \mathrm{d}u = \lambda_g(\hat{\theta}_g)$$
$$\overset{LLN}{=} \mathbf{Pr}(x_{n_g} < \hat{x}_g \,|\, \hat{\theta}_g) = \frac{1}{2\varepsilon_g}\int_{-\varepsilon_g}^{\hat{x}_g-\hat{\theta}_g} \mathrm{d}u,$$

which has the explicit solution,

$$\hat{\theta}_g = \frac{\hat{x}_g+\varepsilon_g}{1+2\varepsilon_g} + \frac{\varepsilon_g}{\varepsilon_b}\frac{\ell N_b(\theta_b+\varepsilon_b-\hat{x}_b)}{1+2\varepsilon_g}.$$

Taken together, the government's failure point is given by the function

$$\hat{\theta}_g(\hat{x}_g, \hat{x}_b, \theta_b) = \begin{cases} \frac{\hat{x}_g+\varepsilon_g}{1+2\varepsilon_g} & \text{if } \theta_b \geq \hat{\theta}_b(\hat{x}_b) \\ \frac{\hat{x}_g+\varepsilon_g}{1+2\varepsilon_g} + \frac{\ell N_b \varepsilon_g}{\varepsilon_b(1+2\varepsilon_g)}(\theta_b+\varepsilon_b-\hat{x}_b) & \text{if } \theta_b < \hat{\theta}_b(\hat{x}_b) \end{cases} \tag{5}$$

The government defaults if and only if $\theta_g < \hat{\theta}_g(\hat{x}_g, \hat{x}_b, \theta_b)$.

4.3. Creditor indifference conditions

Given the default points of bank and government, we now turn to the expected payoff differentials for typical bank and sovereign creditors who observe signals x_{n_i}, $i \in \{b, g\}$, and believe that all other bank creditors are using the threshold strategy around $\hat{x}_b$ and all other sovereign creditors are using the threshold strategy around $\hat{x}_g$.

For the typical bank creditor with signal x_{n_b}, the expected payoff difference between rolling over and foreclosing is given by

$$\pi^b\left(\hat{x}_b, \hat{x}_g, x_{n_b}\right) \equiv \frac{D_b}{2\varepsilon_b}\int_{\hat{\theta}_b(\hat{x}_b)}^{x_{n_b}+\varepsilon_b} \mathrm{d}u + \frac{\ell}{2\varepsilon_b}\int_{x_{n_b}-\varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)}\left(\frac{1}{\sigma_g}\int_{\hat{\theta}_g(\hat{x}_g,\hat{x}_b,u)}^{\tilde{\sigma}_g} \mathrm{d}v\right)\mathrm{d}u - 1, \tag{6}$$

where the second summand is the payment from the guarantee ℓ multiplied by the probability that the bank creditor attaches to the survival of the government. For the sake of notational compactness, we have used $\sigma_g := (\theta_g^0 + 2\eta_g)$ and $\tilde{\sigma}_g := \theta_g^0 + \eta_g$.

The expected payoff difference between rolling over and foreclosing for a typical sovereign creditor with signal x_{n_g} is given by

$$\pi^g\left(\hat{x}_g, \hat{x}_b, x_{n_g}\right) \equiv \frac{D_g}{\sigma_b}\int_{-\eta_b}^{\tilde{\sigma}_b}\left(\frac{1}{2\varepsilon_g}\int_{\hat{\theta}_g(\hat{x}_g,\hat{x}_b,u)}^{x_{n_g}+\varepsilon_g} \mathrm{d}v\right)\mathrm{d}u - C_g, \tag{7}$$

with $\sigma_b := (\theta_b^0 + 2\eta_b)$ and $\tilde{\sigma}_b := \theta_b^0 + \eta_b$. By using the piecewise definition of $\hat{\theta}_g$ from equation 5, we can rewrite the double integral in equation (7) as

$$\frac{(\hat{x}_g + 2\varepsilon_g)}{1+2\varepsilon_g} - \frac{\ell N_b}{(1+2\varepsilon_g)\sigma_b}\int_{-\eta_b}^{\hat{\theta}_b}\int_{\hat{x}_b}^{\hat{\theta}_b+\varepsilon_b}\frac{1}{2\varepsilon_b}\mathrm{d}u.$$

Note further that no guarantee payments come due in case all bank creditors withdraw because they all receive receive signals $x_{n_b} < \hat{x}_b$. Since, by virtue of the uniform distribution, the signals lie in the interval $[\theta_b - \varepsilon_b, \theta_b + \varepsilon_b]$, essentially all bank creditors receive signals below $\hat{x}_b$ and withdraw for realizations $\theta_b < \hat{x}_b - \varepsilon_b$. This implies that the inner integral in the last equation becomes zero for all $\theta_b < \hat{x}_b - \varepsilon_b$. Utilizing this fact, we can finally write the payoff difference between rolling over and withdrawing for a sovereign creditor as

$$\pi^g\left(\hat{x}_g, \hat{x}_b, x_{n_g}\right) = \frac{D_g(\hat{x}_g + 2\varepsilon_g)}{1 + 2\varepsilon_g} - \frac{D_g \ell N_b}{(1 + 2\varepsilon_g)\sigma_b} \int_{\hat{x}_b - \varepsilon_b}^{\hat{\theta}_b} \frac{u + \varepsilon_b - \hat{x}_b}{2\varepsilon_b} \mathrm{d}u - C_g. \tag{8}$$

4.4. Symmetric threshold equilibrium

A symmetric threshold strategy for bank creditors, given the symmetric threshold strategy $\hat{x}_g$ of sovereign creditors is defined as the signal $\hat{x}_b$ such that

$$\pi^b\left(\hat{x}_b, \hat{x}_g, \hat{x}_b\right) \equiv \bar{\pi}^b(\hat{x}_b, \hat{x}_g) = 0, \tag{9}$$

and $\pi^b(\cdot) \gtrless 0$ if and only if $x_{n_b} \gtrless \hat{x}_b$.
Similarly, a symmetric threshold strategy for sovereign creditors, given the symmetric strategy of bank creditors around $\hat{x}_b$, is defined by the signal $\hat{x}_g$ such that

$$\pi^g\left(\hat{x}_g, \hat{x}_b, \hat{x}_g\right) \equiv \bar{\pi}^g(\hat{x}_g, \hat{x}_b) = 0. \tag{10}$$

and $\pi^g(\cdot) \gtrless 0$ if and only if $x_{n_g} \gtrless \hat{x}_g$.
The following Lemma characterizes the two threshold strategies.

Lemma 1. *There exist unique solutions $\hat{x}_b = f_b(\hat{x}_g)$ and $\hat{x}_g = f_g(\hat{x}_b)$ to Equations (9) and (10) respectively.*

Proof. See Appendix. □

The properties of the two solutions are summarized in the following Lemma.

Lemma 2. *The function $f_b(\hat{x}_g)$ is strictly increasing, while the function $f_g(\hat{x}_b)$ is strictly decreasing.*

Proof. See Appendix. □

The equilibrium of the model is given by the intersection of the two curves f_b and f_g, the intersection point constitutes the simultaneous solution to Equations (9) and (10)

Proposition 3. *There exists a unique tuple of threshold signals $(\hat{x}_b^*, \hat{x}_g^*)$ that satisfies $\hat{x}_b^* = f_b(\hat{x}_g^*)$ and $\hat{x}_g^* = f_g(\hat{x}_b^*)$.*

Proof. See Appendix. □

Figure 7 illustrates the equilibrium. That f_b is strictly increasing over the entire range of $\hat{x}_g$ means that sovereign creditors' actions are *strategic complements* for bank creditors. Indeed, if sovereign creditors increase their critical signal, the risk of a government default increases and the likelihood that the guarantee will be paid out decreases. In response, bank creditors increase their critical signal as well. In contrast, f_g is strictly decreasing over the entire range of $\hat{x}_b$, which implies that bank creditors' actions are *strategic substitutes* for sovereign creditors. This deserves some comment. We show in the proof of Lemma 2 that

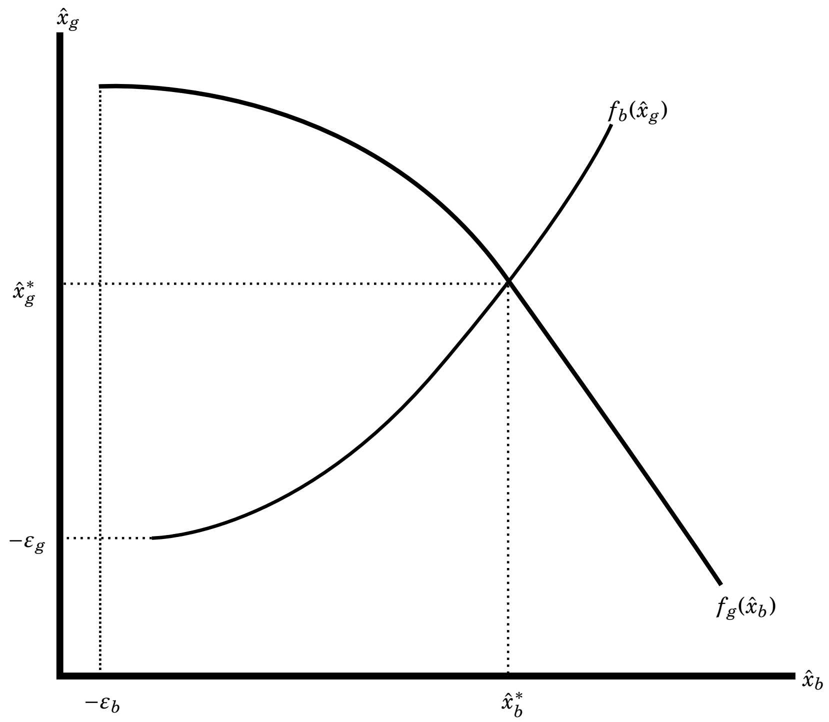


Figure 7: Best reply curves f_b and f_g. The joint equilibrium in the roll over games occurs at the intersection point $(\hat{x}_b^*, \hat{x}_g^*)$.

$$\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\hat{x}_b} = f_g'(\hat{x}_b) \propto \frac{d}{d\hat{x}_b}\left(\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)} (u + \varepsilon_b - \hat{x}_b)\mathrm{d}u\right).$$

Suppose that bank creditors increase their critical signal $\hat{x}_b$. This exerts two opposing effects on sovereign creditors' payoff and thus on their critical signal $\hat{x}_g$. Firstly, a higher $\hat{x}_b$ increases $\hat{\theta}_b$ and therefore the range of θ_b–realizations where the bank may default and the guarantee comes due enlarges. This in turn decreases sovereign creditors' expected payoffs from rolling over and leads them to increase their critical signal as well. From the expression above, this effect is, up to a factor of proportionality, given by

$$(\hat{\theta}_b + \varepsilon_b - \hat{x}_b)\frac{\partial\hat{\theta}_b}{\partial\hat{x}_b}$$

But there exists a second, opposing effect. As $\hat{x}_b$ is larger, fewer bank creditors *mistakenly* rollover their debt whenever the bank fails and consequently the government's liabilities due to the guarantee payout are lower. This is true for all states $\theta_b < \hat{\theta}_b$. In turn, the likelihood that the government survives rises and a typical sovereign creditor's expected payoff from rolling over increases. Formally, this effect is proportional to

$$-(\hat{\theta}_b + \varepsilon_b - \hat{x}_b).$$

But the second effect outweighs the first one as long as $\varepsilon_b > 0$ since

$$\frac{\partial\hat{\theta}_b}{\partial\hat{x}_b} = \frac{N_b}{N_b + 2\varepsilon_b} < 1.$$

4.5. Comparative statics

We are now analyzing the comparative statics properties of the critical signals with respect to parameters $\{\ell, N_b, \theta_b^0, \theta_g^0\}$. Firstly, consider the effect of a marginal increase in the guarantee ℓ, depicted in Figure 8. An increase in the guarantee shifts the f_b–curve to the left since, for any given $\hat{x}_g$, a higher guarantee increases bank creditors expected payoff from rolling over and therefore leads them to lower their critical signal. The f_g–curve is shifted to the right because, for any given $\hat{x}_b$, a higher guarantee promise lowers the probability that the government survives and, in response, sovereign creditors raise their critical signal. The increase in the guarantee thereby exerts a direct effect on payoffs of both types of creditors as well as an indirect effect through the change in the other type of creditors' critical signal. For sovereign creditors both effects work in the same direction, thus producing a clear-cut total effect. For bank creditors, the increase in the critical signal of sovereign creditors lowers expected payoffs and therefore works against the positive effect of the higher guarantee. Yet, if sovereign creditors' signal is not raised too much, and the shift in the f_g–curve is sufficiently small compared to the shift in the f_b–curve, then a marginally higher guarantee decreases bank creditors' critical signal. The following Lemma provides a necessary and sufficient condition for this to hold.

Lemma 3. *A marginal increase in the guarantee lowers bank creditors' critical signals, i.e.* $\frac{\mathrm{d}\hat{x_b}}{\mathrm{d}\ell} < 0$*, if and only if*

$$\tilde{\sigma}_g - \hat{\theta}_g^*(\hat{\theta}_b^*) > \frac{\ell N_b}{\sigma_b} \int_{\hat{x}_b^* - \varepsilon_b}^{\hat{\theta}_b^*} \frac{u + \varepsilon_b - \hat{x}_b^*}{2\varepsilon_b} \mathrm{d}u. \tag{11}$$

Proof. See proof of Lemma A13 in the Appendix. □

The right–hand side of condition (11) is the ex ante expected guarantee payout. The condition then says that bank creditors increase their critical signal if only if the ex ante expected guarantee payout is no larger than the bound on the left–hand side which is negatively dependent on the default point of the government and positively dependent on the upper bound of the government's liquidity distribution. Equation (11) can thus be interpreted as a "credibility condition". If it fails to hold, bank creditors may ex ante judge government's resources to be insufficient to cover guarantee promises and their response to a an increase in the guarantee is to raise their critical signal. Note further that the condition always holds for $\ell = 0$, which implies that upon introduction of the guarantee, bank creditors' critical signal is always lowered.

Secondly, consider the effect of an increase in the bank's exposure to funding illiquidity, which is measured by the mass of bank creditors N_b. This is depicted in Figure 9. As a higher degree of funding illiquidity is associated with a higher probability of bank failure and with larger expected guarantee payments, an increase in N_b shifts both curves to the right. This leads to a higher critical signal of bank creditors. From the graphical analysis alone, the sign of the effect on sovereign creditors' signal is not clear-cut. One the one hand, a larger N_b increases the expected liabilities from the guarantee (for any given ℓ and $\hat{x}_b$) and leads government creditors to increase their critical signal. However, the strategic substitutability implies that a higher critical signal of bank creditors makes sovereign creditors more willing to roll over, thereby inducing them to lower the critical signal. However, we show in the Appendix that the latter effect is smaller than

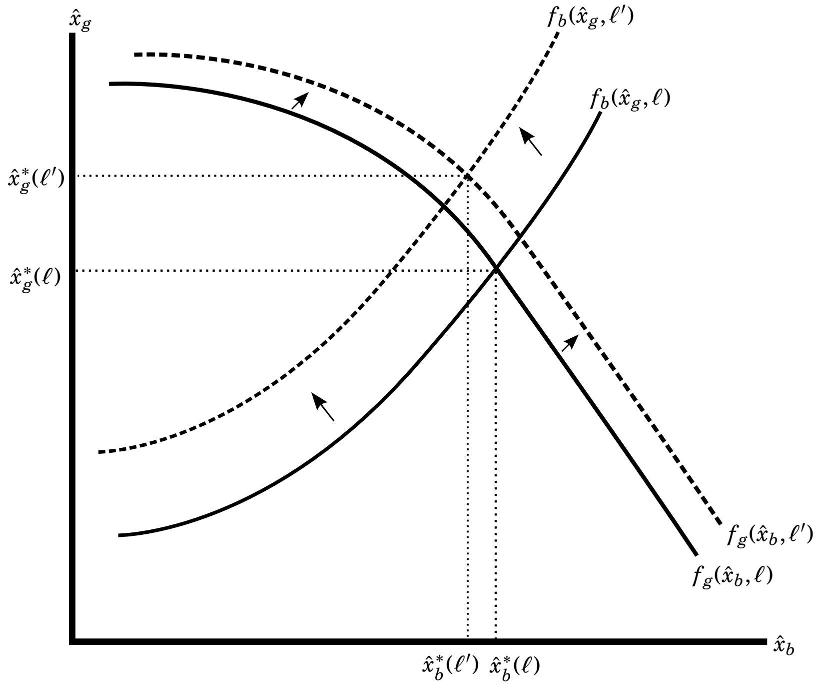


Figure 8: Change in critical signals due to an increase in the guarantee from ℓ to ℓ', given that condition (11) holds.

the former, implying that a larger N_b always leads to an increase in sovereign creditors' signal.[11]

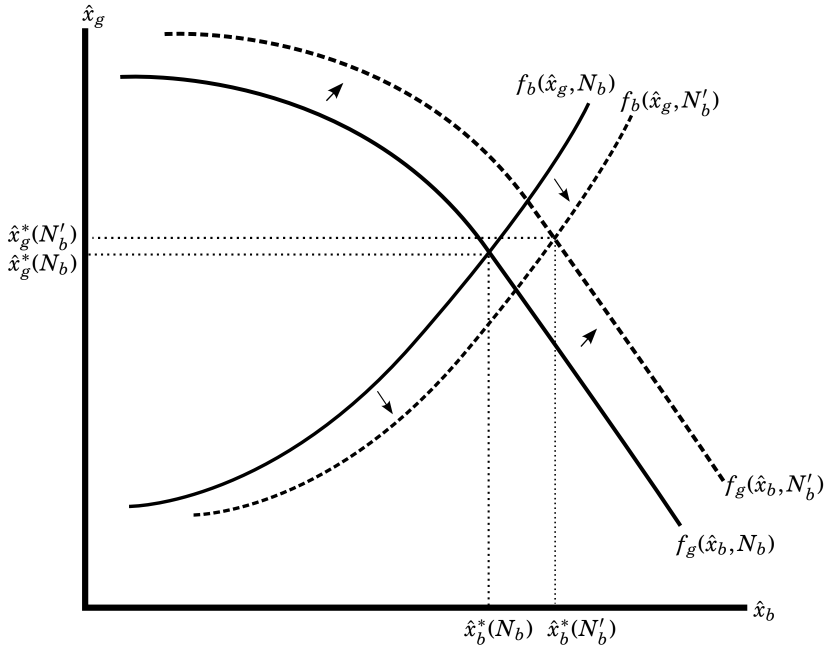


Figure 9: Change in critical signals due to an increase in funding illiquidity from N_b to N_b'.

