Public debt and economic growth through History:

New evidence from a macroeconometric retrospective analysis

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Abstract: We explore the relevance of the exogenous threshold highlighted by Reinhart & Rogoff (2010) using up-to-date econometric techniques and reveal an endogenously-estimated threshold around a debt-to-GDP ratio of 115%, above which the negative debt-growth link changes sign. Based on these incipient findings, we investigate the relation between public debt and growth from a historical perspective. Our econometric retrospective analysis for the 1880-2009 period confirms the presence of nonlinearities in the effect of public debt on growth, showing that high growth and high public debt were associated during remarkable historical episodes. These historical specificities should be accounted for when designing economic policy in a high-debt context as the one currently experienced by an important number of countries worldwide.

Keywords: public debt, economic growth, nonlinear effects, historical perspective.

JEL Codes: H63, E62, N10

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I. Introduction

In their already-famous article "Growth in a Time of Debt", published by The American Economic Review in 2010 (*AER* 100(2), pp. 573-78), Carmen Reinhart and Kenneth Rogoff analyze the relation between economic growth and public debt. Drawing upon a new public debt database, Reinhart & Rogoff (2010, hereafter RR) exhibit the presence of debt-to-GDP ratios that they assess in terms of threshold. Regarding advanced economies, RR show that post-WW2 economic growth is dramatically declining on average once the public debt-to-GDP ratio is above the 90% threshold. Policy recommendations by official institutions, including the OECD, the EU Commission or the French Report on Public Finance (April 2010), seem to have adopted this threshold as an ultimate guide for policy making, by taking for granted the "Reinhart and Rogoff's debt intolerance ratio" and transforming it, rather mechanically, into a "natural" target to shift public spending.

This paper contributes to the current strand of literature on public debt and economic growth,[1] by exploring the relevance of the 90% threshold emphasized by RR.[2] Using up-to-date econometric techniques, which allow dealing properly with complex nonlinearities on panel data, we find the existence of an endogenously-estimated threshold, at roughly 115%, in the public debt to economic growth relation. Below this threshold, a debt increase damages growth; however, this negative effect is declining as public debt is increasing. Above this threshold, the link between public debt and economic growth changes sign.

In light of these estimations, we reproduce the stylized facts emphasized by RR. On the one hand, we show that, similarly to RR, average economic growth is lower for countries with debt levels between 90 and 115%, compared to countries with debt levels between 60 and 90%. On the other hand, we show that average economic growth is higher for countries with public debt above 115%, compared to countries with debt levels between 90 and 115%,

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[1] For a recent survey on this literature, see Di Martino (2010), Panizza & Presbitero (2012) or Parent (2012).
[2] An additional critique of RR concerns the problem of causality, which is below the scope of this paper; see Panizza & Presbitero (2012) for an excellent analysis on this issue.

and more importantly, that average economic growth is not statistically different for the former group compared to countries with debt levels between 60 and 90%.

In addition, we extend the historical dimension of our sample to the period 1880-2009, and emphasize yet again the presence of threshold effects, for a debt ratio of 130%. Countries with debt ratios above 130% do not experience significantly different growth rates compared to countries with a debt ratio between 60 and 90%, confirming and expanding our findings for 1945-2009 to a much wider period. During the 1880-2009 period, we identify several countries experiencing high economic growth in a context of high public debt, in some well-known remarkable periods, namely the beginning of the "*Trente Glorieuses*" (the post WW2 decade), the "Roaring Twenties" and the "*Belle Epoque*".

Our results involve that much attention should be given to the historical dimension. Indeed, the analysis of the relation between public debt and growth from a historical perspective shows that results based on the post-WW2 episodes are not a simple statistical artifact, and are backed up by noteworthy historical episodes: in some countries, the rising of capitalism, in its times of high economic growth, went along with high public debt ratios. Although one should reasonably refrain from concluding that governments should adopt loose fiscal policies leading to high public debt levels to foster economic growth, this latter result provides a new perspective on the "debt intolerance ratio" emphasized by RR. Indeed, an increasing public debt path is not generating bottomless pit growth losses, but is bordered by a high public debt regime in which economic growth may increase, emphasizing that additional evidence is needed before suggesting policy recommendations regarding growth effects of fiscal policy in such high debt regimes, which may be subject to complex nonlinearities.

Section 2 emphasizes the RR result, section 3 presents the econometric method and compares our results to RR, section 4 embraces a historical perspective by providing

estimations on the 1880-2009 period and discusses, in light of this retrospective econometric analysis, economic policy lessons, and section 5 concludes.

II. Public debt and economic growth: A discussion of Reinhart & Rogoff (2010)

The result we aim at discussing regards the correlation between public debt and economic growth. Using a new database, RR distinguish between four regimes, namely advanced economies with low public debt (below 30% in ratio of GDP), medium-low public debt (between 30% and 60% in ratio of GDP), medium-high public debt (between 60% and 90% of GDP), and high public debt (above 90% in ratio of GDP). According to their Figure 2 entitled "Government Debt, Growth and Inflation: Selected Advanced Economies, 1946 – 2009"[3] (page 575), over the period 1946-2009 the average economic growth rate is dramatically lower in countries presenting a debt ratio above 90%, compared to the other countries in the sample.

However, this result is somehow puzzling. Indeed, their Figure 2 shows that compared to the sharp contraction of economic growth (around 3 percentage points higher for medium-high debt with respect to high debt countries), the difference between the median growth rates is considerably lower (around 1 percentage point). This observation could suggest a typical outlier problem, and has been the starting point of our analysis.

