## Financial frictions and robust monetary policy in the models of New Keynesian framework

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#### Abstract

In this paper I study how financial frictions affect robustness of monetary policy in DSGE models in the case of model uncertainty. I consider a basic New Keynesian model, a financial accelerator model and a model with collateral constraints. Modeling monetary policy in terms of optimal interest rate rules, I find that welfare-maximizing policies for these three models are not robust to model uncertainty despite the fact that the models put non-competing perspectives about expectations formation and inflation persistence. Thereby I show that disparity of amplification mechanisms inherent to different financial frictions exerts adverse influence on monetary policy robustness. This finding has implications for monetary policymaking in the case of model uncertainty – when using a model with a particular type of frictions, a policymaker exposes economy to risks of significant welfare losses. To resolve this problem of non-robustness I propose an extension of fault tolerance methodological approach. I find that policy rule that is robust to model uncertainty is the one optimal for the financial accelerator model where output coefficient is sufficiently reduced.

Keywords: optimal monetary policy rules, financial frictions, DSGE models, robustness

JEL Classification: E32, E37, E44, E52

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### **1** Introduction

It has been claimed that model uncertainty is not a trivial problem for monetary policymaking (Greenspan, 2004, King, 2004). In particular, it is often (if not always) the case that central bank does not know the true structure of economy with full certainty, and thus has to allow for the possibility of economy to be represented by several models. The phenomenon of model uncertainty could be illustrated by a situation when the members of monetary policy committee do not agree on a model that represents the true structure of economy. Thus, a decision on the stance of monetary policy that has to be made by the committee has to be acceptable in all the alternative economy representations in order to be supported by all the committee members, i.e. the policy should be robust to model uncertainty.

A particular relevance of model uncertainty is induced by the fact that in aftermath of the financial crisis 2007-2009 there is a growing debate about what amplification mechanisms are conductive to economic distress. It has been widely acknowledged that financial factors have significantly contributed into the recent economic decline. But which of the factors play the principal role in economic developments is a subject to disagreement; considerable uncertainty surrounds the "true" amplification mechanism. While there is a number of studies revealing empirical relevance of financial accelerator mechanism: Bernanke et al. (1999), Carlstrom and Fuerst (1997), Mody and Taylor (2004), Aliaga-Diaz and Olivero (2010), Peersman and Smets (2005), Almeida et al. (2006), Cavalcanti (2010); there is also evidence on significance of collateral constraints as a factor behind aggregate fluctuations: Fazzari et al. (1988), Gertler et al. (1991), Gilchrist and Himmelberg (1995), Hubbard et al. (1995), Kashyap et al. (1994). Financial accelerator as a principal factor behind the financial crisis 2007-2009 has been advocated by Krishnamurthy (2010) and Geanakoplos (2009), whereas collateral constraints are supported by Chatterjee (2010) and Peralta-Alva (2011 a,b).

As a result, there is no consensus about a "correct" model that captures relevant type of frictions. Various models could arguably be used as economy representations for the analysis of monetary policy transmission mechanism, what increases relevance of the issue of model uncertainty with respect to financial frictions.

In the literature robustness of monetary policy to model uncertainty has been addressed within several methodological approaches. The first one proposed by Brainard (1967) and developed by Hansen and Sargent (2001a,b, 2002, 2003, 2007) considers robustness with respect to a benchmark model. Alternative models are supposed to lie around the benchmark at some small distance; thus, the set of alternative models could be thought of as being "local". In this approach optimal policy is found by solving minimax problem for the "cloud" of models surrounding the benchmark. This methodology is employed in a number of works analyzing monetary policy robustness, for example in Brock and Durlauf (2004, 2005), Giannoni (2002), Marcellino and Salmon (2002), Onatski and Stock (2002) and Tetlow et al. (2001). However, the range of alternative models considered in these works is restricted by focus of methodology on the small set of possible models; thus, non-nested models with competing perspectives about inflation persistence and expectations formation could hardly be analyzed in the context of this methodological approach.

An alternative approach to address model uncertainty is model averaging. It was initially advocated by McCallum (1988) who claimed that robust policy should be defined as one that works well enough in all the models considered; a robust rule might not be the best one for any of the models in the set but it should be acceptable (in terms of losses or welfare costs) for all the alternative models. The principal value of this approach is that it does not require alternative models to be close enough to the benchmark. This is important for analysis of monetary policy transmission mechanism, because possible economy representations are not necessarily similar. Indeed, these are disparate models of economy that one would typically want to take into account when looking for robust monetary policy. For example, this is the case of uncertainty about the factors that are behind the financial crisis of 2007-2009. Acknowledging this significance this paper adheres to model averaging methodology.