Finally, we turn to the effects of increases in the parameters governing ex ante expected liquidity, θ_b^0 and θ_g^0. The corresponding diagrams are shown in figures 10 and 11 respectively. An increase in θ_b^0 leaves the f_b–curve unaffected and shifts f_g to the

[11]Lemma A13 in the Appendix provides the formal details.

left, implying lower critical signals for both, bank and sovereign creditors. θ_b^0 does not affect the f_b–curve directly, because bank creditors base their decisions on their updated information about θ_b after observing signal x_{n_b}, which is independent of θ_b^0. However, as sovereign creditors do not receive further information about θ_b, their critical signal depends on θ_b^0. Since a higher θ_b^0 raises the probability that the bank survives and lowers the government's expected payments due to the guarantee promise. This in turn increases sovereign creditors' expected payoffs from rolling over and makes them lower their critical signal. By strategic complementarities, the lower $\hat{x}_g$ leads to a lower $\hat{x}_b$. However, an increase in θ_g^0 leads to a qualitatively different result. For a similar reason as discussed above, θ_g^0 affects only bank creditors' expected payoffs and leaves sovereign creditors' expected payoffs unaffected. An increase in θ_g^0 then increases the likelihood that the government manages to roll over its debt and therefore it increases the probability that the guarantee can be paid out. This leads bank creditors to lower their critical signal. But now, since bank creditors' actions are strategic substitutes for sovereign creditors, their critical signal will be increased.

These results suggest that whenever bank and sovereign are connected through the guarantee promise, a positive spill-over effect exists from the bank's liquid resources to the likelihood that the government manages its debt roll over and survives. Similarly, an improvement in the government's ex ante liquidity also spills over to the likelihood that the bank survives. Yet, this comes at the cost of a higher critical signal of sovereign creditors which, in turn, may jeopardize the beneficial effect of the improved θ_g^0 on the government's likelihood of managing the debt roll over.

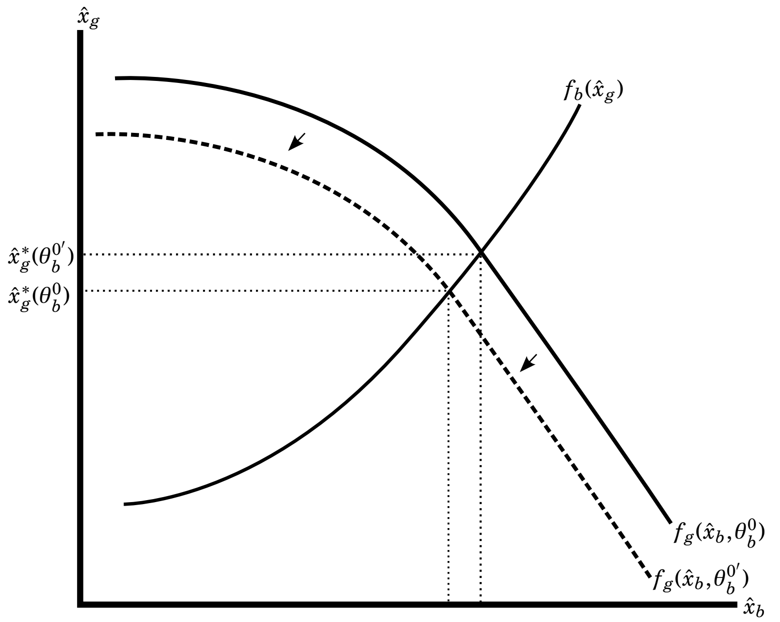


Figure 10: Change in critical signals due an increase in bank's ex ante expected liquidity from $\theta_b^0/2$ to $\theta_b^{0'}/2$.

5. The optimal guarantee and its properties

In this section we determine the optimal guarantee based on a stylized measure for expected costs of crises. Moreover, we discuss how the guarantee affects the probabilities of sovereign default, bank default, and dual default (a systemic crisis).

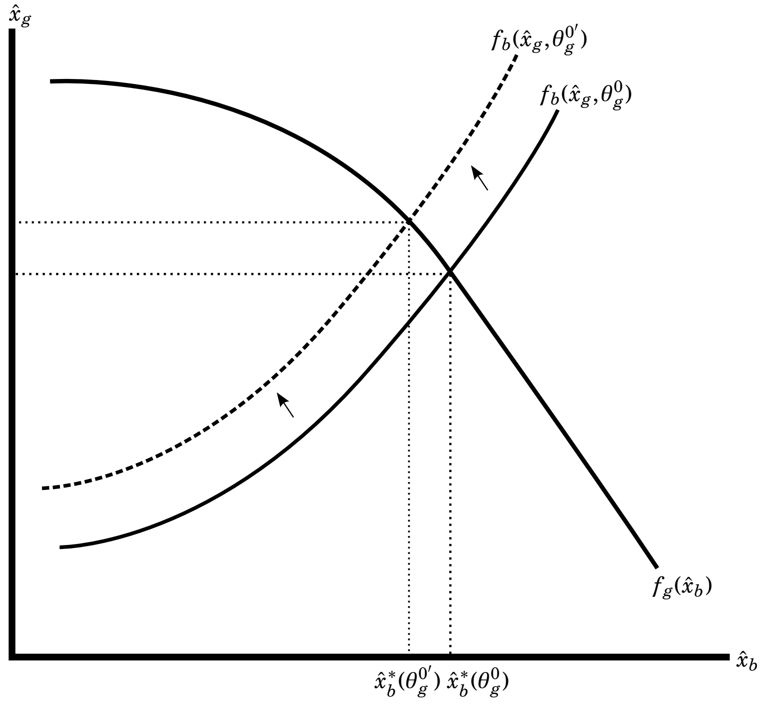


Figure 11: Change in critical signals due an increase in government's ex ante expected liquidity from $\theta_g^0/2$ to $\theta_g^{0'}/2$.

5.1. A measure for expected costs of crises

When setting the guarantee ex ante, the government faces a trade-off between lowering the expected costs of bank default on the one hand, while on the other hand, placing additional strains on its own budget and thus raising the likelihood that it enters into default itself. We formalize this trade-off by defining a measure for the expected costs of crises which the government minimizes by setting the guarantee ℓ.

We denote the cost of a pure bank default (a bank default, when the government survives) by ϕ_b, the cost of a pure government default (given that the bank survives) by ϕ_g and the costs of a systemic crisis, i.e. a crisis where both, government and bank default, by ϕ_s. We normalize these costs by setting $\phi_s \equiv 1$. We interpret the costs as the loss in the economy's output that materializes once the respective crises occur. In particular, ϕ_b results from a disruption in financial intermediation and the reduction in available bank credit in the aftermath of default. Banks make considerable investments into screening and monitoring technologies, and into the build-up of long-term relationships with borrowers (Leland and Pyle (1977)). In case the bank defaults, these investments are lost and have to be acquired anew, which involves costs for the economy as a whole. Moreover, due to the specificity of this information, some of the bank's borrowers cannot easily find a new bank and may become credit constrained. Such constraints may be binding for households and smaller enterprizes who otherwise face high costs when trying to borrow on financial markets and therefore depend to a much larger degree on financial intermediation via the banking sector (Allen and Gale (2001)).

ϕ_g is the foregone output due to a sovereign default. The default may impose reputation costs on the government, implying higher borrowing costs in the future or even a full exclusion from financial markets (Eaton and Gersovitz (1981)). A government default may also exert a negative effect on trade through either sanctions and retaliations, or through reduced access to trade credit. Moreover, empirically, sovereign default is also associated with an immediate effect on economic growth in the default period (Borensztein and Panizza (2009)).

The government's objective is then to

$$\min_{\{\ell\in[0,1]\}} K(\ell) = \phi_g \left(P_g(\ell) - q(\ell)\right) + \phi_b \left(P_b(\ell) - q(\ell)\right) + q(\ell), \tag{12}$$

where $P_g(\ell)$ denotes the probability that the government defaults, $P_b(\ell)$ stands for the probability that the bank defaults and $q(\ell)$ is the probability of a systemic crisis.[12]

We compare the expected costs under the optimally chosen guarantee, denoted by $K^{opt} \equiv K(\ell^{opt})$, to two different benchmarks. The first benchmark is the first-best outcome that occurs in the absence of any coordination problems on the side of sovereign and bank creditors. Without coordination failures, the government and the bank default if and only if $\theta_i < 0$. As the fundamentals are uniformly distributed, the first-best benchmark can be calculated as

$$K^{FB} = \phi_g \frac{\eta_g}{\sigma_g} + \phi_b \frac{\eta_b}{\sigma_b} + (1 - \phi_g - \phi_b)\frac{\eta_b}{\sigma_b}\frac{\eta_g}{\sigma_g}. \tag{13}$$

While K^{FB} provides a floor to the expected costs, the second benchmark is the ceiling, given by

$$K^0 \equiv K(0) = K^{FB} + \phi_g \frac{C_g/D_g}{\sigma_g} + \phi_b \frac{1/D_b}{\sigma_b} + (1 - \phi_g - \phi_b)\frac{(C_g/D_g + \eta_g)(1/D_b + \eta_b) - \eta_g\eta_b}{\sigma_g\sigma_b}, \tag{14}$$

when no guarantee is issued and coordination failures are present in both, the banking and the sovereign game.

5.2. Probabilities of crises

We write the equilibrium critical signals as $(\hat{x}_b^*(\ell), \hat{x}_g^*(\ell))$ in order to emphasize their dependency on the guarantee ℓ. The default points of government and bank are then written as $\hat{\theta}_b^*(\ell) \equiv \hat{\theta}_b(\hat{x}_b^*(\ell))$ and $\hat{\theta}_g^*(\ell, \theta_b) \equiv \hat{\theta}_g(\hat{x}_g^*(\ell), \hat{x}_b^*(\ell), \theta_b)$.

The probabilities in the cost function $K(\ell)$ are then given by

$$P_b(\ell) \equiv \mathbf{Pr}\left(\theta_b < \hat{\theta}_b^*(\ell)\right) \qquad \text{and} \qquad P_g(\ell) \equiv \mathbf{Pr}\left(\theta_g < \hat{\theta}_g^*(\ell)\right),$$

and

$$q(\ell) \equiv \mathbf{Pr}\left(\{\theta_b < \hat{\theta}_b^*(\ell)\} \cap \{\theta_g < \hat{\theta}_g^*(\ell)\}\right).$$

Moreover, the probability that at least one crisis occurs is given by

$$Q(\ell) \equiv \mathbf{Pr}\left(\{\theta_b < \hat{\theta}_b^*(\ell)\} \cup \{\theta_g < \hat{\theta}_g^*(\ell)\}\right).$$

With respect to the likelihood of a bank default, the guarantee exerts its impact on $\hat{\theta}_b^*$ only through the critical signal $\hat{x}_b^*$. The effect of the guarantee corresponds to a level shift in the critical value $\hat{\theta}_b^*$. This can be seen by writing explicitly

$$P_b(\ell) = \frac{1}{\sigma_b}\int_{-\eta_b}^{\hat{\theta}_b^*(\ell)} \mathrm{d}u = \frac{\frac{N_b(\hat{x}_b^*(\ell)+\varepsilon_b)}{N_b+2\varepsilon_b} + \eta_b}{\sigma_b}. \tag{15}$$

The impact on the probability of a government crisis, however, runs through two channels. Firstly, there is the effect on the critical signal $\hat{x}_g^*$, which induces a level-shift in the

[12] Explicit expression of these probabilities are provided in the subsequent section.

default point $\hat{\theta}_g^*$. This effect is of similar nature as the effect on the bank's default point θ_b^*. Secondly, the government's default point depends directly on the payments due to the guarantee. Since these payments vary with the bank's liquidity θ_b, the ex ante probability of a government default depends on the expected payments to bank creditors due to the guarantee. And despite θ_b and θ_g being independently distributed due to Assumption 1, calculation of the government's default probability requires to integrate over θ_g *and* θ_b. Besides the level shift, the guarantee thereby induces a functional dependency between the likelihood of a government default and the bank's liquidity. Formally expressed,

$$\begin{aligned} P_g(\ell) &= \frac{1}{\sigma_b}\int_{-\eta_b}^{\hat{\theta}_b^*(\ell)}\left(\frac{1}{\sigma_g}\int_{-\eta_g}^{\hat{\theta}_g^*(\ell,u)}\mathrm{d}v\right)\mathrm{d}u \quad + \quad \frac{1}{\sigma_b}\int_{\hat{\theta}_b^*(\ell)}^{\tilde{\sigma}_b}\mathrm{d}u \times \frac{1}{\sigma_g}\int_{-\eta_g}^{\hat{\theta}_g^*(\ell)}\mathrm{d}v \\ &= \frac{\frac{\hat{x}_g^*(\ell)+\varepsilon_g}{1+2\varepsilon_g}+\eta_g}{\sigma_g} \quad + \quad \frac{1}{\sigma_b\sigma_g}\frac{\ell N_b 2\varepsilon_g}{(1+2\varepsilon_g)}\int_{\hat{x}_b^*-\varepsilon_b}^{\hat{\theta}_b^*(\ell)}\frac{u+\varepsilon_b-\hat{x}_b^*(\ell)}{2\varepsilon_b}\mathrm{d}u, \end{aligned} \tag{16}$$

where the last term in the second line makes clear the functional dependency between the government's default probability and the bank's fundamental. It illustrates starkly that the government's fate does not exclusively lie in the hand of its own creditors but, through the guarantee, becomes closely tied to that of the bank, even though the liquidity resources that otherwise govern individual default probabilities are fully independent.

In much the same way, the probability of a systemic crisis can be calculated as,

$$\begin{aligned} q(\ell) &= \frac{1}{\sigma_b}\int_{-\eta_b}^{\hat{\theta}_b^*(\ell)}\left(\frac{1}{\sigma_g}\int_{-\eta_g}^{\hat{\theta}_g^*(\ell,u)}\mathrm{d}v\right)\mathrm{d}u \\ &= \frac{\frac{\hat{x}_g^*(\ell)+\varepsilon_g}{1+2\varepsilon_g}+\eta_g}{\sigma_g} \times \frac{\frac{N_b(\hat{x}_b^*(\ell)+\varepsilon_b)}{N_b+2\varepsilon_b}+\eta_b}{\sigma_b} \quad + \quad \frac{1}{\sigma_b\sigma_g}\frac{\ell N_b 2\varepsilon_g}{(1+2\varepsilon_g)}\int_{\hat{x}_b^*-\varepsilon_b}^{\hat{\theta}_b^*(\ell)}\frac{u+\varepsilon_b-\hat{x}_b^*(\ell)}{2\varepsilon_b}\mathrm{d}u, \end{aligned} \tag{17}$$

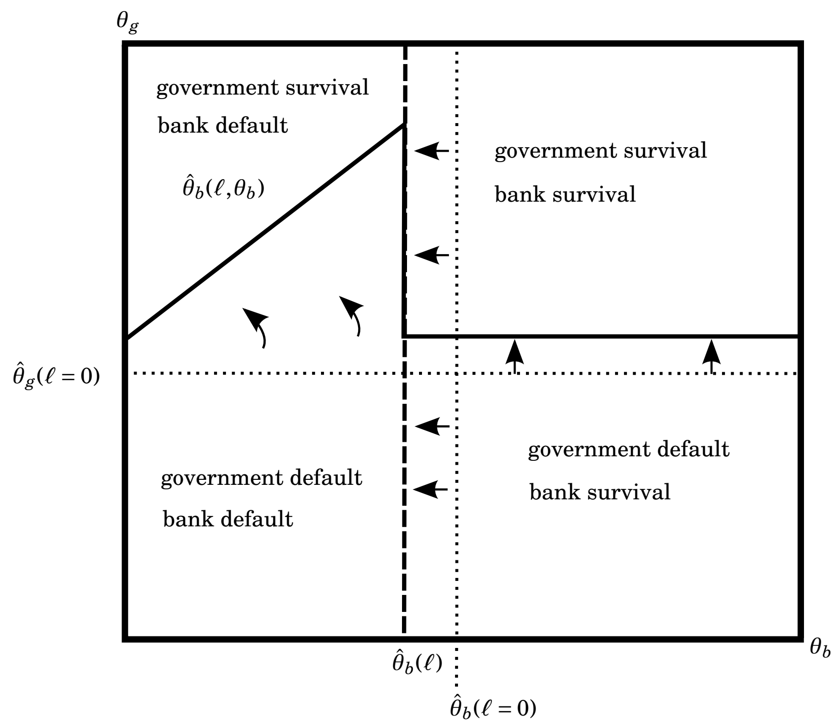


Figure 12: Regions of bank and / or sovereign default in θ_b–θ_g–space.