To investigate the robustness of the findings of RR, we begin by reproducing their Figure 2. The data we are using cover the same time period (1945-2009) and come from two sources. First, using real GDP data from Maddison (2007), we compute the real economic growth rate. Second, we use the very recent Ali Abbas et al. (2010) database on public debt. The reason for using this database is double. On the one hand, we aim at investigating if RR

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[3] The twenty advanced economies included are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, the United Kingdom and the United States.

findings still hold when using a different database on public debt.[4] On the other hand, we aim at exploring the robustness of our results from a historical perspective, namely for the 1880-2009 period. To ease comparison, we focus on the same sample of 20 advanced economies.

Figure 1. GDP growth and the debt-to-GDP ratio[5]
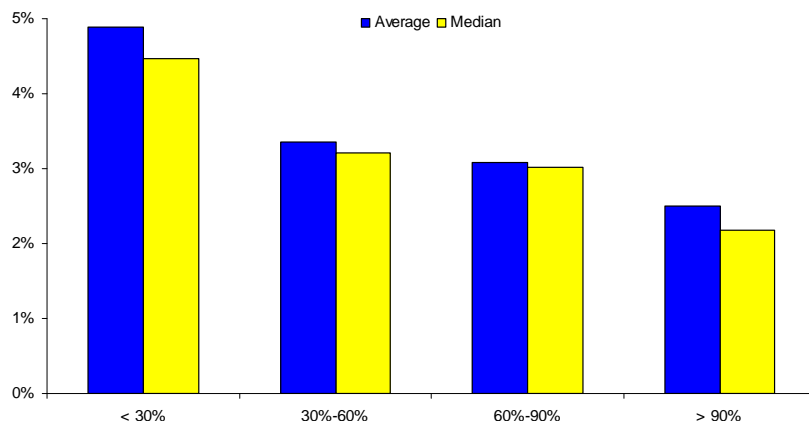
Similar to RR, we find that a debt-to-GDP ratio over 90% is reducing average economic growth. However, contrary to RR, the contraction in economic growth is much less obvious, as suggested by simple equality tests which fail to support the presence of a statistically significant difference between the average growth rates of the last two regimes.[6] Consequently, one should be cautious in deriving economic policy implications from this chart alone, before exploring more in detail the properties of the data.

III. Public debt and economic growth: An econometric reappraisal

The previous section emphasized the difficulties of establishing a clear-cut relation between public debt and economic growth, on the basis of simple statistics. In addition, the results that could be derived from Figure 1 suffer from major econometric shortcomings, including (i) the specification of exogenous thresholds in the public debt-to-GDP ratio, (ii) the

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[4] Our efforts for obtaining the database used by RR were, up until now, unfortunately unsuccessful. In addition, the database we use is an official database from the IMF and is, to the best of our knowledge, the most complete data collection on public debt.
[5] Figure 1 is based on the same countries as RR. The number of observations in our sample (in RR sample) in the four regimes is 307 (443), 452 (442), 246 (199) and 129 (96), for a total of 1134 (1180) observations.
[6] Under the null hypothesis of an identical economic growth rate in the last two regimes, the Anova and Welch F-tests equal 1.305 and 0.836 respectively, and the associated p-values are 0.25 and 0.36 respectively.

absence of econometric tests for the relevance of the regimes, and (iii) the presence of brutal transitions (i.e. sudden jumps) in the debt-growth relation around the debt thresholds. To overcome these issues, we perform in this section an analysis based on up-to-date econometric techniques, namely the Panel Smooth Threshold Regression (PSTR) method, recently coined by Gonzalez et al. (2005). Let us consider the following PSTR model[7]

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\begin{cases}
Y_{it} = \alpha_i + \beta_1 B_{it} + \sum_{k=1}^{K} \beta_2^k B_{it} \Gamma^k \left(B_{it}; . \right) + \varepsilon_{it} \\
\Gamma^k \left(B_{it}; \gamma^k, \overline{B}_h^k \right) = \left[1 + \exp\left(-\gamma^k \prod_{h=1}^{H_k} \left(B_{it} - \overline{B}_h^k \right) \right) \right]^{-1}
\end{cases} \tag{1}
$$

measuring the effect of the debt-to-GDP ratio (B) on growth (Y), depending on the debt ratio level (B), with $i = \overline{1, N}$ countries (α_i stand for country fixed effects), $t = \overline{1, T}$ years, and with Γ^k ($k = \overline{1, K}$) transition functions, each with different $h = \overline{1, H_k}$ thresholds, depending on public debt-to-GDP (B), the thresholds ($\overline{B}$), and a smoothing parameter ($\gamma^k > 0$).

According to RR, public debt is found to reduce economic growth above the 90% threshold, while this relation seems to be less obvious and not significant in our Figure 1. In addition, even RR cast doubts on the shape of the effect of public debt on economic growth below the 90% level. Consequently, it is difficult to infer a clear-cut shape of the public debt-economic growth link, and particularly regarding the presence or the absence of nonlinearities. To overcome these shortcomings, we first present the results of the identification procedure, which, as usual with nonlinearities, tests their merely existence; provided the presence of nonlinearities, this phase also establishes the number of transition functions and thresholds of the general model (1). Based on these findings, we estimate in the second phase the PSTR model.

Table 1 presents the identification procedure of the PSTR model (1).

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[7] The Appendix contains a detailed presentation of the PSTR technique.