The model averaging approach is adopted in a number of works with the aim of arriving at interest rate rule - Taylor rule or another type of simple rules, - which is robust across a particular set of models. Brock et al. (2007) examine uncertainty about the suite of backward-looking models in style of Rudebusch and Svensson (1999) and hybrid models a la Rudebusch (2002) analyzing model uncertainty with respect to formulations of expectations and lag strength structure. Levine et al. (2008) study different variants of Smets and Wouters model (2003). Levin and Williams (2003) search for a simple rule which is robust to model uncertainty across the set of non-nested models: the basic New Keynesian model, backward-looking model in style of Rudebusch and Svensson (1999) and a hybrid New Keynesian model with backward-looking elements (Fuhrer, 2000). Thus, the idea of these works is to analyse the sets of models with competing perspectives about expectations formation and inflation persistence.

In this paper the focus is different. I look at the models that are equivalent in some aspects: for example, with respect to presence of nominal rigidities in imperfect competition setup, of inflation persistence and expectations formation. The motivation for this work comes from pronounced uncertainty about financial factors behind the recent economic decline; therefore I study the models that are different with respect to financial frictions incorporated in them. This setup allows seeing, first, what is the discrepancy in the models' transmission mechanisms arising exactly from this difference in frictions, and second, whether the contribution of difference in financial factors is big enough to generate non-robustness of monetary policy rules. The relevance of investigating robustness in this context is stipulated by the fact that adhering to a model that does not capture the "true" type of financial frictions might entail harmful welfare consequences.

The aim of this work is to establish how financial frictions affect robustness of monetary policy to uncertainty about types of frictions that drive aggregate fluctuations. I also attempt to find monetary policy that produces acceptable welfare outcomes in three models: basic New Keynesian model, financial accelerator model and a model with collateral constraints, assuming that all the models have equal weights as possible economy representations. First, I characterize policy rules that are optimal for each of the models. Second, I evaluate welfare consequences of adopting suboptimal policy rules in all the model economies. I demonstrate that no optimal policy rule is robust to model uncertainty: the welfare losses of adhering to optimal policies in alternative model economies are significant; this happens despite the fact that New Keynesian models I consider put non-competing perspectives about expectations formation and inflation persistence. I show that it is exactly due to the presence of financial frictions in the structure of the models why optimal policy rules are not robust. I demonstrate that following an interest rate rule optimal for the basic New Keynesian model - a model with no financial frictions embedded in it, - is welfare superior to adopting a rule which is optimal for a model with any type of frictions, because in the case of model uncertainty there is a positive probability of these frictions being "incorrect" (incorrect in the sense that they do not capture "true" amplification mechanism). Finally, by employing fault tolerance methodology, i.e. by considering welfare implications of deviations from the optimal policy rules parameter values, I obtain policy formulation that is robust to model uncertainty across the set of New Keynesian models examined here. I demonstrate that significant reduction of output coefficient in the policy rule optimal for financial accelerator model results in this rule being robust.

Monetary policy is modeled here in terms of optimal simple implementable interest rate rules (as in Schmitt-Grohe and Uribe, 2006, Faia and Monacelli, 2005 and Mendicino and Pescatori, 2005). I assume that a policymaker is able to commit to a rule. Optimality requires welfare maximization; simplicity means that interest rate should be a function of a small number of easily observable variables; implementability calls for unique rational expectations equilibrium delivered by a policy rule. The optimality criterion I use is welfare maximization (as in Schmitt-Grohe and Uribe, 2006 and Faia and Monacelli, 2005). This criterion differs from a conventional approach applied in literature on model uncertainty, which is quadratic loss function minimization (for example, in Cogley and Sargent, 2005, Levin and Williams, 2003, Cogley et al., 2011). On one hand, using welfare maximization to estimate parameters of optimal simple rules enables to be consistent with the microfoundations of DGSE models. On the other hand, this method is not perfect, because, first, it does not allow to account for distribution of welfare across households, as all the models considered feature representative agent framework. Second, within welfare maximization framework it is complicated to incorporate preferences of a policymaker that it might have due to institutional reasons (preference to smooth interest rate reversals that Caplin and Leahy, 1996 and Goodhart, 1996 argue for), due to concerns about financial market fragility (advocated by Lowe and Ellis, 1997), or due to political issues. Thus, choosing welfare maximization as an optimality criterion has its advantages and disadvantages; there seems to be no unobjectionable way to incorporate all empirically relevant aspects into methodology of optimal policy rule estimation. Nonetheless, pursuing the aim of consistency, staying in line with microfoundations and trying to avoid Lucas critique, this paper uses maximization of social welfare as a criterion of optimality.