Figure 12 illustrates the impact of the guarantee on the default points $\hat{\theta}_g^*$ and $\hat{\theta}_b^*$. The guarantee decreases $\hat{x}_b^*$ and increases $\hat{x}_g^*$. The dotted lines separate the regions of default and no default in the absence of the guarantee. The introduction of guarantee ℓ shifts the bank's default point to the left (dashed line) and enlarges the region where the bank survives. Moreover, it increases the sovereign creditors' critical signal and the dotted horizontal line moves up to become the solid line. In the region where the bank defaults (to the left of the dashed line), the government's default point is a function of θ_b and therefore the solid line slopes upwards.

5.3. The influence of transparency on the optimal guarantee

The influence of the guarantee in reducing the likelihood of bank default depends on how 'credible' the guarantee is, which in turn is determined by the risk of sovereign default. The pertinent question is then whether and to what degree a particular guarantee promise itself undermines the guarantee's credibility by putting additional strains on the government's refinancing situation and increasing its likelihood of default. We explained in Section 3.6 that the eventual costs of a guarantee promise are crucially dependent on the degree of balance sheet transparency. To better understand the intuition behind the effects of changes in the degrees of balance sheet transparency, ε_b and ε_g, on the optimal policy in the numerical analysis in Section 6 below, we provide a brief digression on two border cases where either one degree of fundamental uncertainty vanishes.

5.3.1. Case of fully transparent bank.

With a high degree of balance sheet transparency and ε_b becoming negligibly small, bank creditors face only strategic uncertainty about the behavior of other bank creditors. The resulting coordination failure can be mitigated by the guarantee at zero cost for the government.[13] Reiterating the logic from Section 3.6: With $\varepsilon_b = 0$, all bank creditors receive exactly the same signal. Either all run, in which case the bank fails and no payout due to the guarantee promise have to be made; or, all roll over, the bank survives and the guarantee is not used. Consequentially, the prevailing strategic uncertainty can be reduced by a sufficiently large guarantee promise at zero cost for the government.

The critical thresholds for vanishing ε_b are provided in the following lemma.

Lemma 4. *Whenever the bank is fully transparent, $\varepsilon_b \to 0$, independent of the degree of transparency of the government, $\varepsilon_g \geq 0$, the default points of bank and government are given by*

$$\hat{\theta}_b^*(\ell) = \frac{N_b\left(1-\ell(1-P_g)\right)}{D_b - \ell(1-P_g)} \quad \text{and} \quad \hat{\theta}_g^* = \frac{C_g}{D_g},$$

where $P_g := \frac{C_g/D_g + \eta_g}{\sigma_g}$.

Proof. See Appendix. □

While sovereign default risk influences the critical threshold θ_b^*, the guarantee does not put any additional strains on the government and its threshold converges to the one in the canonical model. This implies that there is a clear-cut negative effect of the guarantee on the costs of crises $K(\ell)$. The government's program has its solution at a corner.

[13] This is basically the result obtained by Bebchuk and Goldstein (2010).

Lemma 5. *If the bank is fully transparent, $\varepsilon_b \to 0$, the first-order necessary condition of the government's program is given by*

$$K'(\ell) = -\frac{N_b}{\sigma_b} \frac{(1-P_g)(D_b - C_b)}{(D_b - \ell(1-P_g))^2} \left((1-P_g)\phi_b + P_g \left(1-\phi_g\right)\right) < 0, \tag{18}$$

where $P_g := \frac{C_g/D_g + \eta_g}{\sigma_g}$.

Proof. See Appendix. □

Equation (18) implies that under a regime of high transparency in the banking sector, it becomes optimal for the government to provide the maximal possible guarantee, which corresponds to a 100%–coverage of bank creditors' unit claims. However, while by setting $\ell = 1$ the government shrinks the range of fundamentals where inefficient bank runs can occur, it does not completely remove the possibility of inefficient bank failures. As the government itself defaults with probability P_g, $\ell = 1$ is not high enough to achieve $\hat{\theta}_b(1) = 0$. To accomplish this, the government would need to set

$$\ell = \frac{1}{1 - P_g} > 1, \tag{19}$$

in order to remove inefficient bank failures. Yet, this would amount to paying a subsidy to bank creditors in case the guarantee falls due.

5.3.2. Case of fully transparent government and intransparent bank.

Transparency of the government plays an entirely different role. ε_g has no decisive influence on whether the guarantee creates an actual cost or not. Yet, as can be seen from Equation (16), if ε_g vanishes, the functional dependency between the government's likelihood of default and the bank's liquidity disappears, which in turn lowers the magnitude of the guarantee's effect on the government's critical threshold.

Lemma 6. *Whenever the government is fully transparent, $\varepsilon_g \to 0$, and the bank is intransparent, $\varepsilon_b > 0$, the default points of bank and government take the form*

$$\hat{\theta}_b^*(\ell) = \frac{N_b(\hat{x}_b^*(\ell) + \varepsilon_b)}{N_b + 2\varepsilon_b} \quad \textit{and} \quad \hat{\theta}_g^*(\ell) = \hat{x}_g^*(\ell).$$

Proof. See Appendix. □

The different effects of bank and government balance sheet transparency can also be seen from the change in the optimal policy. The policy outlined in Lemma 5 may change when the bank is intransparent, albeit the government is fully transparent.

Lemma 7. *If the government is fully transparent, $\varepsilon_g \to 0$, and the bank is intransparent, $\varepsilon_b > 0$, then the first-order necessary condition of the government's program is given by*

$$\begin{aligned} K'(\ell) = & \frac{1}{\sigma_b} \left(\phi_b(1 - P_g(\ell)) + (1-\phi_g) P_g(\ell)\right) \frac{N_b}{N_b + 2\varepsilon_b} \frac{\partial \hat{x}_b^*(\ell)}{\partial \ell} \\ & + \frac{1}{\sigma_g} \left(\phi_g(1 - P_b(\ell)) + (1-\phi_b) P_b(\ell)\right) \frac{\partial \hat{x}_g^*(\ell)}{\partial \ell}, \end{aligned} \tag{20}$$

with $P_g(\ell) := \frac{\hat{x}_g^*(\ell) + \eta_g}{\sigma_g}$ *and* $P_b(\ell) := \frac{N_b(\hat{x}_b^*(\ell) + \varepsilon_b + \eta_b) + 2\varepsilon_b \eta_b}{\sigma_b(N_b + 2\varepsilon_b)}$.

Proof. See Appendix. □

Comparing equations (18) and (20), the coefficient of $\partial\hat{x}_b^*/\partial\ell$ in equation (20) is the equivalent to the second factor in equation (18). There is, however, a second term in equation (20), which does not have a counterpart in equation (18) since, by virtue of the highly transparent bank, the probability P_g did not change with ℓ. The sign of $K'(\ell)$ in (20) may therefore be different. The guarantee may not only lower the bank creditors' critical signal but it now increases the sovereign creditors' critical signal. Whether there will be a corner solution (at the upper or the lower bound of the support of ℓ) or an interior solution depends crucially on the remaining parameters governing the model and essentially on whether these render the critical signals increasing or decreasing in ℓ.

6. Numerical analysis

Although the government's minimization problem is conceptually very simple, tractable analytical solutions are not available and we must resort to a numerical analysis to determine the optimal guarantee and examine its dependence on the degrees of transparency and the parameters governing the liquidity situations of government and bank.

Our numerical analysis consists of two parts. First, we investigate how the optimal guarantee depends on the exogenous cost parameters ϕ_b and ϕ_g. Specifically, for combinations of $(\phi_b,\phi_g) \in \mathscr{C} = \{\phi_b,\phi_g | \phi_b+\phi_g \le 1\}$, we determine whether the optimal policy is (i) to have no guarantee, $\ell^{opt} = 0$, (ii) to set the guarantee at it's maximum value, $\ell^{opt} = 1$, or (iii) at an interior value $\ell \in (0,1)$. We show this diagrammatically by dividing the space $\mathscr{C}$ into distinct regions where the different optimal policies are found. Using the diagrams, we investigate how the areas occupied by the different policies change as the other exogenous model parameters, N_b, ϵ_b and ϵ_g are varied. Secondly, in Section 6.2, we pick specific values for (ϕ_b,ϕ_g) and we investigate how the optimal guarantee, welfare gain and probabilities of crises vary with the degrees of transparency.

6.1. *Properties of the optimal guarantee*

Table 5 lists the benchmark parametrization for the exogenous parameters. Since the global game requires the existence of dominance regions, we have to ensure that for all values under consideration, $N_b+2\varepsilon_b \le \theta_b^0+\eta_b$ and $\eta_b \ge 2\varepsilon_b$, which we guarantee by taking $\eta_b = 3.01$ and $\theta_b^0 = 3$. Similarly, the dominance regions in the sovereign game require $\ell N_b + 1 + 2\varepsilon_g \le \theta_g^0 + \eta_g$ and $\eta_g > 2\varepsilon_g$. We therefore set $\eta_g = 0.51$ and $\theta_g^0 = 4$. Assuming that bank claims are more liquid than claims on the government, we set $C_g = 0.9$ and $C_b = 1$. Moreover, we suppose that government claims pay a higher interest rate than bank claims and therefore set $D_b = 1.5$ and $D_g = 1.75$.[14] We then vary parameters ε_g, ε_b, and N_b throughout the different treatments.

[14]Our choices of payoff parameters amount to a rate a return for bank and sovereign creditors of 50% and 83.33%, respectively. At the time Lehman Brothers collapses in September 2008, the LIBOR-OIS rate, which is an industry benchmark for interbank lending rates, was more than 40 times that of rates at the start of the year. Thus a rate of return of 50% may be expected during crises periods. Similarly, prior to default, spreads on Irish government debt rose to over 200 basis points relative to the German Bund. Thus our estimate of an 83.33% return, while extreme, is plausible.

Parameter	Numerical Value
η_b	3.01
η_g	0.51
D_b	1.5
D_g	1.75
C_b	1
C_g	0.9

Table 5: Parameterization

6.1.1. Benchmark case

We begin our analysis by studying a benchmark case with high transparency of bank and government, $\varepsilon_b = \varepsilon_g = 0.025$ and the bank's degree of funding illiquidity set at $N_b = 1$.

With high transparency, there are few bank creditors who rollover their loans when the bank fails. Consequently, the total guarantee payout is negligibly small and does not place an adverse burden on the government's finances. As such, the sovereign creditors rollover their loans, leading to the government's continued solvency. Thus, the government can credibly set the guarantee at its maximum value. We can readily glean this result from the Figure (), where for all combinations of ϕ_b and ϕ_g the optimal policy is to have the maximum guarantee.

6.1.2. Impact of transparency

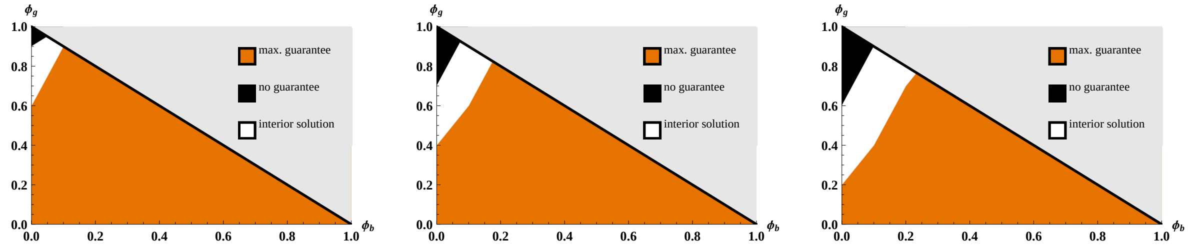
Next, we investigate how changes in bank transparency influence the optimal guarantee. As ϵ_b increases, the fraction of bank creditors who get signals above $\hat{x}_b^*$ despite $\theta_b < \hat{\theta}_b^*$ becomes larger. If the bank were to now fail, the guarantee payout due would be larger, thus placing a burden on the government's finances. This leads to an increasing fraction of sovereign creditors foreclosing on their loans. Consequently, in equilibrium, the government's ability to credibly provide the maximum guarantee is diminished. We see this from the panels in Figure **??**, where on increasing ϵ_b, the areas where the optimal policy is to have no guarantee or an interior solution increase.

Decreases in the government's degree of transparency has a similar, albeit nuanced impact. As ϵ_g increases, there are more sovereign creditors with disparate signals. Specifically, there are more creditors who believe that the government's fundamentals are weak and decide to withdraw their loans. The government's credibility to provide the maximal guarantee is diminished. From Figure 13 we note that as ϵ_g increases, the area where the optimal policy is to provide the maximum guarantee is reduced, while that for the interior solution increases. However, the area for having no guarantee remains almost unchanged.

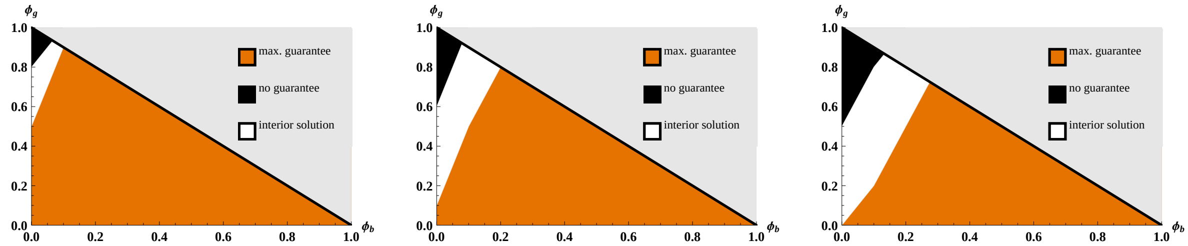
6.1.3. Impact of funding illiquidity

The size of the pool of bank creditors is governed by N_b. When N_b is large, the bank's funding illiquidity, is exacerbated. From an ex ante point of view, an increase in N_b for a given θ_b^0 implies that the bank has to cover a larger number of maturing claims with given expected resources. Its liquidity mismatch becomes larger and the expected guarantee payout increases. The larger payment places additional strains on the government's resources, thus making it more likely that sovereign creditors will withdraw their loans, leading to the sovereign's default. Under these circumstances, the government becomes less able to credibly provide the maximum guarantee. As suggested, these effects are more pronounced when ϵ_b is large, which is demonstrated in Figure 14. As

Figure 13: Optimal guarantee in space $\mathscr{C}$. Mass of bank creditors set to $N_b = 1$.



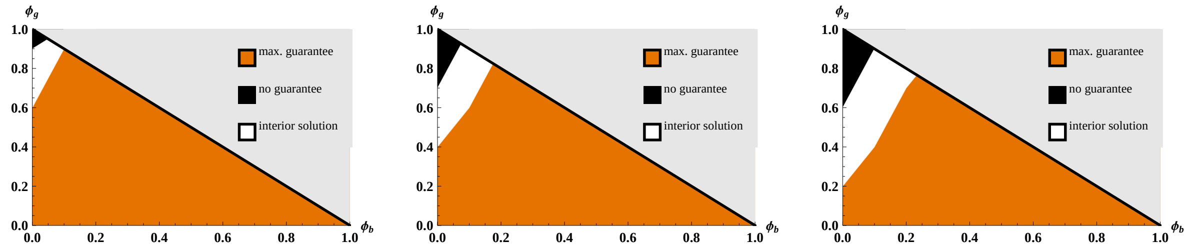
Panel 1: $\epsilon_g = 0.025, \epsilon_b = 0.5$. Panel 2: $\epsilon_g = 0.025, \epsilon_b = 1.0$. Panel 3: $\epsilon_g = 0.025, \epsilon_b = 1.5$.



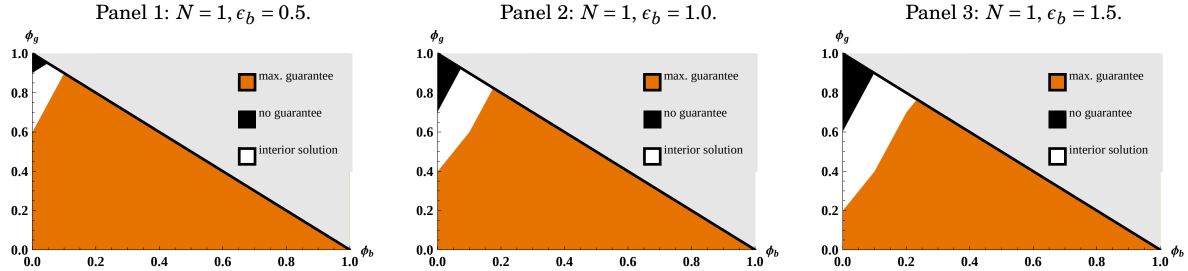
Panel 4: $\epsilon_g = 0.25, \epsilon_b = 0.5$. Panel 5: $\epsilon_g = 0.25, \epsilon_b = 1.0$. Panel 6: $\epsilon_g = 0.25, \epsilon_b = 1.5$.