Table 1. Identification of the PSTR model (1): nonlinearities in the public debt ratio

		First transition Function	Second transition function (First transition function has three thresholds)
1 threshold	LM Test	44.6 (2.38e-011)	2.57 (0.109)
	F Test	45.6 (2.33e-011)	2.52 (0.113)
2 thresholds	LM Test	50.1 (1.34e-011)	5.29 (0.071)
	F Test	25.7 (1.25e-011)	2.60 (0.075)
3 thresholds	LM Test	**59.5 (7.53e-013)**	8.84 (0.032)
	F Test	**20.5 (6.17e-013)**	2.90 (0.034)

The tests are based on the linearized form of model (1) (see the Appendix). We considered up to $H = 3$ thresholds, as suggested by Gonzalez et al. (2005). Bolded values signal the strongest rejection of the null hypothesis (namely, a linear panel). p-values are reproduced in brackets. Since this is a sequential procedure, the significance level (1% or 5%) is reduced after each sequence by a constant factor $\tau = 0.5$ (see Gonzalez et al., 2005).

The first column of Table 1 depicts the values of the tests for the first transition function with up to three thresholds, as suggested by Gonzalez et al. (2005). The low p-values confirm the existence of strong nonlinearities in the effect of public debt on economic growth. Moreover, both LM and F tests show that the strongest rejection of the null hypothesis criterion (see Gonzalez et al., 2005) is consistent with a first transition function with $H = 3$ thresholds. Thus, we block the number of thresholds to three in this first transition function, and search next for a second transition function. Irrespective of the number of thresholds considered, the rejection of the null hypothesis arises for p-values largely lower than for the first transition function, and in particular lower than half of the significance level considered in the first sequence (namely, 1% or 5%, see Gonzalez et al., 2005). Consequently, the PSTR model that comes out from the identification procedure consists of one transition function with three thresholds, namely

$$Y_{it} = -\underset{(0.013)}{0.119}^{***} B_{it} + \underset{(0.016)}{0.120}^{***} B_{it}\Gamma(.) + \varepsilon_{it}$$

$$\text{with } \Gamma\left(B_{it}; \gamma = 0.415, \overline{B}_1 = \overline{B}_2 = \overline{B}_3 = 0.067\right) = \left[1 + \exp\left(-\gamma\prod_{h=1}^{3}\left(B_{it} - \overline{B}_h\right)\right)\right]^{-1}. \tag{2}$$

The results of the PSTR regression (2) were estimated following the algorithm presented in the Appendix, with standard errors (corrected for heteroskedasticity) reproduced in brackets, and the stars *** denoting the significance at the 1% level. In line with the results emphasized in the identification procedure, the presence of a nonlinear relation between public debt and economic growth is confirmed by the strong significance of the β_2 coefficient.

Let us explore in the following the effect of the public debt ratio on economic growth, namely

$$\frac{dY}{dB} = \beta_1 + \beta_2\Gamma(.) + \beta_2\gamma\,\Gamma(1-\Gamma)B_{it}\left(3B_{it}^2 - 2B_{it}\left(\overline{B}_1 + \overline{B}_2 + \overline{B}_3\right) + \overline{B}_1\overline{B}_2 + \overline{B}_2\overline{B}_3 + \overline{B}_3\overline{B}_1\right). \tag{3}$$

According to (3), the influence of the level of public debt on economic growth transits through a rather complex nonlinear way. To have a better look at these effects, we present in Figure 2 a graphical representation of (3), based on results from regression (2).

Figure 2. The effect of the public debt-to-GDP ratio on economic growth
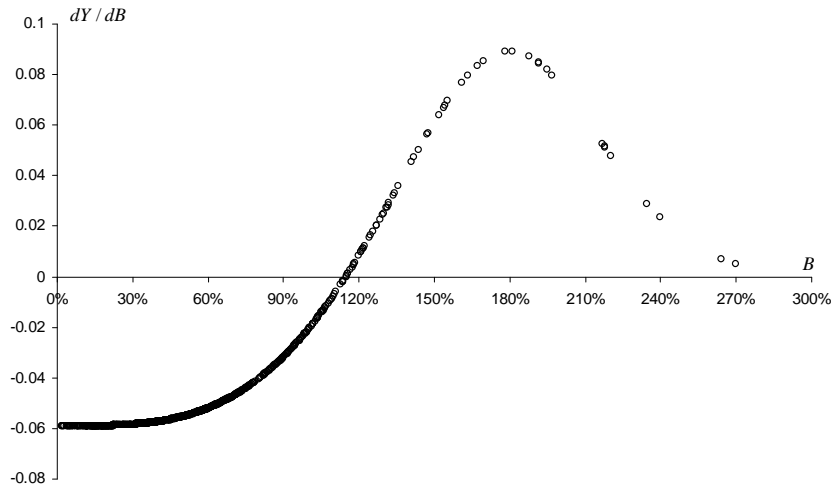
The results depicted in Figure 2 confirm some of the findings of RR, and equally go beyond them. Indeed, according to our estimations, an increase in the debt ratio decreases

economic growth (the derivative is negative). This result, in accordance with our Figure 1 illustrating a negative correlation between economic growth and debt, extends the analysis of RR who fail to derive a clear-cut relation between public debt and growth for debt ratios below 90%. Besides, although economic growth declines as public debt increases, its reduction progressively vanishes, contrary to the conclusions of RR who support a dramatic reduction in economic growth when public debt is above 90%.