To show that these are exactly financial frictions that drive the result of nonrobustness of optimal policy rules across the set of New Keynesian models, I consider the case of the models with the frictions in them being inactive. This results in the estimated optimal rules being robust – welfare costs of adopting them are small enough in all the models. Thereby I demonstrate that frictions are the reason of the optimal policy rules' non-robustness. I establish that policy rule optimal for the basic New Keynesian model is the closest one to be robust across the set of the models. The policy rules optimal for the models of financial accelerator and collateral constraints are not robust because adopting them in alternative model economies entails unacceptable welfare losses. So, there is no way to implement any simple rule that is optimal for a model with financial frictions without the risk of inducing unacceptably high welfare costs in the alternative model with different amplification mechanism. I conclude that following an interest rate rule that is optimal for the model with no financial frictions is welfare superior comparing to adopting a rule optimal for the model with the "wrong" type of frictions. In short, the consequences of a possible mistake of adopting the model with "wrong" type of financial frictions are significantly damaging.

I am using here an extension of the model averaging approach that has been proposed by Brock et al. (2007). This extension consists in reporting not only the robust policy rule, but also the effects of model uncertainty, so that a policymaker knows how the form of robust rule is affected by specific characteristics of divergent models taken into account. Incorporating this extension here I disclose degrees of outcome dispersion - how losses associated with optimal policy depend on a model, - and action dispersion - how optimal policy differs across alternative models.

The optimal rules are not robust to model uncertainty due to substantial differences between the model of financial accelerator and the model of collateral constraints. In addition, amplification mechanism of collateral constraints is stronger than the one of financial accelerator in present calibration. Partly the reason for this is that the model of collateral constraints features sizable asset price shock. This shock improves asset price performance in the housing and collateral constraints model, what enhances amplification of output (Iacoviello, 2005). As a result, collateral constraints model calls for substantial output stabilization comparing to the financial accelerator model, whereas the latter one requires to focus on stabilization of inflation. To sum, different type of credit frictions entail policy transmission mechanisms in DSGE models that are disparate to a large extent. As a result, there is no robust rule that performs reasonably well in all of the models considered.

Another extension of the model averaging approach adopted in this paper is the fault tolerance methodology suggested by Levin and Williams (2003). Its goal is to ascertain whether a robust rule is attainable across the set of models by estimating welfare implications of deviations from optimal policy. This paper proposes an extension to this approach. I estimate how tolerant are model economies in welfare sense to deviations from optimal policy for all the policy rules' parameters and all the models' optimal rules. This enables to establish whether a robust rule across the set of models is attainable and how optimal rules of the certain models are to be modified so that they are robust across the set of models. I find that to obtain a policy rule that is robust to model uncertainty with respect to financial frictions one should reduce an output coefficient in the rule optimal for the financial accelerator model. Though, this rule is not optimal for any of the models (it does not deliver the highest social welfare in any of the models), it yields acceptable performance in welfare sense in all the model economies.

This paper is organized as follows. Section 2 presents three models analyzed for robustness of monetary policy. Section 3 presents the monetary policy setup, welfare measure and discusses the results of robustness analysis. Section 4 investigates fault tolerance of the models. Section 5 concludes.

### 2 The Models

To analyse the impact of different amplification mechanisms on monetary policy robustness the models studied here are similar in many respects but not in financial frictions. The suite of models includes a basic New Keynesian model, a financial accelerator model and a model of collateral constraints. All the models are forward-looking, contain no inflation persistence and account for nominal stickiness and monopolistic competition. In all the models studied here monetary policy plays an active role in stabilizing economy because of long-term money neutrality and short-term nominal inertia. All the models incorporate shock to government spending, monetary policy shock and productivity shock. Additionally, models with frictions account for some specific shocks; contribution of these particular shocks is evaluated by simulating the models with and without these particular shocks. In what follows I introduce main characteristics of the models. Equilibrium conditions and parameters' calibration used for simulations are presented in the Appendices 1-4.

#### 2.1 Basic New Keynesian model

The standard forward-looking basic New Keynesian model (BNK) (Clarida et al., 1999) is as a benchmark model here, as it does not incorporate any financial factors. The model accounts for purely forward-looking output and inflation; dynamics is entirely due to exogenous force processes without endogenous persistence; outcomes depend on agents' expectations. The baseline BNK model features no investment and no capital. The version of the BNK model studied here is taken unaltered in its standard form (Walsh, 2010). Equilibrium conditions and calibrated parameter values for BNK model are given in Appendix 1 and 4.