N_b increases, the region where an interior solution is the optimal response widens, while the regions for no guarantee and maximal guarantee are almost unaffected and decrease, respectively.

Figure 14: Optimal guarantee in space $\mathscr{C}$. Degree of government transparency set to $\epsilon_g = 0.025$.



Panel 1: $N = 1, \epsilon_b = 0.5$. Panel 2: $N = 1, \epsilon_b = 1.0$. Panel 3: $N = 1, \epsilon_b = 1.5$.



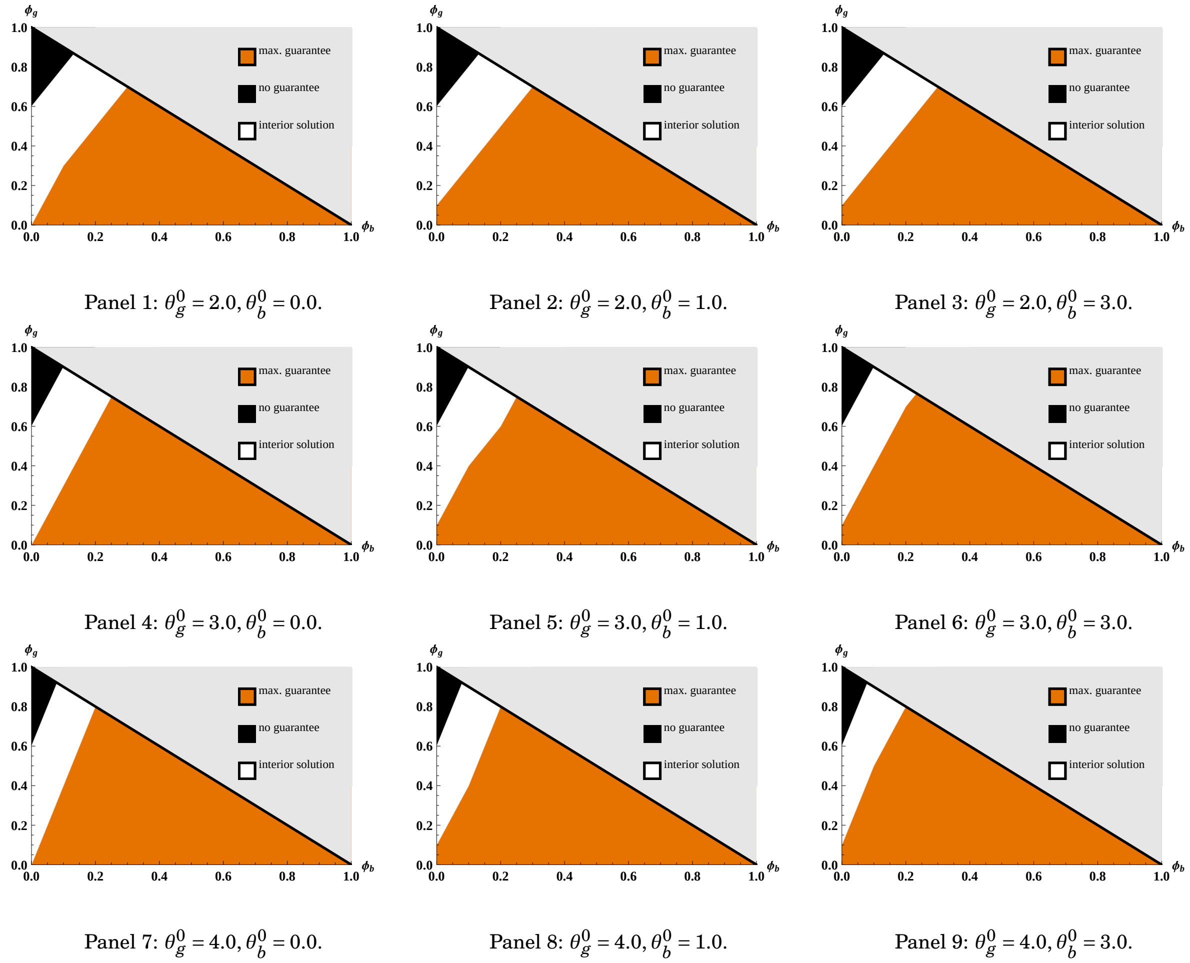
Panel 4: $N = 3, \epsilon_b = 0.5$. Panel 5: $N = 3, \epsilon_b = 1.0$. Panel 6: $N = 2, \epsilon_b = 1.5$.

6.1.4. Impact of increased expected liquidity

In the previous exercises, the parameters governing ex ante expected liquidity, θ_b^0 and θ_g^0, were fixed. To conclude this section, we finally investigate how variations in these parameters affect the optimal guarantee. θ_b^0 increases the expected liquid resources of the bank. As was shown in Section 4.5, this decreases the critical signals of both types of creditors. Moreover, a higher θ_b^0 also decreases the ex ante probability of bank default directly (see equation (**??**)). The likelihood of a guarantee payout diminishes and, as Figure **??** shows, the region where the maximal guarantee is optimal increases, while the region for no guarantee decreases.

As explained in Section 4.5, an increase in θ_g^0 decreases the critical signal of bank creditors, but it increases the critical signal of sovereign creditors. Thereby, the probability that the bank defaults decreases, while the probability that the government defaults increases. However, an increase in θ_g^0 affects the probabilities not only through its impact on the critical signals, but it also decreases the likelihood of sovereign default directly through its change in the support of the θ_g (see equation (16)). For our parametrization of the model, the effect on the sovereign creditors' critical signal is dominated by the other two effects. Figure 15 demonstrates that, as θ_g^0 increases, the regions where the government's optimal policy is to provide the maximal guarantee increases, and the region where no guarantee is optimal decreases.

Figure 15: Optimal guarantee in space $\mathscr{C}$. Mass of bank creditors set to $N_b = 1$ and degree of government transparency set to $\varepsilon_g = 0.25$.



Panel 1: $\theta_g^0 = 2.0, \theta_b^0 = 0.0$.

Panel 2: $\theta_g^0 = 2.0, \theta_b^0 = 1.0$.

Panel 3: $\theta_g^0 = 2.0, \theta_b^0 = 3.0$.

Panel 4: $\theta_g^0 = 3.0, \theta_b^0 = 0.0$.

Panel 5: $\theta_g^0 = 3.0, \theta_b^0 = 1.0$.

Panel 6: $\theta_g^0 = 3.0, \theta_b^0 = 3.0$.

Panel 7: $\theta_g^0 = 4.0, \theta_b^0 = 0.0$.

Panel 8: $\theta_g^0 = 4.0, \theta_b^0 = 1.0$.

Panel 9: $\theta_g^0 = 4.0, \theta_b^0 = 3.0$.

6.2. ...

In this section we focus on empirically plausible pairs of costs (ϕ_b, ϕ_g) and consider the impact of changes in the degree degrees of transparency ε_b and ε_g. In principle, the degrees of transparency may as well be influenced by authorities and regulatory bodies. The Basel Committee on Banking Supervision in their progress report on resolution policies and frameworks notes that several countries, while strengthening safety nets for banks are also mandating for greater disclosure of balance sheet information to the public.[15] To better understand the implications of transparency for the optimal guarantee, and to determine its possible welfare gains, we resort to numerical experiments where the broad brushed parametrization is drawn from the recent Irish banking and sovereign debt crisis.

6.2.1. Parametrization

According to IMF (2011), the gross debt of Irish financial institutions totaled 664% of Irish GDP in 2011. Moreover, the refinancing needs of the Irish banks amounted to 25% of their total liabilities. This roughly equates to the Irish banks having to obtain funding in the order of 166% of GDP. In contrast, the Irish government faced financing needs of only 19.5% of GDP in 2011. This implies that the amount of maturing claims of Irish banks was approximately 8.5 times that of the Irish government, implying $N_b = 8.5$, where we continue to maintain our normalization of $N_g = 1$.

To further ensure that the dominance regions of the two games are well-defined, we now take $\eta_b = 5.6$, $\eta_g = 0.51$, $\theta_b^0 = 6$ and $\theta_g^0 = 9.5$. Consequently, the banking sector faces a considerable ex ante rollover problem, with expected liquidity being only around 35% of maturing claims. The government, in contrast, has expected liquidity almost five times that of its own maturing claims. The remaining payoff parameters are kept unchanged compared to the previous section.

Cost parameters ϕ_b and ϕ_g are interpreted as, respectively, the output losses due to a solo bank or sovereign crisis relative to the loss due to a dual crisis. Table 6 provides a brief overview over available empirical estimates of such losses. It can be seen that the output costs of a sovereign default ex banking crisis are around 10% (when no currency crisis occurs at the same time) up to around 50% (when also a currency crisis occurs) of GDP. The output losses due to a solo banking crisis are in the range of around 6% to 25% while the costs of a dual crisis are around 54% of GDP. For the exercises in this section we set (ϕ_b, ϕ_g) to $(0.05, 0.3)$, corresponding to output losses of a pure bank default of around 2.7% and costs of a pure sovereign crisis of around 16% of GDP, and alternatively to $(0.1, 0.2)$, implying costs of 5% and 10% of GDP respectively.

Source	Type of crisis	Duration	Output loss
Hoggarth et al. (2002)	Banking	3.2	6.3% (c)
	Twin (Banking and Currency)	4.2	29.9% (c)
Honohan and Klingebiel (2000)	Banking	3.5	12.5% (c)
Hutchison and Noy (2005)	Banking	3.3	10% (c)
	Twin (Banking and Currency)	3-4	13%-18% (c)
De Paoli et al. (2009)	Sovereign	4	2.5%
	Twin (Sovereign and Banking)	11	4.9%
	Twin (Sovereign and Currency)	8.1	6.2%
	Triple	12.5	17.1%
John H. Boyd (2005)	Banking	5.1	5.4%

Table 6: Costs of different types of crises. Output loss in percent of annual GDP or cumulated (c). Reported values are either average losses reported in the respective studies or indicate the range of losses found in these studies.

[15] See Basel Committee on Banking Supervision (2011).

In what follows, we measure the welfare gain (in percentage points of GDP) from introducing the guarantee as

$$\text{welfare} = K^0 - K^{opt}.$$

Moreover, in order to assess the impact of the guarantee on the likelihood of crises, we consider the differences

$$\Delta P_b \equiv P_b(\ell^{opt}) - P_b(0) \quad \text{and} \quad \Delta P_g \equiv P_g(\ell^{opt}) - P_g(0),$$

as well as

$$\Delta Q \equiv Q(\ell^{opt}) - Q(0) \quad \text{and} \quad \Delta q \equiv q(\ell^{opt}) - q(0).$$

6.2.2. Results

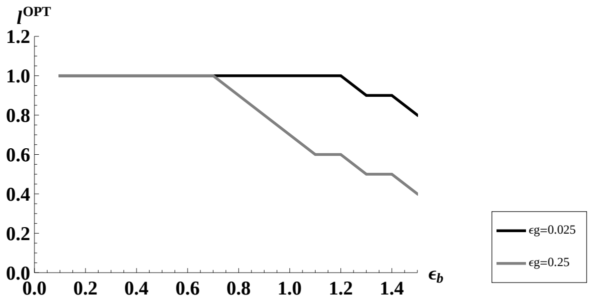
Figure 16 shows a first set of comparative statics exercises with respect to ε_b where $\phi_b = 0.05$ and $\phi_g = 0.3$. As the difference between the grey and the black line indicates, a lower transparency of the government reduces the optimal guarantee and, as shown in Panel 2, is associated with a sizeable welfare loss. A tenfold increase in ε_g reduces the welfare gain by around 30%. However, the welfare effects are generally rather low. In our numerical example the expected welfare gains are equivalent to at most 3% of GDP *over the course of the crisis*. This is negligibly small, given the large output losses presented in Table 6. Panels 3 to 6 show the probability differences ΔQ, Δq, ΔP_b and ΔP_g. As one would expect, the probability of a sovereign crisis rises. Moreover, it rises less than the reduction in the probability of a banking crisis, which in turn explains why probabilities q and Q are strongly decreasing. Higher bank balance sheet transparency is clearly enhancing the effect of the guarantee on the different crisis probabilities. Similarly, a less transparent government significantly dampens the effect of the guarantee on all probabilities.

Figure 17 shows the numerical results when $\phi_b = 0.1$ and $\phi_g = 0.2$. An important difference to the previous exercise is that a highly transparent government finds it now optimal to provide a maximal guarantee independent of the degree of bank transparency. Hence, even the though costs of a bank default are still below the costs of a sovereign crisis, the significantly smaller impact of the guarantee on P_g allows the government to provide full coverage of bank claims even if the bank is intransparent.

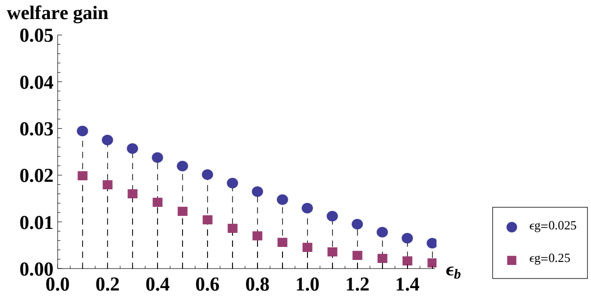
Finally, when the costs of a banking crisis exceed the costs of a sovereign crisis, the optimal guarantee provides 100%–coverage independent of both degrees of transparency. This can be seen from figure 6.2.2 where we have set $\phi_b = 0.2$ and $\phi_g = 0.1$. However, the welfare effects associated with bank and sovereign transparency are still significant.

A robust finding throughout these numerical exercises is that the increase in the government's default probability is, in absolute magnitude, significantly smaller than the reduction in the bank's default probability. This replicates the empirical behavior of CDS–spreads that we alluded to in the introduction (see Figure 4) and allows us to put forward an interpretation of this stylized fact. Recall that in our model, under a regime of full bank transparency, no guarantee payout will ever come due. This implies, as can be seen from the corresponding panels in Figures 16 and 17, that the sovereign's default probability remains almost unchanged, whereas the impact on the bank's default probability is large. The guarantee removes strategic uncertainty, thereby serving as a device to coordinate bank creditors on the efficient equilibrium. When the degree of bank transparency becomes smaller, the mass of bank creditors who may eventually claim the guarantee increases and, in case the bank defaults, the guarantee creates an actual cost burden for the government. As a result, the government's default probability begins to increase. The large decrease in CDS spreads across countries, that was observed right

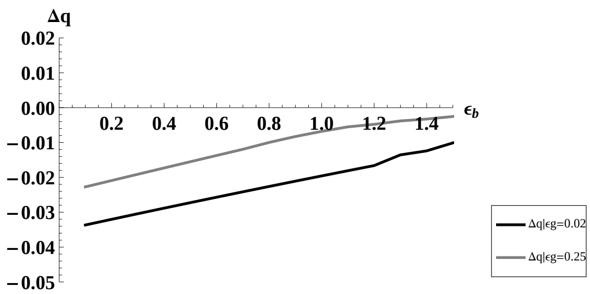
Figure 16: Comparative statics of ε_b with $\phi_b = 0.05$ and $\phi_g = 0.3$



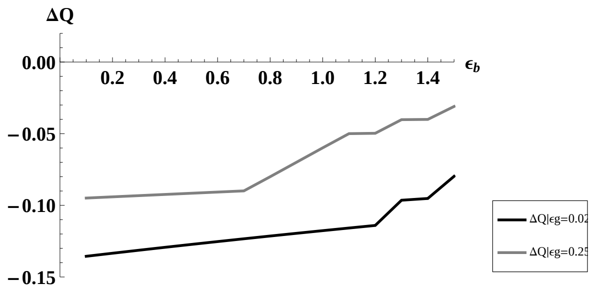
Panel 1: Optimal guarantee



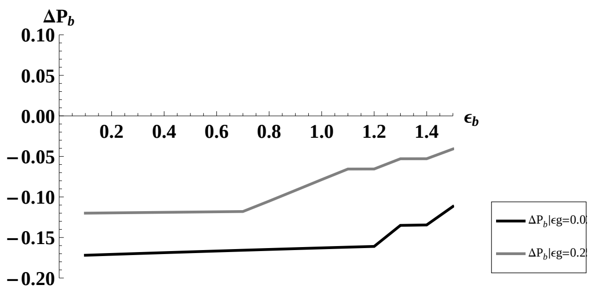
Panel 2: Welfare gain from introduction of guarantee, $K^0 - K^{opt}$



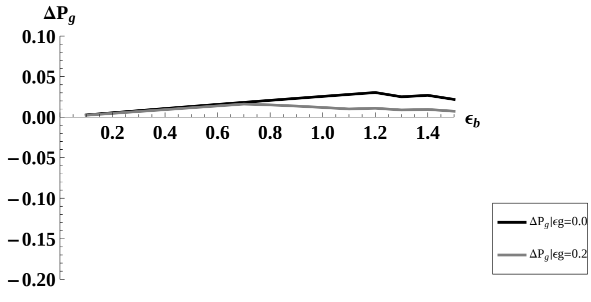
Panel 3: Change in probability of systemic crisis due to introduction of guarantee, Δq