Moreover, the effect of high (above 90%) public debt on growth displays much more complex nonlinearities that stressed by RR. Indeed, an increase of public debt above the 90% ratio reduces economic growth, as emphasized by RR. But in addition we depict the existence of a threshold, around a public debt-to-GDP ratio of 115%, above which the correlation between public debt and growth changes sign.[8] Consequently, contrary to RR, countries with public debt ratios above 90% not only do not see their economic growth plunging, but they could even experience an increase in their growth, once public debt is above the endogenously-estimated threshold of 115%.[9]

Figure 3 appends our Figure 1 above in light of these results, by accounting for the presence of a threshold at around 115% for the public debt ratio (for simplicity, we focus on public debt ratios above 60%).

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[8] Compared to RR, where the 90% debt threshold brutally distinguishes between countries with high, respectively low growth rates, the transition between negative and positive effects of debt on growth occurs smoothly, around the 115% debt threshold.
[9] Besides, the magnitude of positive effect of public debt on economic growth is rather important, and may be even stronger (in absolute value) than the negative effect illustrated for low public debt ratios.

Figure 3. GDP growth and the debt-to-GDP ratio: accounting for threshold effects
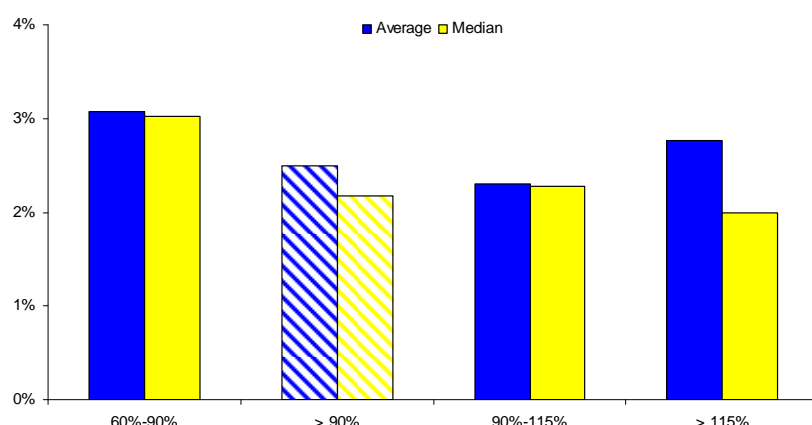
The first two charts, reproduced from our Figure 1, recall that average growth is lower in countries with a public debt ratio above 90%, compared to countries with a debt ratio between 60 and 90%.[10] The last two charts develop the analysis by accounting for the presence of threshold effects for a debt ratio of 115%; thus, we distinguish between high public debt countries below this threshold (namely, with a debt ratio between 90 and 115%) and respectively above it, and compute average and median economic growth rates.

In the spirit of RR, we find that compared to countries with a debt ratio between 60 and 90%, countries with a debt ratio between 90 and 115% experience a decline in their average economic growth rates. Although this decline is statistically significant,[11] notice that the economic growth contraction is much less pronounced than acknowledged by RR. In addition, contrary to RR, we find that countries with a public debt ratio above 115% present an average growth rate which is higher compared to the average growth rate of countries with a public debt ratio between 90 and 115%. More important, the growth rate of countries with a debt ratio above 115% is not found to significantly decline compared to the growth rate of countries with a debt ratio between 60 and 90% (the values of Anova and Welch mean

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[10] However, remind that equality tests show that there is statistically no difference between average growth rates for the two regimes.
[11] The values of Anova and Welch mean equality F-tests equal 4.568 and 6.250 respectively, and the associated p-values are 0.03 and 0.01 respectively.

equality F-tests equal 0.169 and 0.050 respectively, and the associated p-values are 0.68 and 0.82 respectively).

IV. Public debt and economic growth: Economic policy lessons from a historical perspective (1880-2009)

Our previous findings revealed an alternative perspective for the period 1945-2009. Indeed, according to RR, countries should be cautious regarding their fiscal policy, since loose fiscal policies, raising the public debt level, may drive them towards high debt ratio regimes (characterized by a public debt above 90%), associated with dramatically lower economic growth. Our results do not go against this general idea, since we find that increasing public debt is detrimental to economic growth, when public debt is relatively low. However, we also find that the negative effect of public debt on growth is declining, and thus the growth contraction might be less important that acknowledged by RR. In addition, we even show that raising public debt can even increase economic growth, in a context of high debt levels, namely when the public debt-to-GDP ratio is above a threshold level estimated at around 115%.

An immediate question regarding our findings concerns this last regime, namely countries with high public debt ratios (above 115%) and high economic growth. Among them, we can enumerate Belgium, Canada, New Zealand or United Kingdom, and the periods concerned are the decade just after the WW2 and, to a lesser extent, the last decade of the 20th century. However, one may argue that these events are singular, in the sense that they are not likely to have been observed in other time periods. In this case, the positive association between high public debt and growth may not be more but a simple artifact conditional to the considered time sample. To explore the importance of this possible effect, we investigate if our results still hold when considering a historical perspective. To this end, we perform our

analysis on the same sample of countries, but on the period 1880-2009, since this period encompasses periods previously documented by historians of high debt levels.[12]

The results of the identification procedure performed on the general PSTR model (1) for the 1880-2009 period are presented in Table 2.