Panel 4: Change in probability of crisis due to introduction of guarantee, ΔQ

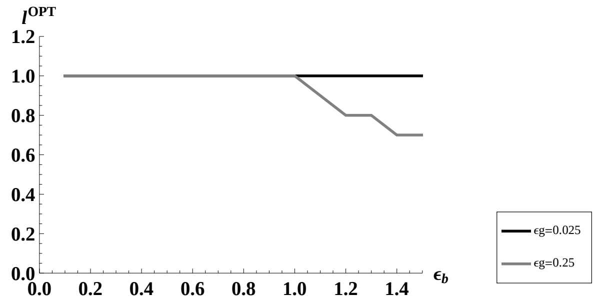


Panel 5: Change in probability of banking crisis due to introduction of guarantee, ΔP_b

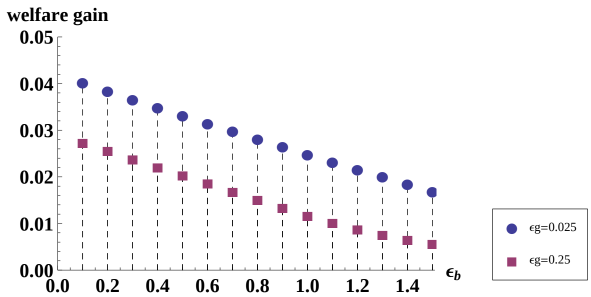


Panel 6: Change in probability of sovereign crisis, ΔP_g

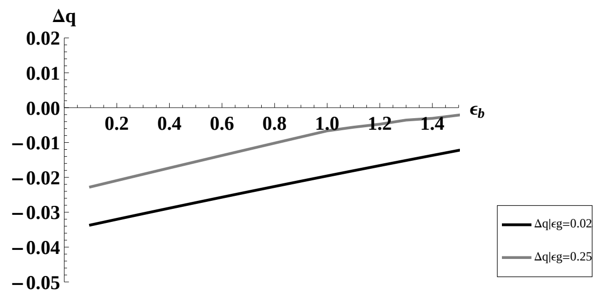
Figure 17: Comparative statics of ε_b with $\phi_b = 0.1$ and $\phi_g = 0.2$



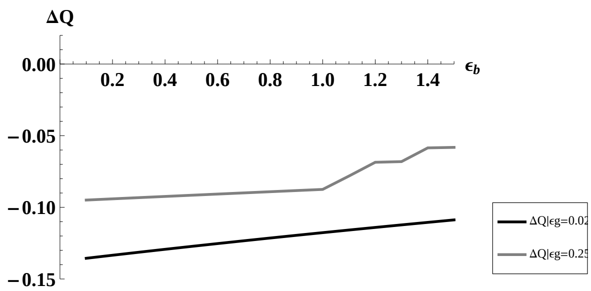
Panel 1: Optimal guarantee



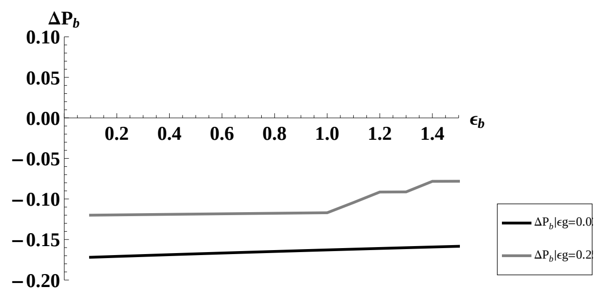
Panel 2: Welfare gain from introduction of guarantee, $K^0 - K^{opt}$



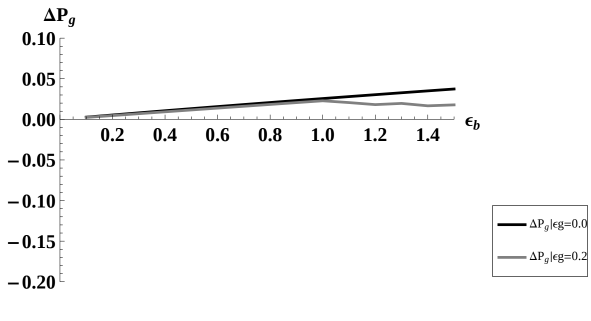
Panel 3: Change in probability of systemic crisis due to introduction of guarantee, Δq



Panel 4: Change in probability of crisis due to introduction of guarantee, ΔQ

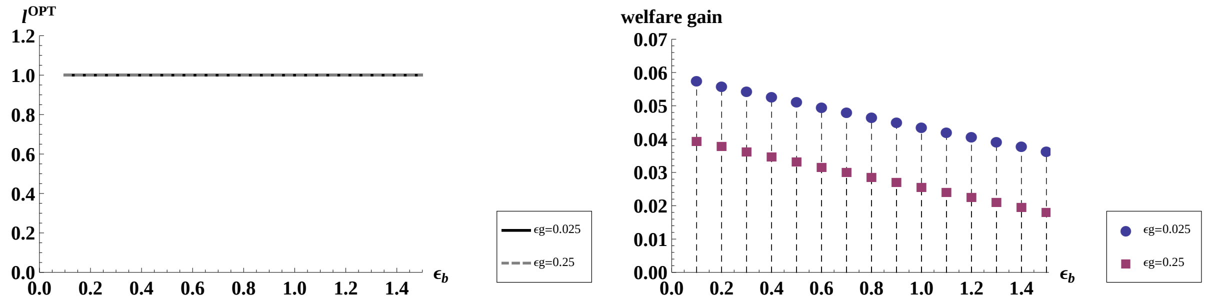


Panel 5: Change in probability of banking crisis due to introduction of guarantee, ΔP_b



Panel 6: Change in probability of sovereign crisis, ΔP_g

Figure 18: Comparative statics of ϵ_b with $\phi_b = 0.2$ and $\phi_g = 0.1$



Panel 1: Optimal guarantee

Panel 2: Welfare gain from introduction of guarantee, $K^0 - K^{opt}$

after the issuance of bank debt guarantees, may therefore mirror the removal of strategic uncertainty among bank creditors. However, sovereign CDS–spreads increased at the same time, which suggests that the corresponding banking sectors may not have operated under a regime of full transparency. Market participants in sovereign funding markets may have conjectured that the guarantees would create an actual cost for the sovereign and therefore withdrew funding.

7. Conclusion

In this paper we have analyzed the effects of a bank debt guarantee provided by the government and the role played by the degree of balance sheet transparency in making the guarantee costly. To examine this phenomenon, we used a stylized global games framework to address the following questions: (i) How does the introduction of a bank liability guarantee by a government affect the behavior of banking and sovereign creditors? (ii) How does the guarantee affect the likelihood of crises? (iii) What is the optimal guarantee that trades-off the expected costs associated with the different types of crises? and (iv) How do changes in the parameters governing fundamental uncertainty / transparency and liquidity impact on the optimal guarantee?

As the guarantee promise increases the sovereign's expected liabilities, sovereign creditors may lend to the government less often, thereby increasing the government's own likelihood of default. This in turn can jeopardize the effectiveness of the guarantee as bank creditors become less eager to rely on the guarantee when they expect that the government becomes unable to fund its promises.

In our model, it turned out that equation (11) is a necessary and sufficient condition for the guarantee to be effective in raising the incentives of bank creditors to roll over their loans. Moreover, our model provides a theoretical foundation for the empirically observed behavior of credit default spreads during the recent crisis across the different countries that issued bank debt guarantees. By resorting to a stylized measure for the expected costs of crises, we were able to characterize the optimal guarantee in the space of relevant cost parameters as a function of the key parameters that reflect bank's funding illiquidity and the degrees of transparency of bank and sovereign creditors respectively.

Our results also show a clear cut welfare improvement with greater transparency, which lowers fundamental uncertainty. This would suggest that in designing guarantee schemes, authorities can improve on their credibility by mandating greater disclosure on the part of the banks. These findings are in line with the new approaches being sought by several countries, as discussed in the Basel Committee for Banking Supervision (2011) report. Moreover, by improving on the government's own transparency, these gains can be further enhanced.

While reduced form, the model captures key strategic interactions across sovereign and bank creditors in the design of optimal guarantee schemes, that have previously been assumed exogenous. Such cautionary tales equally apply to the design of new regulations, where authorities focus on effects in partial, rather than general equilibrium models.

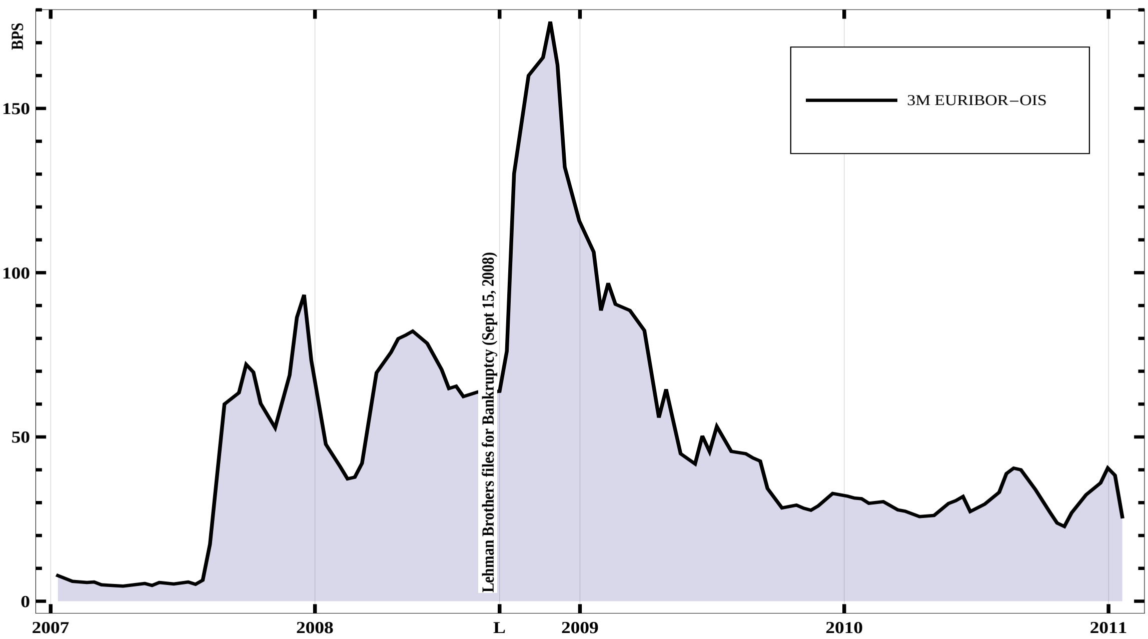


Figure 1: 3m-Euribor-OIS spread in basis points. Source: authors' calculation, data taken from Bloomberg.

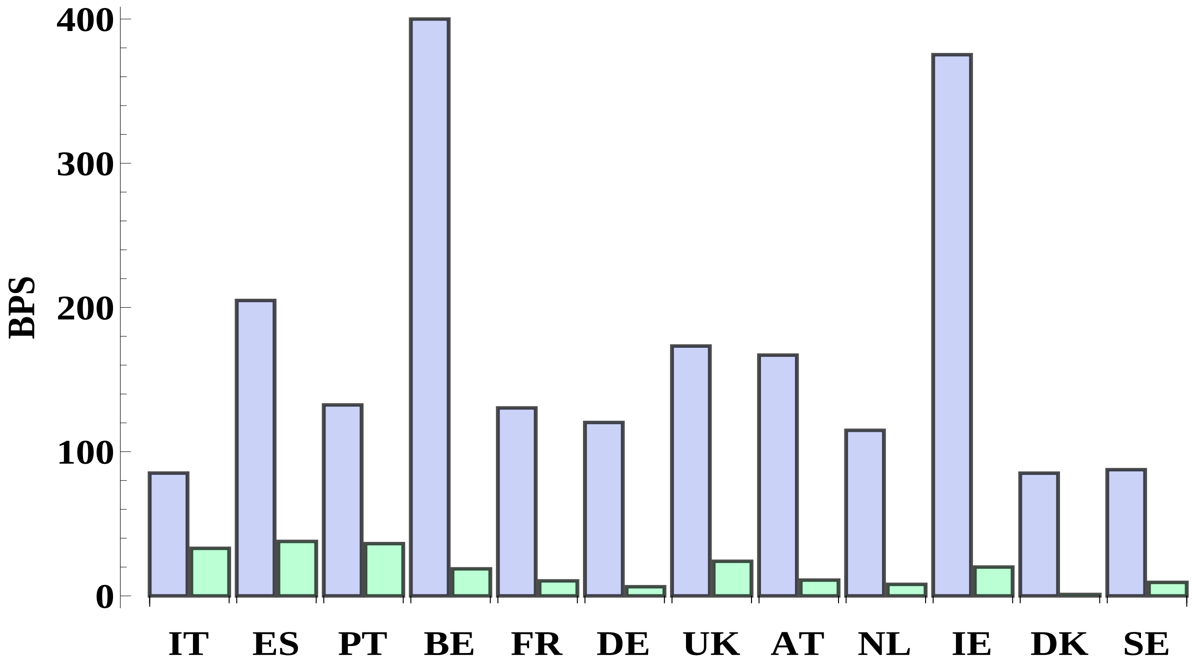


Figure 2: Change in CDS–spreads for banks and sovereigns between 1/1/2007 and 9/25/2008. Bank CDSs are unweighted averages of banks with headquarter in respective country. Figure taken from Acharya et al. (2011).

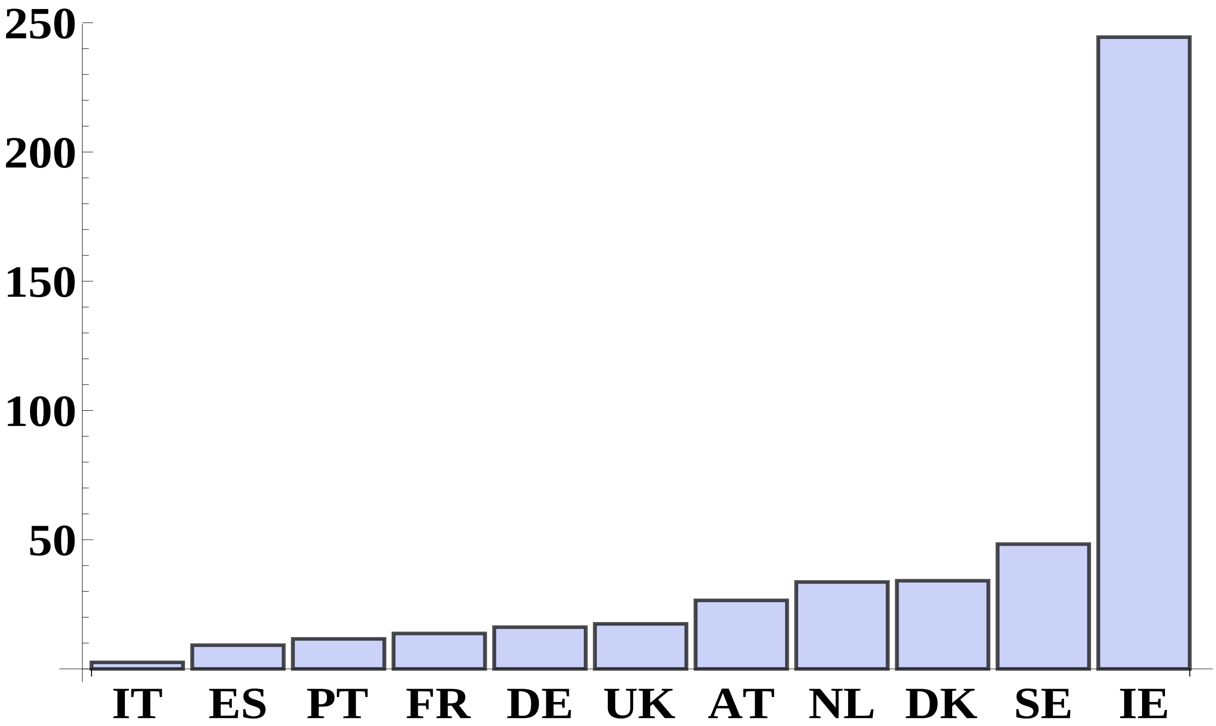


Figure 3: Guarantee sizes in % of GDP. Source: authors' calculation, data taken from OECD.

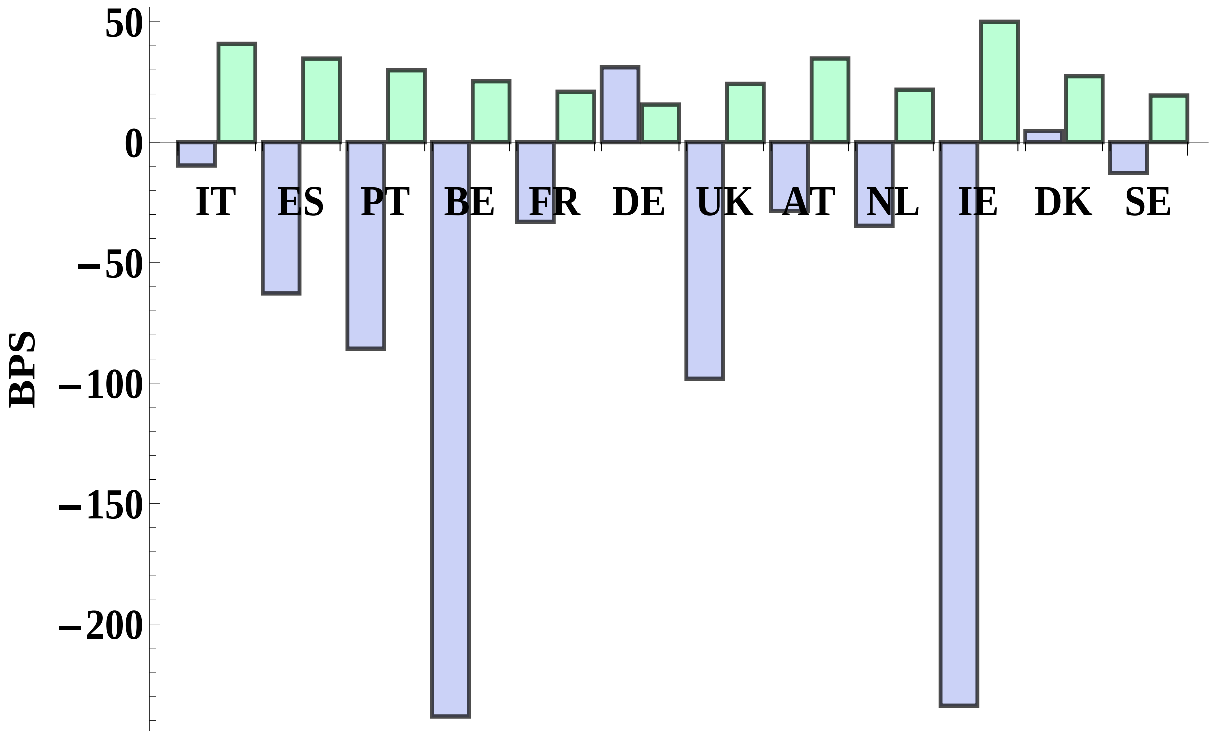


Figure 4: Change in CDS–spreads for banks and sovereigns between 9/26/2008 and 10/21/2008. Bank CDSs are unweighted averages of banks with headquarter in respective country. Figure taken from Acharya et al. (2011).

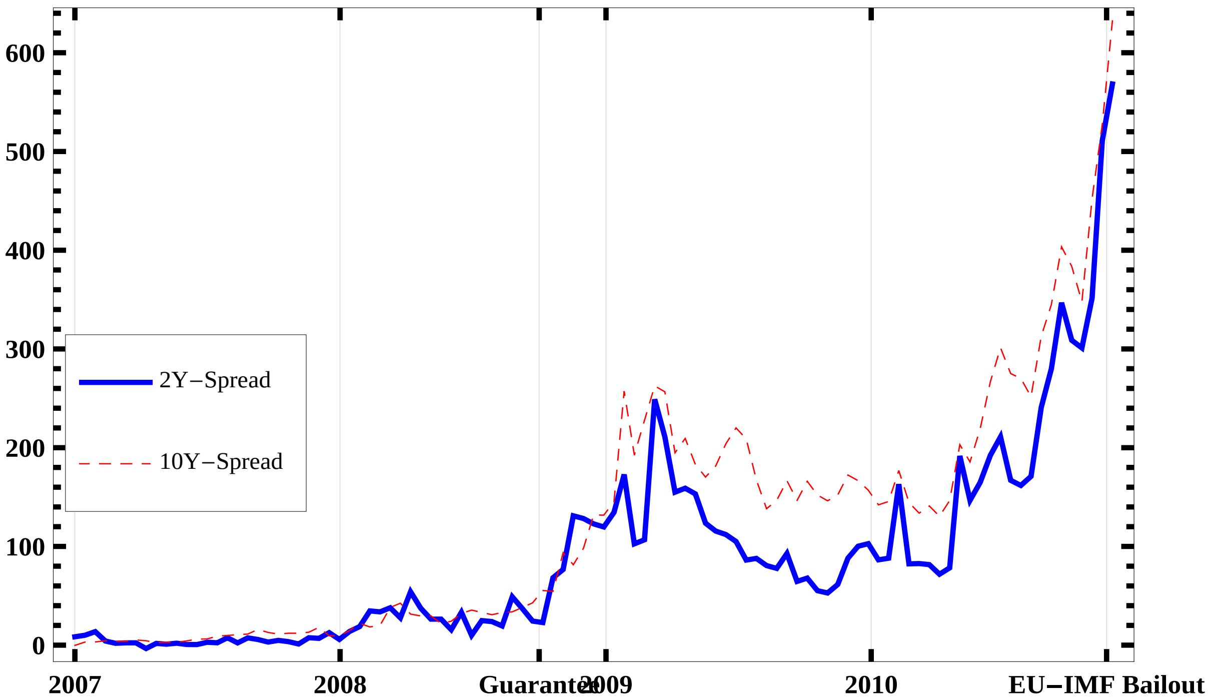


Figure 5: Irish spreads over German bund, in percent. Source: Bloomberg

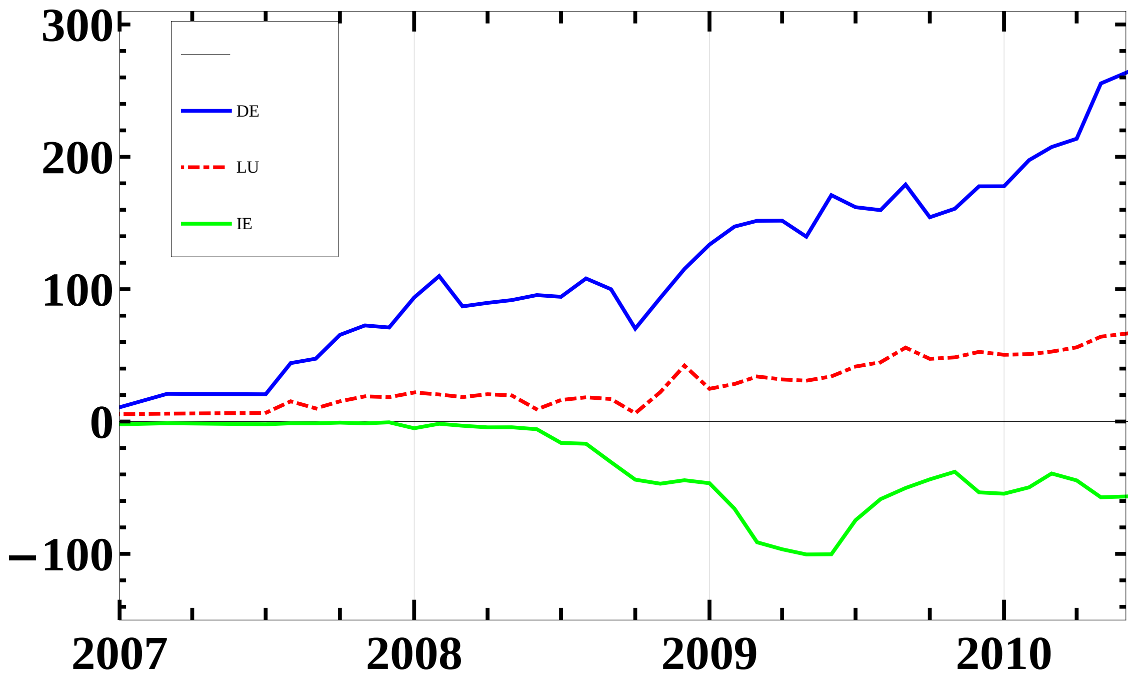


Figure 6: Net TARGET2 Liabilities of selected euro area national central banks against the Eurosystem in millions of euro. Negative values reflect a Target2-liability, positive numbers a Target2-asset. Source: University of Osnabrück, EconoCrisisMonitor

Country	Name of program	Issued	Tenor	Size	GDP	Size	GSD	Size
				(bn of domestic currency)	(bn of domestic currency)	(% of GDP)	(bn of domestic currency)	(% of GSD)
Austria	Interbank Market Support Act (Finanzmarktstabilisierungsgesetz)	Oct 2008	3-5 years	75	283	27%	181	42%
France	Société de Refinancement des Activités des Etablissements de Crédit	Oct 2008	5 years	265	1,931	14%	1,319	20%
Ireland	Financial Support Act 2008	Oct 2008	2 years	440	180	244%	80	553%
Ireland	Eligible Liabilities Guarantee Scheme 2009	Dec 2009	5 years	150	161	93%	105	143%
Italy	Italian Guarantee Scheme	Nov 2008	5 years	40	1,575	3%	1,667	2%
Netherlands	2008 Credit Guarantee Scheme	Oct 2008	5 years	200	594	34%	348	58%
Portugal	Portuguese State Guarantee Scheme 2008	Oct 2008	3 years	20	172	12%	123	16%
Spain	Spanish Guarantee Scheme	Dec2008	5 years	100	1,088	9%	437	23%
United Kingdom	2008 Credit Guarantee Scheme	Oct 2008	3 years	250	1,434	17%	752	33%

Table 1: Summary of guarantee schemes introduced in several developed economies following the collapse of Lehman Brothers. All monetary figures are provided in the country of origin's local currency. The column labeled 'Size' refers to the size of the guarantee; 'GDP' refers to the Gross Domestic Product; 'GSD' stands for Gross Sovereign Debt.

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Appendix

Proof of Proposition 1. Morris and Shin (2003) show that the model has a unique symmetric threshold equilibrium where creditors use the strategy around $\hat{x}_b$ and the bank defaults whenever $\theta_b < \hat{\theta}_b$. The creditor who observes $x_{n_b} = \hat{x}_b$ must therefore be indifferent between rolling over and foreclosing. Thus, the expected payoff difference between rolling over and foreclosing is given by

$$D_b \mathbf{Pr}\left(\theta_b > \hat{\theta}_b \middle| \hat{x}_b\right) + \ell \mathbf{Pr}\left(\theta_b \le \hat{\theta}_b \middle| \hat{x}_b\right) - C_b = 0, \tag{A21}$$

which, by using the assumed uniform distributions, can be written as

$$\frac{D_b - C_b}{D_b - \ell} = \frac{1}{2\varepsilon_b} \int_{\hat{x}_b - \varepsilon_b}^{\hat{\theta}_b} \mathrm{d}u. \tag{A22}$$

Due to the law of large numbers, $\lambda_b(\theta_b) = \mathbf{Pr}\left(x_{n_b} \le \hat{x}_b \middle| \theta_b\right) = \frac{\int_{\theta_b - \varepsilon_b}^{\hat{x}_b} \mathrm{d}u}{2\varepsilon_b}$, and by utilizing the failure condition,

$$\frac{1}{2\varepsilon_b} \int_{\hat{\theta}_b - \varepsilon_b}^{\hat{x}_b} \mathrm{d}u = \frac{\hat{\theta}_b}{N_b}. \tag{A23}$$

From Equation (A22),

$$1 - \frac{D_b - C_b}{D_b - \ell} = \frac{C_b - \ell}{D_b - \ell} = 1 - \frac{1}{2\varepsilon_b} \int_{\hat{x}_b - \varepsilon_b}^{\hat{\theta}_b} \mathrm{d}u = \frac{1}{2\varepsilon_b} \int_{\hat{\theta}_b - \varepsilon_b}^{\hat{x}_b} \mathrm{d}u,$$

and combining the latter with Equation (A23) gives Equation (2) in the text,

$$\frac{N_b(C_b - \ell)}{D_b - \ell} = \hat{\theta}_b.$$

Moreover, solving Equation (A23) for $\hat{x}_b$, gives Equation (1) in the text,

$$\frac{1}{2\varepsilon_b} \int_{\hat{\theta}_b - \varepsilon_b}^{\hat{x}_b} \mathrm{d}u = \frac{\hat{x}_b - \hat{\theta}_b + \varepsilon_b}{2\varepsilon_b} = \frac{\hat{\theta}_b}{N_b} \quad \Rightarrow \hat{x}_b = \hat{\theta}_b \left(1 + \frac{2\varepsilon_b}{N_b}\right) - \varepsilon_b.$$

□

Proof of Proposition 2. Without loss of generality, we can concentrate on symmetric strategies in both groups $i \in \{g, b\}$. We will show that for any symmetric strategy $s_i : x_{n_i} \mapsto \{0, 1\}$ played by group i, the only equilibrium in the rollover game between agents of group $j \neq i$ is a threshold equilibrium and that there are no other equilibria in non-threshold strategies. As a corollary to this, the simultaneous equilibrium in the game with a guarantee is an equilibrium where bank and government creditors play threshold strategies and there are no other equilibria. The proof is simplified considerably due to Assumption 1, which, since it implies that agents' signals are completely uninformative about the fundamental in the other game, allows us to treat the fundamental and the strategy in the respective other game as exogenously given. We begin with the rollover game played by bank creditors. Denote the fraction of bank creditors who withdraw by λ_b and suppose that government creditors play any symmetric strategy $s_g(x_{n_g})$. Given the government's liquidity θ_g, we can then write the fraction of government creditors who withdraw as $\int_{\theta_g - \varepsilon_g}^{\theta_g + \varepsilon_g} s(x_{n_g}) \mathrm{d}x_{n_g}$. The payoff differential between rolling over and withdrawing for a typical bank creditor is then given by

$$\pi^b(\theta_b, \lambda_b, \theta_g, s_g(\cdot)) = \begin{cases} D_b - C_b & \text{if } \lambda_b < \theta_b,\ \forall \theta_g \\ \ell - C_b & \text{if } \lambda_b > \theta_b,\ \int_{\theta_g - \varepsilon_g}^{\theta_g + \varepsilon_g} s_g(x_{n_g}) \mathrm{d}x_{n_g} < \theta_g - (1 - \lambda_b)\ell \\ -C_b & \text{if } \lambda_b > \theta_b,\ \int_{\theta_g - \varepsilon_g}^{\theta_g + \varepsilon_g} s_g(x_{n_g}) \mathrm{d}x_{n_g} > \theta_g - (1 - \lambda_b)\ell \end{cases} \tag{A24}$$

Lemma A8. *Bank creditors' payoff differential* (A24) *has the following properties.*

1. *Action single-crossing in λ_b: For any θ_b, there exists λ_b^* such that $\pi^b > 0$ for any $\lambda_b < \lambda^*$ and $\pi^b < 0$ for any $\lambda_b > \lambda_b^*$.*
2. *State monotonicity in θ_b: π^b is non-decreasing in θ_b.*
3. *Laplacian State Monotonicity: There exists a unique θ_b^* such that*

$$\int_0^1 \pi(\theta_b^*, \lambda_b, \theta_g, s_g(\cdot))\mathrm{d}\lambda_b = 0.$$

4. *Uniform Limit Dominance: There exist $\underline{\theta}_b$ and $\overline{\theta}_b$ such that $\pi^b < -\delta$ for $\theta_b < \underline{\theta}_b$ and $\pi^b > \delta$ for $\theta_b > \overline{\theta}_b$ for some $\delta > 0$.*

Moreover, the noise distribution satisfies

5. *Monotone Likelihood Property.*
6. *Finite expectations of signals.*

Proof of Lemma A8.

1. Note that $D_b - C_b > 0 > \ell - C_b > -C_b$. Action single-crossing then follows by setting $\lambda_b^* = \theta_b$.
2. Can be inferred immediately from Equation (A24).
3. We can write the integral $\int_0^1 \pi(\theta_b, \lambda_b, \theta_g, s_g(\cdot))\mathrm{d}\lambda_b$ as follows

$$(D_b - C_b)\int_0^{\theta_b} \mathrm{d}\lambda_b - C_b \int_{\theta_b}^{\min\left\{1, 1-\ell^{-1}\left(\int_{\theta_g-\varepsilon_g}^{\theta_g+\varepsilon_g} s_g(x_{n_g})\mathrm{d}x_{n_g} - \theta_g\right)\right\}} \mathrm{d}\lambda_b$$
$$+ (\ell - C_b)\int_{\min\left\{1, 1-\ell^{-1}\left(\int_{\theta_g-\varepsilon_g}^{\theta_g+\varepsilon_g} s_g(x_{n_g})\mathrm{d}x_{n_g} - \theta_g\right)\right\}}^{1} \mathrm{d}\lambda_b.$$

   As the left hand side is negative for $\theta_b = 0$, positive for $\theta_b = 1$ and otherwise strictly increasing in θ_b, there exists a unique θ_b^* such that $\int_0^1 \pi(\theta_b^*, \lambda_b, \theta_g, s_g(\cdot))\mathrm{d}\lambda_b = 0$.

4. The claim follows by setting $\underline{\theta}_b = 0$, $\overline{\theta}_b = 1$ and $\delta = \min\{C_b - \ell, D_b - C_b\}$.
5. Uniform noise satisfies MLRP, see (Shao, 2003, p. 399).
6. This follows immediately from the assumption of a uniform distribution with bounded support.