Table 2. Identification of the PSTR model (1): public debt nonlinearities (1880-2009)

		First transition Function	Second transition function (First transition function has one threshold)
1 threshold	LM Test	**27.8 (1.32e-007)**	1.29e-005 (0.997)
	F Test	**27.9 (1.39e-007)**	1.27e-005 (0.997)
2 thresholds	LM Test	28.1 (8.09e-007)	3.23 (0.199)
	F Test	14.1 (8.54e-007)	1.60 (0.203)
3 thresholds	LM Test	31.7 (5.94e-007)	3.54 (0.316)
	F Test	10.6 (6.24e-007)	1.17 (0.321)

The tests are based on the linearized form of model (1) (see the Appendix). We considered up to $H = 3$ thresholds, as suggested by Gonzalez et al. (2005). Bolded values signal the strongest rejection of the null hypothesis (namely, a linear panel). p-values are reproduced in brackets. Since this is a sequential procedure, the significance level (1% or 5%) is reduced after each sequence by a constant factor $\tau = 0.5$ (see Gonzalez et al., 2005).

The above identification procedure supports the presence of one transition function with one threshold, and the results of the estimation of this model are the following

$$Y_{it} = -\underset{(0.004)}{0.029^{***}} B_{it} + \underset{(0.005)}{0.031^{***}} B_{it}\Gamma(.) + \varepsilon_{it}$$
$$\text{with } \Gamma\left(B_{it}; \gamma = 4.866, \overline{B}_1 = 1.670\right) = \left[1 + \exp\left(-\gamma\left(B_{it} - \overline{B}_1\right)\right)\right]^{-1},$$

(4)

with the associated effect of public debt on economic growth illustrated in Figure 4.

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[12] Unfortunately, panel data are not available before 1880; however, remark that considering the 1880-2009 period doubles the time dimension used in the analysis of RR (namely, 1945-2009).

Figure 4. Public debt and economic growth from a historical perspective (1880-2009)
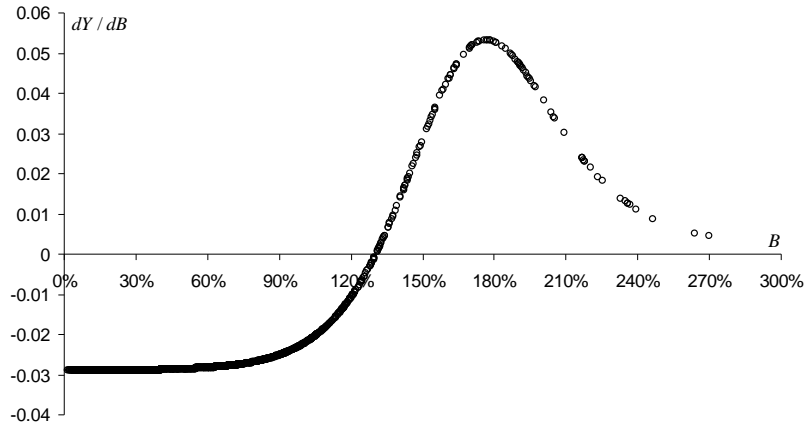

Figure 4 confirms that the results obtained for the 1945-2009 period are robust when considering a historical perspective, by looking at the relation between debt and economic growth over the period 1880-2009. In particular, remark the presence, yet again, of a threshold for a public debt-to-GDP ratio around 130%, above which the correlation between public debt and growth becomes positive. The following chart appends Figure 3, derived for the 1945-2009 period, to the findings for the 1880-2009 period.

Figure 5. GDP growth and the debt-to-GDP ratio:

Accounting for threshold effects from a historical perspective (1880-2009)
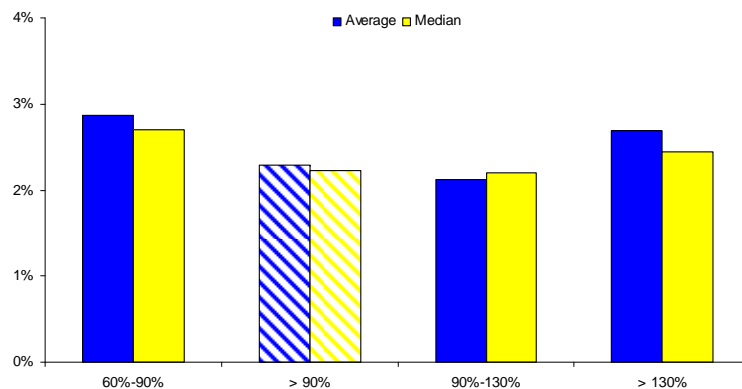
Simple equality tests show that (i) average growth declines for countries with a debt ratio between 90 and 130% (the Anova and Welch mean equality F-tests equal 5.917 and 5.385, and the associated p-values equal 0.01 and 0.02 respectively), and (ii) countries with high debt ratios, namely above 130%, do not experience significantly different growth rates

compared to countries with a debt ratio between 60 and 90% (the Anova and Welch mean equality F-tests equal 0.100 and 0.045, and the associated p-values equal 0.74 and 0.83 respectively). These findings extend and confirm our conclusions for the 1945-2009 period.

In addition to the episodes emphasized for the 1945-2009 period, this positive correlation equally captures historical features that occurred during the 1880-1945 period, namely the "Roaring Twenties" and the "*Belle Epoque*". During these periods, many countries from our sample, including Belgium, France, Italy, Spain or UK, experienced remarkably high economic growth rates in a context of elevated public debt ratios. Our retrospective econometric analysis for the 1880-2009 period confirms that the positive association between debt and growth is not singular to the recent period (1945-2009), but equally applies to the outstanding periods of the "Roaring Twenties" and of the "*Belle Epoque*". It is important to keep in mind that lessons from retrospective econometric analysis do not make the case for the use of loose fiscal policies as an engine to promote economic growth, when public debt reaches high levels. Our results show that an increasing public debt path is not generating bottomless pit growth losses, but is bordered by a regime in which economic growth and debt may be positively correlated. According to our analysis, countries in this regime, characterized by a public debt-to-GDP ratio above 130%, present on average growth rates that are comparable to the growth rates of countries with significantly lower public debt ratios, namely between 60 and 90%.

V. Conclusion

We investigated in this paper whether high public debt ratios are consistent with substantial economic growth declines in advanced economies, as supported by Reinhart & Rogoff (2010). Using the new IMF database on public debt for the period 1945-2009, we emphasize the presence of nonlinearities in the effect of debt on growth for high debt ratios.

On the one hand, debt reduces growth for countries with a debt-to-GDP ratio below a threshold estimated at around 115%. On the other hand, we reveal that economic growth and public debt are positively associated for debt ratios above 115%; for countries in this regime, simple equality tests support that average economic growth is not significantly different from the average growth rate of countries with a public debt ratio between 60 and 90%, developing the findings of RR.

Starting from these incipient results, we extended our analysis to a historical perspective and revealed a public debt-to-GDP ratio threshold around 130% for the period 1880-2009. Our analysis shows that an increasing public debt path is not generating bottomless pit growth losses, but is bordered by a regime is which economic growth can be positively correlated with high public debt levels. Retrospective econometric analysis goes hand in hand with History to support the presence of threshold effects of public debt on economic growth during crucial historical periods of rising capitalism, namely the "*Belle Epoque*", the "Roaring Twenties" and the beginning of the "*Trente Glorieuses*". Given the presence of these important nonlinearities, our findings should not be over-interpreted, and loose fiscal policies should not be considered as an unambiguous engine for fostering economic growth in a historical perspective. Instead, the historical lessons of our retrospective econometric analysis support the presence and the importance of historical singularities in the historical relation between public debt and economic growth. Accounting for the presence of such historical specificities should fuel the design of economic policy in a high-debt context, as the one currently experienced by an important number of countries worldwide.

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Appendix: A detailed presentation of the PSTR method
General considerations

The Panel Smooth Threshold Regression (PSTR) model, recently coined by Gonzalez et al. (2005), can be seen as an upgrading of two existing techniques. On the one hand, as a generalization to panel data of thresholds with smooth transition used in time series (see the seminal paper on STAR models by Chan & Tong, 1986). On the other hand, as a generalization to smooth transitions of panel threshold models (PTR) with brutal transitions. In the following, we present the PSTR method in the light of the latter view.

PTR models were introduced by Hansen (1999), in an attempt for providing a tool for estimating threshold effects on panel data. Indeed, recall that RR illustrate the presence of a threshold at a debt-to-GDP ratio of 90%, above which growth declines dramatically. Consequently, assuming a panel model of dimension $i = \overline{1,N}$ countries and $t = \overline{1,T}$ years, it is appropriate to test the RR result in the following PTR model

$$\begin{cases} Y_{it} = \alpha_i + \beta_1 B_{it} + \beta_2 B_{it} \Gamma\left(B_{it}; \overline{B}\right) + \varepsilon_{it} \\ \Gamma(.) = \begin{cases} 0, \; if \; B_{it} < \overline{B} \\ 1, \; if \; B_{it} \geq \overline{B} \end{cases} \end{cases}, \tag{A1}$$

with α_i country fixed effects, and ε_{it} an error term. According to (A1), the impact of the public debt-to-GDP ratio B on the economic growth rate Y depends nonlinearly on the values of the debt ratio with respect to the threshold $\overline{B}$, namely $dY/dB = \beta_1$ if $B_{it} < \overline{B}$ (regime 1) and $dY/dB = \beta_1 + \beta_2$ if $B_{it} \geq \overline{B}$ (regime 2). If we follow RR, the threshold $\overline{B}$ should be exogenously determined and equals to $\overline{B} = 90\%$. However, Hansen (1999) developed a procedure to *endogenously* estimate the value of the threshold $\overline{B}$, as well as for testing its relevance (namely, the absence of nonlinearities described by the null hypothesis $H_0 : \beta_1 = \beta_2$) through a pertinent bootstrapping analysis. Besides, a sequential identification procedure can be implemented to extend the PTR model to multiple thresholds.[13]

Despite its novelty compared to standard polynomial approaches for modeling nonlinearities, the PTR model suffers however of two important shortcomings. First, the presence of a brutal transition between regimes, involving that either the elasticity fully depends on the transition variable or is completely independent of it. This is the case in the Figure 1 of RR, according to which the correlation between public debt and economic growth is significantly different for countries with a public debt ratio on the left and respectively the right hand side of the 90% threshold. However, such structural differences are hard to justify for countries with fairly close public debt-to-GDP levels. Second, even if we develop (A1) to the presence of multiple thresholds, the obtained PTR model would still allow for a reduced number of regimes. For example, a two-threshold PTR model yields three different regimes (i.e. three different elasticities), which may be considered as rather limited when one deals with panel data with (i) an important time dimension (in this paper, 65 and up until 130 years), and (ii) important heterogeneity among countries (since the large time period we consider may cover different stages of economic development). To overcome these shortcomings in an adequate manner, we focus in the following on the PSTR technique.