□

Lemma A9. *For any strategy $s_g(\cdot)$ played by government creditors, the rollover game between bank creditors has a unique threshold equilibrium.*

Proof of Lemma A9. Since the payoff differential satisfies properties (1) to (6) in Lemma A8, the claim follows from (Morris and Shin, 2003, Lemma 2.3). □

Lemma A10. *There are no other equilibria in non-threshold strategies.*

Proof of Lemma A10. Since noise terms are uniformly distributed and the payoff differential satisfies action single-crossing , the claim follows immediately from the proof to (Goldstein and Pauzner, 2005, Theorem 1). □

Next, we turn to the rollover game played by government creditors. Suppose that bank creditors play any strategy $s_b(x_{n_b})$. Given any θ_b, we can then write the fraction of bank creditors who withdraw as $\int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b}$. The payoff differential between rolling over and withdrawing for a typical government creditor is then given by

$$\pi^g(\theta_g, \lambda_g, \theta_b, s_b(\cdot)) = \begin{cases} D_g - C_g & \text{if } \lambda_g < \theta_g,\ \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b} < \theta_b \\ D_g - C_g & \text{if } \lambda_g < \theta_g - (1 - \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b})\ell,\ \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b} > \theta_b \\ -C_g & \text{if } \lambda_g > \theta_g - (1 - \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b})\ell,\ \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b} > \theta_b \\ -C_g & \text{if } \lambda_g > \theta_g,\ \forall \theta_b \end{cases} \quad \text{(A25)}$$

Lemma A11. *Government creditors' payoff differential* (A25) *has the following properties.*

1. *Action monotonicity in λ_g: π^g is non-increasing in λ_g.*
2. *State monotonicity in θ_g: π^g is non-decreasing in θ_g.*
3. *Laplacian State Monotonicity: There exists a unique θ_g^* such that*

$$\int_0^1 \pi(\theta_g^*, \lambda_g, \theta_b, s_b(\cdot))\mathrm{d}\lambda_g = 0.$$

4. *Uniform Limit Dominance: There exist $\underline{\theta}_g$ and $\overline{\theta}_g$ such that $\pi_g < -\delta$ for $\theta_g < \underline{\theta}_g$ and $\pi_g > \delta$ for $\theta_g > \overline{\theta}_g$ for some $\delta > 0$.*

Moreover, the noise distribution satisfies

5. *Monotone Likelihood Property.*
6. *Finite expectations of signals.*

Proof of Lemma A11.

1. Suppose $\theta_b > \lambda_b$, then, since $D_g - C_g > -C_g$, π^g is clearly non-increasing in λ_g for any θ_g. Similarly for the case where $\theta_b < \lambda_b$.
2. Suppose $\theta_b > \lambda_b$, then π^g is increasing in θ_g for any λ_g. Similarly for $\theta_b < \lambda_b$.
3. If $\theta_b > \lambda_b$, then $\theta_g^* = C_g/D_g$. If $\theta_b < \lambda_b$, then $\theta_g^* = C_g/D_g + (1-\lambda_b)\ell$.
4. This follows by setting $\overline{\theta}_g = 1 + \ell$ and $\underline{\theta}_g = 0$ and $\delta = D_g - C_g$.
5. Uniform noise satisfies MLRP, see (Shao, 2003, p. 399).
6. This follows immediately from the assumption of a uniform distribution with bounded support.

□

Lemma A12. *For any strategy $s_b(\cdot)$ played by bank creditors, the rollover game between government creditors has a unique equilibrium in threshold strategies. Moreover, there are no equilibria in non-threshold strategies.*

Proof of Lemma A12. Since the payoff differential satisfies properties (1) to (6) in Lemma A11, the claim follows immediately from (Morris and Shin, 2003, Proposition 2.2). □

Proposition 2 follows since by Lemmas A9 and A10, bank creditors will respond to any strategy s_g by using a threshold strategy and since by Lemma A12, government creditors will respond to any strategy s_b by using a threshold strategy. Hence, the only equilibrium is a threshold equilibrium. This completes the proof. □

Proof of Lemma 1. Fix $\hat{x}_g$. Due to the existence of dominance regions there exist $\underline{\hat{x}}_b$ and $\overline{\hat{x}}_b$ such that $\bar{\pi}^b(\hat{x}_b, \hat{x}_g) < 0$ for any $\hat{x}_b < \underline{\hat{x}}_b$, and $\bar{\pi}^b(\hat{x}_b, \hat{x}_g) > 0$ for any $\hat{x}_b > \overline{\hat{x}}_b$. Similarly, fix $\hat{x}_b$, then there exist $\underline{\hat{x}}_g$ and $\overline{\hat{x}}_g$ such that $\bar{\pi}^g(\hat{x}_g, \hat{x}_b) < 0$ for any $\hat{x}_g < \underline{\hat{x}}_g$, and $\bar{\pi}^g(\hat{x}_g, \hat{x}_b) > 0$ for any $\hat{x}_g > \overline{\hat{x}}_g$. And since $\pi^b(\cdot)$ and $\pi^g(\cdot)$ are continuous they both cross the x-axis at least once.

To show that the crossing points are indeed threshold equilibria, we must show that $\pi^i(\cdot) > 0$ if and only if $x_{n_i} > \hat{x}_i$ and $\pi^i(\cdot) < 0$ if and only if $x_{n_i} < \hat{x}_i$ for $i \in \{b, g\}$. Fix the crossing point $\hat{x}_i$. Observe that increasing (decreasing) x_{n_i} above (below) $\hat{x}_i$ increases (decreases) the part of the payoff differential that is multiplied by D_i (for $i = b$, it also decreases (increases) the part multiplied by ℓ). Since for $x_{n_i} = \hat{x}_i$, the positive and the negative part of the payoff differential exactly offset each other, we can conclude that $\pi^i(\cdot) > 0$ if and only if $x_{n_i} > \hat{x}_i$ and conversely for $x_{n_i} < \hat{x}_i$ (for $i = b$, this follows because $D_b > \ell$). This establishes the existence of at least one threshold equilibrium in each game.

In order to show that there is exactly one threshold equilibrium in each game, it suffices to show that $\bar{\pi}^b(\hat{x}_b, \hat{x}_g)$ is strictly increasing in $\hat{x}_b$ and $\bar{\pi}^g(\hat{x}_g, \hat{x}_b)$ is strictly increasing in $\hat{x}_g$.

From Equation (**??**) in the text follows

$$\frac{\partial \hat{\theta}_g}{\partial \hat{x}_g} = \frac{1}{1 + 2\varepsilon_g}$$

for all θ_b. Hence,

$$\frac{\partial \bar{\pi}^g(\hat{x}_g, \hat{x}_b)}{\partial \hat{x}_g} = \frac{D_g}{1 + 2\varepsilon_g} > 0. \tag{A26}$$

Next, we turn to the derivative of $\bar{\pi}^b(\cdot)$ with respect to $\hat{x}_b$. Observe first that $\hat{\theta}'_b(\hat{x}_b) = N_b(N_b + 2\varepsilon_b)^{-1}$ and $(1-\hat{\theta}'_b(\hat{x}_b)) = 2\varepsilon_b(N_b + 2\varepsilon_b)^{-1}$. Moreover, if $\theta_b < \hat{\theta}_b$, then $\partial\hat{\theta}_g/\partial\hat{x}_b = -\ell N_b \varepsilon_g(\varepsilon_b(1+2\varepsilon_g))^{-1}$. Let $\hat{\theta}_g^T := (\hat{x}_g + \varepsilon_g)(1+2\varepsilon_g)^{-1}$, so that we can write $\hat{\theta}_g(\hat{x}_g, \hat{x}_b, \hat{\theta}_b) = \hat{\theta}_g^T + \frac{\ell N_b 2\varepsilon_g}{1+2\varepsilon_g}\frac{N_b - \hat{x}_b + \varepsilon_b}{N_b + 2\varepsilon_b}$, while $\hat{\theta}_g(\hat{x}_g, \hat{x}_b, \hat{x}_b - \varepsilon_b) = \hat{\theta}_g^T$. With these definitions in mind, we can write the derivative of $\bar{\pi}^b(\cdot)$ with respect to $\hat{x}_b$ as follows

$$\begin{aligned}\frac{\partial\bar{\pi}^b\left(\hat{x}_b,\hat{x}_g\right)}{\partial\hat{x}_b} &= \frac{D_b}{2\varepsilon_b}\left(1-\hat{\theta}'_b(\hat{x}_b)\right) + \frac{\ell}{2\varepsilon_b}\left(\frac{\hat{\theta}'_b(\hat{x}_b)}{\sigma_g}\int_{\hat{\theta}_g(\hat{x}_g,\hat{x}_b,\hat{\theta}_b)}^{\tilde{\sigma}_g}\mathrm{d}v - \frac{1}{\sigma_g}\int_{\hat{\theta}_g(\hat{x}_g,\hat{x}_b,\hat{x}_b-\varepsilon_b)}^{\tilde{\sigma}_g}\mathrm{d}v - \int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{\frac{\partial\hat{\theta}_g(\cdot)}{\partial\hat{x}_b}}{\sigma_g}\mathrm{d}u\right)\\
&= \frac{D_b}{N_b+2\varepsilon_b} + \frac{\ell}{2\varepsilon_b\sigma_g}\left(\frac{N_b}{N_b+2\varepsilon_b}\left(\theta_g^0+\eta_g-\hat{\theta}_g(\hat{\theta}_b)\right) - \left(\theta_g^0+\eta_g-\hat{\theta}_g^T\right) + \left(\frac{\ell N_b 2\varepsilon_g}{1+2\varepsilon_g}\frac{(N_b-\hat{x}_b+\varepsilon_b)}{N_b+2\varepsilon_b}\right)\right)\\
&= ((N_b+2\varepsilon_b)\sigma_g)^{-1}\left[\sigma_g D_b - \ell(\tilde{\sigma}_g - \hat{\theta}_g^T) + \frac{\ell}{2\varepsilon_b}\left(\frac{\ell N_b 2\varepsilon_g}{1+2\varepsilon_g}\left(1-\frac{N_b}{N_b+2\varepsilon_b}\right)(N_b-\hat{x}_b+\varepsilon_b)\right)\right]\end{aligned}$$

Now note that, $\sigma_g D_b - \ell(\tilde{\sigma}_g - \hat{\theta}_g^T) = \sigma_g\left(D_b - \ell\frac{(\tilde{\sigma}_g-\hat{\theta}_g^T)}{\sigma_g}\right) > 0$ since $D_b > \ell$ and $\frac{(\tilde{\sigma}_g-\hat{\theta}_g^T)}{\sigma_g} \le 1$ because it is a probability. Further, note that $N_b + \varepsilon_b - \hat{x}_b \ge 0$ because the existence of an upper dominance region implies that $\hat{x}_b$ is bounded above by $N_b + \varepsilon_b$. Thus, $\frac{\partial\bar{\pi}^b(\hat{x}_b,\hat{x}_g)}{\partial\hat{x}_b} > 0$.

where we used that, due to the existence of dominance regions, $\hat{x}_g \ge \varepsilon_b$.

Note further that it is straightforward to show that $\partial\pi^b/\partial\hat{x}_g < 0$. To eventually establish the Lemma, apply the implicit function theorem separately to equations $\pi^b(\hat{x}_b, \hat{x}_g, \hat{x}_b) = 0$ and $\pi^g(\hat{x}_g, \hat{x}_b, \hat{x}_g) = 0$. This yields $\hat{x}'_b(\hat{x}_g) = -\frac{\partial\pi^b/\partial\hat{x}_g}{\partial\pi^b/\partial\hat{x}_b} > 0$ and $\hat{x}'_g(\hat{x}_b) = -\frac{\partial\pi^g/\partial\hat{x}_b}{\partial\pi^g/\partial\hat{x}_g} < 0$. □

Proof of Lemma 2. We have shown that $\partial\bar{\pi}^b/\partial\hat{x}_b > 0$. Moreover, we have

$$\frac{\partial\pi^b}{\partial\hat{x}_g} = -\frac{\ell}{2\varepsilon_b}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{1}{\sigma_g}\mathrm{d}u < 0. \tag{A27}$$

The function $\hat{x}_b = f_b(\hat{x}_g)$ is implicitly defined by $\bar{\pi}^b(\hat{x}_b, \hat{x}_g) = 0$. Application of the implicit function theorem yields $f'_b(\hat{x}_g) = -\frac{\partial\bar{\pi}^b/\partial\hat{x}_g}{\partial\bar{\pi}^b/\partial\hat{x}_b} > 0$.

The function $\hat{x}_g = f_g(\hat{x}_b)$ is implicitly defined by $\bar{\pi}^g(\hat{x}_g, \hat{x}_b) = 0$. From the implicit function theorem follows $f'_g(\hat{x}_b) = -\frac{\partial\pi^g/\partial\hat{x}_b}{\partial\pi^g/\partial\hat{x}_g}$. Since $\partial\bar{\pi}^g/\partial\hat{x}_g > 0$, the sign of $f'_g(\hat{x}_b)$ is determined by the sign of $-\partial\pi^g/\partial\hat{x}_b$. Consider the derivative of π^g with respect to $\hat{x}_b$. We can write π^g as

$$\frac{D_g(\hat{x}_g+\varepsilon_g)}{1+2\varepsilon_g} - \frac{D_g\ell N_b}{2\varepsilon_b\sigma_b(1+2\varepsilon_g)}\int_{-\eta_b}^{\hat{\theta}_b(\hat{x}_b)}(u+\varepsilon_b-\hat{x}_b)\mathrm{d}u.$$

Then, the derivative is given by

$$\begin{aligned}\frac{\partial\pi^g}{\partial\hat{x}_b} &= -\frac{D_g\ell N_b}{2\varepsilon_b\sigma_b(1+2\varepsilon_g)}\frac{\partial}{\partial\hat{x}_b}\left(\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)}(u+\varepsilon_b-\hat{x}_b)\mathrm{d}u\right)\\
&\propto -\frac{\partial}{\partial\hat{x}_b}\left(\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)}(u+\varepsilon_b-\hat{x}_b)\mathrm{d}u\right)\\
&= (\hat{\theta}_b+\varepsilon_b-\hat{x}_b)\left(1-\frac{\partial\hat{\theta}_b}{\partial\hat{x}_b}\right) > 0,\end{aligned}$$

since $\frac{\partial\hat{\theta}_b}{\partial\hat{x}_b} = \frac{N_b}{N_b+2\varepsilon_b} < 1$.

□

Lemma A13. *The signs of the derivatives of the critical signals $\hat{x}_b$ and $\hat{x}_g$ with respect to parameters $\{\ell, N_b, \theta_b^0, \theta_g^0\}$ are given by*

$$\begin{aligned}&\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\ell} > 0 \quad \textit{and} \quad \frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\ell} \lesseqgtr 0;\\
&\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}N_b} > 0, \quad \textit{and} \quad \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}N_b} > 0;\\
&\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\theta_b^0} < 0 \quad \textit{and} \quad \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\theta_b^0} < 0;\\
&\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\theta_g^0} < 0 \quad \textit{and} \quad \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\theta_g^0} > 0.\end{aligned}$$

Proof of Lemma A13. Let $\xi = (\ell, N_b, \theta_b^0, \theta_g^0)$ with typical element ξ_k. The total effects $\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\xi_k}$ and $\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\xi_k}$ can be found by applying the implicit function theorem to the set of equations

$$\bar{\pi}^g(\hat{x}_g, \hat{x}_b, \xi) = 0$$
$$\bar{\pi}^b(\hat{x}_b, \hat{x}_g, \xi) = 0$$

The Jacobian of this system is given by

$$\mathbf{J} = \begin{pmatrix} \frac{\partial \pi^b}{\partial \hat{x}_b} & \frac{\partial \pi^b}{\partial \hat{x}_g} \\ \frac{\partial \pi^g}{\partial \hat{x}_b} & \frac{\partial \pi^g}{\partial \hat{x}_g} \end{pmatrix} = \begin{pmatrix} (+) & (-) \\ (+) & (+) \end{pmatrix},$$

and thus its determinant is positive, $|\mathbf{J}| > 0$.