Keeping the same notations as in (A1), let us assume the following PSTR model (the model described by equation (1) in the main text)

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[13] As the number of thresholds increases, the estimation of the PTR model is technically more difficult (for convergence issues, for example).

$$
\begin{cases}
Y_{it} = \alpha_i + \beta_1 B_{it} + \sum_{k=1}^{K} \beta_2^k B_{it} \Gamma^k \left(B_{it}; . \right) + \varepsilon_{it} \\
\Gamma^k \left(B_{it}; \gamma^k, \overline{B}_h^k \right) = \left[1 + \exp\left(-\gamma^k \prod_{h=1}^{H_k} \left(B_{it} - \overline{B}_h^k \right) \right) \right]^{-1}
\end{cases} , \tag{A2}
$$

with Γ^k ($k = \overline{1, K}$) transition functions depending on the level of the public debt-to-GDP ratio (B), the threshold ($\overline{B}$), and the smoothing parameter ($\gamma^k > 0$) to be discussed below. In this general formulation, we consider that each transition function Γ^k has different $h = \overline{1, H_k}$ thresholds.

The properties of the transition function, and of the PSTR model, crucially depend upon the (non-negative) transition parameter γ. Indeed, when $\gamma = 0$, the transition function is constant (it equals 0.5), and the PSTR model collapses to a linear model (since the dY / dB elasticity does not depend on the level of the public debt ratio B anymore). In the other extreme case ($\gamma \to \infty$), the transition function takes only two independent values, and the PSTR model collapses to a PTR model with brutal transition between regimes.[14] Moreover, the intermediate case ($0 < \gamma < \infty$) portrays a PSTR model with smooth transition between regimes (the lower the smoothing parameter, the smoother the transition), as depicted by the left-hand chart of the Figure A below.

Figure A. Different smooth transition functions
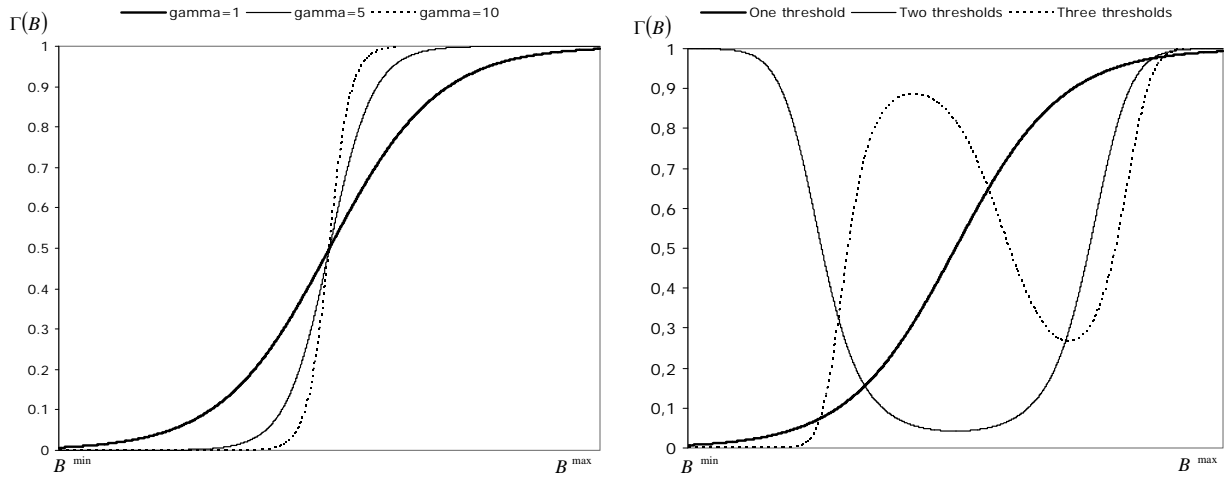
Furthermore, the transition function also depends on the number of thresholds. The right-hand chart of Figure A illustrates several transition functions, with up to $H = 3$ thresholds, as suggested by the seminal contribution of Gonzalez et al. (2005). Irrespective of the number of thresholds, the value of Γ is close to one when public debt approaches its maximum ($B^{\max}$), while its value for low levels of the public debt ratio (around the minimum $B^{\min}$) depends on number of thresholds. In addition, increasing the number of thresholds impacts on the monotonicity of the transition function. Finally, even in the presence of a unique threshold, the model is still much more general than a simple PTR model (provided that the smoothing parameter is low enough, as this is the case in all simulations of Figure A above), since it virtually generates an infinity of regimes, namely an infinity of different values for the effect of public debt on economic growth, depending on the public debt ratio in every point of time and for every country.

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[14] Consequently, the PTR model may be viewed as a special case of a PSTR model, namely when $\gamma \to \infty$.

In search of nonlinearities: the identification procedure

According to RR, the relation between public debt and economic growth is nonlinear, since an increase of the public debt ratio over the 90% threshold reduces considerably growth. However, as usual with nonlinearities, the first operation should consists in testing their merely existence. In a PTR setup, such tests suffer from a nuisance problem (see Hansen, 1999), which however may be overcome by an adapted bootstrapping procedure. Fortunately, such complications may be avoided in a PSTR model; given that the transition function $\Gamma(.)$ satisfies the necessary condition of continuity and presents sufficient conditions for derivability, a simple way to check for the existence of nonlinear effects is to compute the first-order Taylor-linearization of the transition function around the smoothing parameter $\gamma \to 0$. For simplicity, consider a simple one-transition function one-threshold model

$$\begin{cases} Y_{it} = \alpha_i + \beta_1 B_{it} + \beta_2 B_{it} \Gamma(B_{it};.) + \varepsilon_{it} \\ \Gamma(B_{it};\gamma,\overline{B}) = [1 + \exp(-\gamma(B_{it} - \overline{B}))]^{-1} \end{cases}. \tag{A3}$$