The total effects can be computed as

$$\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\xi_k} = \frac{\begin{vmatrix} -\frac{\partial \bar{\pi}^b}{\partial \xi_k} & \frac{\partial \bar{\pi}^b}{\partial \hat{x}_g} \\ -\frac{\partial \bar{\pi}^g}{\partial \xi_k} & \frac{\partial \bar{\pi}^g}{\partial \hat{x}_g} \end{vmatrix}}{|\mathbf{J}|} = \frac{-\frac{\partial \bar{\pi}^b}{\partial \xi_k}\frac{\partial \bar{\pi}^g}{\partial \hat{x}_g} + \frac{\partial \bar{\pi}^b}{\partial \hat{x}_g}\frac{\partial \bar{\pi}^g}{\partial \xi_k}}{|\mathbf{J}|}. \tag{A28}$$

and

$$\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\xi_k} = \frac{\begin{vmatrix} \frac{\partial \bar{\pi}^b}{\partial \hat{x}_b} & -\frac{\partial \bar{\pi}^b}{\partial \xi_k} \\ \frac{\partial \bar{\pi}^b}{\partial \hat{x}_g} & -\frac{\partial \bar{\pi}^g}{\partial \xi_k} \end{vmatrix}}{|\mathbf{J}|} = \frac{-\frac{\partial \bar{\pi}^g}{\partial \xi_k}\frac{\partial \bar{\pi}^b}{\partial \hat{x}_b} + \frac{\partial \bar{\pi}^g}{\partial \hat{x}_g}\frac{\partial \bar{\pi}^b}{\partial \xi_k}}{|\mathbf{J}|}. \tag{A29}$$

It follows that

$$\begin{aligned}
\frac{\partial \bar{\pi}^b}{\partial \xi_k} \times \frac{\partial \bar{\pi}^g}{\partial \xi_k} > 0 &\Rightarrow \frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\xi_k} > 0, \\
\frac{\partial \bar{\pi}^b}{\partial \xi_k} \times \frac{\partial \bar{\pi}^g}{\partial \xi_k} < 0 &\Rightarrow \frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\xi_k} \gtrless 0, \\
\frac{\partial \bar{\pi}^b}{\partial \xi_k} \times \frac{\partial \bar{\pi}^g}{\partial \xi_k} < 0 &\Rightarrow \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\xi_k} > 0, \\
\frac{\partial \bar{\pi}^b}{\partial \xi_k} \times \frac{\partial \bar{\pi}^g}{\partial \xi_k} > 0 &\Rightarrow \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\xi_k} \gtrless 0.
\end{aligned} \tag{A30}$$

The partial derivatives with respect to ℓ are given by

$$\begin{aligned}
\frac{\partial \bar{\pi}^b}{\partial \ell} &= \frac{1}{2\varepsilon_b}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b} \frac{1}{\sigma_g}\int_{\hat{\theta}_g(u)}^{\tilde{\sigma}_g} \mathrm{d}v\,\mathrm{d}u - \frac{\ell}{2\varepsilon_b}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b} \frac{\varepsilon_g N_b}{\varepsilon_b(1+2\varepsilon_g)}\frac{(u+\varepsilon_b-\hat{x}_b)}{\sigma_g}\mathrm{d}u \\
&= \frac{1}{2\varepsilon_b\sigma_g}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\left[\int_{\hat{\theta}_g(u)}^{\tilde{\sigma}_g}\mathrm{d}v - \frac{\varepsilon_g \ell N_b}{\varepsilon_b(1+2\varepsilon_g)}(u+\varepsilon_b-\hat{x}_b)\right]\mathrm{d}u \\
&= \frac{1}{2\varepsilon_b\sigma_g}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\left[\tilde{\sigma}_g - \hat{\theta}_g^T - \frac{2\varepsilon_g \ell N_b}{\varepsilon_b(1+2\varepsilon_g)}(u+\varepsilon_b-\hat{x}_b)\right]\mathrm{d}u \\
&= \frac{\hat{\theta}_b - \hat{x}_b + \varepsilon_b}{2\varepsilon_b}\left\{\left(\frac{\tilde{\sigma}_g - \hat{\theta}_g^T}{\sigma_g}\right) + \frac{2\ell\varepsilon_g N_b(\hat{x}_b-\varepsilon_b)}{\sigma_g\varepsilon_b(1+2\varepsilon_g)} - \frac{2\ell\varepsilon_g N_b(\hat{\theta}_b+\hat{x}_b-\varepsilon_b)}{2\sigma_g\varepsilon_b(1+2\varepsilon_g)}\right\} \\
&= \frac{\hat{\theta}_b - \hat{x}_b + \varepsilon_b}{2\varepsilon_b}\left\{\frac{\tilde{\sigma}_g - \hat{\theta}_g^T}{\sigma_g} + \frac{\ell\varepsilon_g N_b(\hat{x}_b-\varepsilon_b-\hat{\theta}_b)}{\sigma_g\varepsilon_b(1+2\varepsilon_g)}\right\} \\
&= \frac{\hat{\theta}_b - \hat{x}_b + \varepsilon_b}{2\varepsilon_b}\left\{\frac{\tilde{\sigma}_g - \hat{\theta}_g(\hat{\theta}_b)}{\sigma_g}\right\} \geq 0,
\end{aligned} \tag{A31}$$

where we have used the abbreviation $\hat{\theta}_g(u) := \hat{\theta}_g(\hat{x}_g, \hat{x}_b, u)$.

Furthermore,

$$\begin{aligned}
\frac{\partial \bar{\pi}^g}{\partial \ell} &= \frac{-D_g}{2\varepsilon_g\sigma_b}\int_{-\eta_b}^{\hat{\theta}_b}\frac{\varepsilon_g N_b(u+\varepsilon_b-\hat{x}_b)}{\varepsilon_b(1+2\varepsilon_g)}\mathrm{d}u \\
&= \frac{-D_g N_b}{\sigma_b(1+2\varepsilon_g)}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{u+\varepsilon_g-\hat{x}_b}{2\varepsilon_b}\mathrm{d}u < 0.
\end{aligned} \tag{A32}$$

Applying conditions (A30), it follows immediately from equations (**??**) and (**??**) that

$$\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\ell} > 0 \quad \text{and} \quad \frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\ell} \lesseqgtr 0.$$

Condition (11) in the text can be derived by explicitly calculating

$$-\frac{\partial \bar{\pi}^b}{\partial \ell}\frac{\partial \bar{\pi}^g}{\partial \hat{x}_g} + \frac{\partial \bar{\pi}^b}{\partial \hat{x}_g}\frac{\partial \bar{\pi}^g}{\partial \ell}.$$

Using equations (A26), (A27), (A31) and (A32), we obtain

$$-\frac{D_g}{1+2\varepsilon_g}\left(\frac{\hat{\theta}_b - \hat{x}_b + \varepsilon_b}{2\varepsilon_b}\right)\left\{\frac{\tilde{\sigma}_g - \hat{\theta}_g(\hat{\theta}_b)}{\sigma_g}\right\} + \left(\frac{\hat{\theta}_b - \hat{x}_b + \varepsilon_b}{2\varepsilon_b}\right)\frac{D_g \ell N_b}{\sigma_g \sigma_b (1+2\varepsilon_g)}\int_{\hat{x}_b - \varepsilon_b}^{\hat{\theta}_b} \frac{u + \varepsilon_b - \hat{x}_b}{2\varepsilon_b}\mathrm{d}u,$$

which is negative if and only if

$$\tilde{\sigma}_g - \hat{\theta}^*_g(\hat{\theta}^*_b) > \frac{\ell N_b}{\sigma_b}\int_{\hat{x}_b - \varepsilon_b}^{\hat{\theta}_b} \frac{u + \varepsilon_b - \hat{x}_b}{2\varepsilon_b}\mathrm{d}u,$$

where the right-hand side are the ex ante expected guarantee payments $L^e(\hat{\theta}^*_b, \hat{x}^*_b, \ell)$. Subtracting $\hat{\theta}^*_g(\hat{\theta}^*_b)$ from both sides of the equation, dividing through by $\tilde{\sigma}_g - \hat{\theta}^*_g(\hat{\theta}^*_b)$ and rearranging yields condition (11) in the text.

The derivatives with respect to N_b are given by

$$\frac{\partial \bar{\pi}^b}{\partial N_b} = \frac{1}{(N_b + 2\varepsilon_b)^2}\left[\left(-D_b + \ell\frac{\tilde{\sigma}_g - \hat{\theta}_g(\hat{\theta}_b)}{\sigma_g}\right) - \frac{\ell^2}{\sigma_g(1+2\varepsilon_g)}\frac{(N_b + \varepsilon_b - \hat{x}_b)}{(N_b + 2\varepsilon_b)}\left((N_b + 2\varepsilon_b)^2 + N_b(N_b + 4\varepsilon_b)\right)\right] < 0 \tag{A33}$$

and

$$\frac{\partial \bar{\pi}^g}{\partial N_b} = -\frac{D_g \ell}{2\varepsilon_b(1+2\varepsilon_g)\sigma_b}\left[\int_{-\eta_b}^{\hat{\theta}_b}(u + \varepsilon_b - \hat{x}_b)\mathrm{d}u + \frac{2\varepsilon_b(\hat{x}_b + \varepsilon_b)}{(N_b + 2\varepsilon_b)^2}\left(\hat{\theta}_b - \hat{x}_b + \varepsilon_b\right)\right] < 0. \tag{A34}$$

Using condition (A30), we obtain

$$\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}N_b} > 0, \quad \text{and} \quad \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}N_b} \gtreqless 0.$$

To show that $\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}N_b} > 0$, we calculate

$$-\frac{\partial \bar{\pi}^g}{\partial N_b}\frac{\partial \bar{\pi}^b}{\partial \hat{x}_b} + \frac{\partial \bar{\pi}^g}{\partial \hat{x}_b}\frac{\partial \bar{\pi}^b}{\partial N_b}.$$

Using equations (**??**), (A27), (A33) and (A34), we obtain

$$\Omega\left(\frac{\hat{\theta}_b + \varepsilon_b - \hat{x}_b}{4\varepsilon_b} + N_b(\hat{x}_b + \varepsilon_b)\right) - \frac{\Omega N_b}{N_b + 2\varepsilon_b}\frac{\hat{x}_b + \varepsilon_b}{N_b + 2\varepsilon_b} - \frac{\ell^2 N_b \varepsilon_g(\hat{\theta}_b + \varepsilon_b - \hat{x}_b)^2}{4\varepsilon_b^2\sigma_g(N_b + 2\varepsilon_b)(1+2\varepsilon_g)},$$

where $\Omega := \frac{D_b - \ell\frac{\tilde{\sigma} - \hat{\theta}_g(\hat{\theta}_b)}{\sigma_g}}{N_b + 2\varepsilon_b}$. Since $N_b \geq 1$, we have

$$\Omega N_b(\hat{x}_b + \varepsilon_b) > \frac{\Omega N_b}{N_b + 2\varepsilon_b}\frac{\hat{x}_b + \varepsilon_b}{N_b + 2\varepsilon_b}.$$

Moreover,

$$\begin{aligned}
&\Omega\left(\frac{\hat{\theta}_b + \varepsilon_b - \hat{x}_b}{4\varepsilon_b}\right) - \frac{\ell^2 N_b \varepsilon_g(\hat{\theta}_b + \varepsilon_b - \hat{x}_b)^2}{4\varepsilon_b^2\sigma_g(N_b + 2\varepsilon_b)(1+2\varepsilon_g)} > 0 \\
&\Leftrightarrow \Omega > \frac{\ell^2 N_b \varepsilon_g(\hat{\theta}_b + \varepsilon_b - \hat{x}_b)}{\varepsilon_b \sigma_g(N_b + 2\varepsilon_b)(1+2\varepsilon_g)} \\
&\Leftrightarrow D_b - \ell\frac{\tilde{\sigma} - \hat{\theta}^T_g}{\sigma_g} + \frac{\ell^2 N_b \varepsilon_g(\hat{\theta}_b + \varepsilon_b - \hat{x}_b)}{\varepsilon_b(1+2\varepsilon_g)\sigma_g} > \frac{\ell^2 N_b \varepsilon_g(\hat{\theta}_b + \varepsilon_b - \hat{x}_b)}{\varepsilon_b \sigma_g(1+2\varepsilon_g)} \\
&\Leftrightarrow D_b - \ell\frac{\tilde{\sigma} - \hat{\theta}^T_g}{\sigma_g} > 0.
\end{aligned}$$

We thus have $-\frac{\partial \bar{\pi}^g}{\partial N_b}\frac{\partial \bar{\pi}^b}{\partial \hat{x}_b}+\frac{\partial \bar{\pi}^g}{\partial \hat{x}_b}\frac{\partial \bar{\pi}^b}{\partial N_b}>0$, which implies $\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}N_b}>0$.

Finally, the derivatives with respect to θ_b^0 and θ_g^0 are given by

$$\frac{\partial \bar{\pi}^b}{\partial \theta_g^0}=\frac{\ell}{2\varepsilon_b}\int_{x_{n_b}-\varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)}\left(\frac{1}{\sigma_g^2}\int_{\hat{\theta}_g(\hat{x}_g,\hat{x}_b,u)}^{\tilde{\sigma}_g}\mathrm{d}v\right)\mathrm{d}u>0,$$

$$\frac{\partial \bar{\pi}^b}{\partial \theta_b^0}=0,$$

$$\frac{\partial \bar{\pi}^g}{\partial \theta_g^0}=0,$$

$$\frac{\partial \bar{\pi}^g}{\partial \theta_b^0}=\frac{D_g\ell N_b}{(1+2\varepsilon_g)\sigma_b^2}\int_{-\eta_b}^{\hat{\theta}_b}\frac{u+\varepsilon_b-\hat{x}_b}{2\varepsilon_b}>0.$$

Combining these with equations (A28) and (A29), we obtain

$$\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\theta_b^0}<0,\ \frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\theta_g^0}<0,\ \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\theta_b^0}<0,\ \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\theta_g^0}>0.$$

□

Proof of Lemma 4. Observe that for given θ_b, total guarantee payments are given by

$$\begin{cases}\frac{N_b\ell}{2\varepsilon_b}\int_{\hat{x}_b}^{\theta_n+\varepsilon_b}\mathrm{d}u & \text{if } \theta_b<\hat{\theta}_b^* \\ 0 & \text{else}\end{cases}.$$

Hence, whenever $\varepsilon_b\to 0$, $\hat{x}_b^*\to\hat{\theta}_b^*$ and the integral collapses to zero. But then, the guarantee does not appear anymore in the government's default condition and the threshold for government default converges to $\hat{\theta}_g^*=C_g/D_g$, as in the canonical model in Lemma **??**. The probability of a government default can then be calculated as $P_g\equiv\mathbf{Pr}\left(\theta_g<\hat{\theta}_g^*\right)=\frac{C_g/D_g+\eta_g}{\sigma_g}$.

The critical bank creditor's indifference condition can be explicitly written as

$$\bar{\pi}^b(\hat{x}_b,\hat{x}_g)=\frac{D_b(\hat{x}_b+2\varepsilon_b)}{N_b+2\varepsilon_b}+\frac{\ell(\tilde{\sigma}_g-\hat{\theta}_g^T)(\hat{\theta}_b-\hat{x}_b+\varepsilon_b)}{\sigma_g 2\varepsilon_b}-\frac{\ell\varepsilon_g N_b(\hat{\theta}_b-\hat{x}_b+\varepsilon_b)^2}{4\varepsilon_b\sigma_g(1+2\varepsilon_g)}-1=0.$$

Observe that $\hat{\theta}_b-\hat{x}_b+\varepsilon_b=2\varepsilon_b(N_b-\hat{x}_b+\varepsilon_b)/(N_b+2\varepsilon_b)$. Substituting this into the indifference condition and taking the limit $\varepsilon_b\to 0$ leads to

$$\bar{\pi}^b(\hat{x}_b)=D_b\hat{x}_b+(1-P_g)\ell(N_b-\hat{x}_b)-N_b=0,$$

which can be solved for the critical signal,

$$\hat{x}_b=\hat{\theta}_b=\frac{N_b(1-\ell(1-P_g))}{D_b-\ell(1-P_g)}. \tag{A35}$$

□

Proof of Lemma 5. We obtain from equation (A35)

$$\frac{\partial\hat{\theta}_b^*}{\partial\ell}=\frac{N_b(1-P_g)(1-D_b)}{(1-\ell(1-P_g))^2}<0.$$

The probability of a systemic crisis can be computed as

$$q(\ell)=P_g\times P_b(\ell),$$

with $P_b(\ell)=\frac{\hat{\theta}_b^*+\eta_b}{\sigma_b}$. Since P_g does not depend on ℓ, the derivative of the cost of crises measure with respect to ℓ can then be computed as

$$K'(\ell)=(1-P_g)\phi_b\frac{\partial P_b}{\partial\ell}+P_g(1-\phi_g)\frac{\partial P_b}{\partial\ell}.$$

Substituting

$$\frac{\partial P_b}{\partial\ell}=\frac{1}{\sigma_b}\left(\frac{N_b(1-P_g)(1-D_b)}{(1-\ell(1-P_g))^2}\right)$$

gives the expression in the text. □

Proof of Lemma 6. For $\varepsilon_g \to 0$ follows from Equation (5)

$$\hat{\theta}_g^* = \hat{x}_g^*(\ell) \qquad \forall \theta_b.$$

The expression for $\hat{\theta}_b^*$ is given by Equation (4). Since $\varepsilon_g \to 0$, we have $q(\ell) = P_b(\ell) \times P_g(\ell)$ where $P_b(\ell)$ is given in equation (15) and $P_g(\ell) = \frac{\hat{x}_g^*(\ell)+\eta_g}{\sigma_g}$. We have

$$P_b'(\ell) = \frac{1}{\sigma_b}\frac{N_b}{N_b + 2\varepsilon_b}\frac{\mathrm{d}\hat{x}_b^*(\ell)}{\mathrm{d}\ell} \qquad \text{and} \qquad P_g'(\ell) = \frac{1}{\sigma_g}\frac{\mathrm{d}\hat{x}_g^*(\ell)}{\mathrm{d}\ell}.$$

The FOC condition is given by

$$\begin{aligned} K'(\ell) &= \phi_b P_b'(\ell) + \phi_g P_g'(\ell) + (1-\phi_b-\phi_g)\Big(P_b(\ell)P_g'(\ell) + P_g(\ell)P_b'(\ell)\Big) \\ &= P_b'(\ell)\big(\phi_b(1-P_g(\ell)) + (1-\phi_g)P_g(\ell)\big) + P_g'(\ell)\big(\phi_g(1-P_b(\ell)) + (1-\phi_b)P_b(\ell)\big) \\ &= 0. \end{aligned}$$

□