Using the first-order linearization, namely $\Gamma(.;\gamma \to 0) = 0.5 + \gamma(B_{it} - \overline{B})/4$, (A3) becomes

$$\begin{cases} Y_{it} = \alpha_i + \beta_1^* B_{it} + \beta_2^* B_{it}^2 + \varepsilon_{it} \\ \beta_1^* \equiv \beta_1 + 0.5\beta_2 - \gamma\beta_2\overline{B}/4; \quad \beta_2^* \equiv \gamma\beta_2/4 \end{cases}. \tag{A4}$$

Under this transformation, the PSTR model (A3) collapses to a simple second-order polynomial nonlinear panel model. Moreover, since β_2^* is a multiple of γ, we can test the absence of nonlinearities in the PSTR model ($H_0 : \gamma = 0$) using the test $H_0 : \beta_2^* = 0$. This result also holds in the presence of two thresholds, as the parameters associated respectively with B^2 and B^3 in the linearized model are also multiples of γ.[15] To check for the existence of such nonlinearities, we use the two tests developed by the related literature, namely the Lagrange Multiplier-based (LM) test and its Fisher (F) version.[16]

To identify the structure of the PSTR model, we use the following cascade procedure. In the first phase, we consider a PSTR model with one transition function, depending on up to three thresholds. Based on the linearized form of the assumed PSTR model (A1) (or (1) in the main text), we compute the above-mentioned linearity tests, for each number of thresholds. Following Gonzalez et al. (2005), the number of thresholds for the first transition function is chosen according to the strongest rejection of the null hypothesis (linear model). Once the number of thresholds for the first transition function identified, we check, in phase two, for the presence of a second transition function again with up to three thresholds. The procedure stops when the null hypothesis of linear effects can no longer be rejected. Consequently, contrary to traditional polynomial forms used to account for nonlinearities, this identification procedure puts no ex-ante constraint on the functional form of the nonlinear relations. Indeed, given the possibility of virtually combining multiple transition functions, with as much as three thresholds for each one of them, provides sufficient place for obtaining dY/dB

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[15] Namely, $\beta_2^{1*} = -\gamma\beta_2(\overline{B}_1 + \overline{B}_2)/4$ and $\beta_2^{2*} = \gamma\beta_2/4$ respectively. In this case, the absence of nonlinear effects $H_0 : \gamma = 0$ can be checked based on a joint nullity test $H_0 : \beta_2^{1*} = \beta_2^{2*} = 0$. The algebra for a model with three thresholds is easily computable and available upon request.

[16] Denoting by S_0 and S_1 the sum of squared residuals under the null hypothesis (linear panel) and the alternative hypothesis (PSTR model) respectively, the tests are computed as $LM = TN(S_0 - S_1)/S_1$ and $F = [(S_0 - S_1)/H]/[S_1/(TN - N - H)]$ (under the null hypothesis, LM follows a $\chi(H)$ distribution, while F follows a $F(H, TN - N - H)$ distribution).

elasticities that can depend in a rather complex manner on the level of the public debt-to-GDP ratio.

The estimation of a PSTR model

At the end of the identification procedure, the number of transition functions, as well as the number of thresholds for each transition function, is known. Next, we proceed to the estimation of the PSTR model (i.e. the threshold(s), the transition parameter(s), and the slopes), in a three-step procedure. For convergence issues, the first step consists of eliminating fixed effects, an operation which, contrary to a traditional linear panel model, can lead to some complications.[17] Since the quality of the initial values for the parameters γ and $\overline{B}_h$ is crucial for the convergence of the estimation, the second step focuses on this problem. Among several available methods, we use a "grid search" technique, which consists in generating combinations of vectors $(\gamma; \overline{B}_h)$, with arbitrary strictly positive γ-values and $\overline{B}_h$-values from the sample. For each vector, we use the traditional panel least squares technique to estimate the elasticities $\beta_{1;2}$, as well as the associated sum of squared residuals, which we employ to find the vector of initial values for $(\gamma; \overline{B}_h)$, as the one minimizing the sum of squared residuals. The third and last step of the algorithm consists in estimating the PSTR model using the Non-Linear Least Squares (NLLS) technique, based on iterations and consequently strongly dependant on the quality of the initial values. The final estimators for the parameters $(\gamma; \overline{B}_h)$ are the one that minimize the variance of residuals in the NLLS regression, and are being used in the panel least squares regression to estimate the slopes $\beta_{1;2}$.

Supplementary references for the Appendix

- Chan, K., Tong, H. (1986), "On estimating thresholds in autoregressive models", *Journal of Time Series Analysis* 7, 179-190.
- Hansen, B., (1999), "Threshold effects in non-dynamic panels: Estimation, testing, and inference", *Journal of Econometrics* 93, 345-368.

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[17] To eliminate fixed effects, all variables of the model (A1) (or (1) in the main text) are centered as usual around their means, except for the term including the transition function $G(.) \equiv B_{it}\Gamma(.)$, which becomes: $\tilde{G}(.) = G(.) - \overline{G}(.)$, with $\overline{G}(.) = T^{-1}\sum_{t=1}^{T}G(.) = T^{-1}\sum_{t=1}^{T}(B_{it}\Gamma(.))$. Consequently, the centered variable $\tilde{G}(.)$, used in the PSTR model without fixed effects, depends on the γ and $\overline{B}_h$ parameters in both $G(.)$ and $\overline{G}(.)$, which is why $\tilde{G}$ must be computed at each iteration, i.e. each time that γ and $\overline{B}_h$ take different values.