BNK model features a negative effect of interest rate on output. Current output depends on the expectations of future consumption, what is consistent with the preference of the agents to smooth consumption. Nominal prices are set based on future marginal costs; this indicates no inertia or lagged dependence in inflation. Inflation ultimately depends on movements in marginal costs, associated with variation in excess demand. The monetary policy rule that closes the model is presented in the section 3.

The BNK model here incorporates three types of exogenous disturbances: shock to government spending, productivity and monetary policy shock.

#### 2.2 Financial accelerator model

This model adopts the financial accelerator (FA) framework developed in Bernanke et al. (1989). It incorporates credit market frictions by modeling borrowers and lenders of capital explicitly. Frictions arise from an agency problem that comes from informational asymmetries (profitability of borrowers is private information) and entailed agency costs between borrowers and lenders. In the costly state verification setup (Townsend, 1979) the optimal contract is a standard debt contract where entrepreneur's payment is independent of realization of her idiosyncratic productivity. When entrepreneur cannot repay, the lender pays verification cost as a share of entrepreneur's assets and takes over her entire project.

The model manifests the cost of external funds higher than the cost of internal funds. It also sets out how procyclical net worth of borrowers affects demand for investments, thus giving rise to the amplification of the shocks. Thus, financial sector in the FA model propagates exogenous disturbances because net worth depends on return to capital disproportionally due to the leverage effect.

Here the FA model specification of that features nominal stickiness follows Christensen and Dib (2008). Apart from the standard monetary policy, productivity and government spending shocks, the FA model features preference shock, money demand shock and investment specific shock. As argued in Bernanke et al. (1999) and Christensen and Dib (2008), investment specific shock is a crucial one to improve the performance of financial accelerator as an amplification mechanism to produce empirically relevant results.

As results of simulations show, the role of financial accelerator mechanism in investment fluctuations depends on the nature of the shock. Financial accelerator amplifies and propagates the effects of demand shocks – monetary policy, money demand and preference shocks – on investment. At the same time financial accelerator pushes down the response of investment to supply side shocks – technology and investment-efficiency shocks (Christensen and Dib, 2008).

Equilibrium conditions and calibrated parameter values for the FA model are in Appendix 2 and 4 respectively.

#### 2.3 Model with collateral constraints

The third model is a New Keynesian model with housing and collateral constraints (HCC). There are three types of agents in the HCC model: entrepreneurs, liquidity-constrained households and unconstrained households. As proposed by Iacoviello (2005) this model incorporates housing used by the borrowers (entrepreneurs and constrained households) as collateral.

The HCC model incorporates a rich endogenous propagation mechanism that conducts exogenous disturbances to affect output: beyond workings of financial accelerator the change of asset prices affects borrowing capacity of the debtors. Assuming that constrained households have a strong preference for current consumption, growing housing prices induce more than proportional rise of borrowing and consumption, which in its turn has an influence on aggregate demand. Debt deflation also contributes to the changes in value of the borrowers' net worth. Thus the demand shocks are amplified in the HCC model. At the same time inflation depresses the influence of supply shocks that induce negative correlation between output and inflation. So, the influence of the supply shocks in this model is contracted in the same way as in the FA model. In addition to standard shocks (monetary policy, productivity and government spending), the HCC model accounts for cost-push shock, housing price and preference for housing shocks. The specification of the HCC model used in this work is taken from Iacoviello (2005) unaltered. Equilibrium conditions and calibrated parameter values for HCC model are in Appendix 3 and 4 respectively.

#### 3 Monetary Policy and Welfare Measure

I assume that monetary policy is conducted by means of interest rate reaction function that is simple, optimal and implementable in style of Schmitt-Grohe and Uribe (2006). It implies that, first, interest rate should be a function of a small number of easily observable variables. Second, this reaction function should maximize social welfare. Third, the rule should deliver a unique rational expectations equilibrium.

Thus, I assume that monetary policy is conducted by means of policy rule of the following form:

$$\ln\left(\frac{r_t}{r}\right) = \rho * \ln\left(\frac{r_{t-1}}{r}\right) + \alpha_\pi * \ln\left(\frac{\pi_t}{\pi}\right) + \alpha_y * \ln\left(\frac{y_t}{y}\right), \tag{1}$$

where  $r_t$  is the gross nominal interest rate,  $\pi_t$  is inflation rate and  $y_t$  is output. The variables without subscripts denote their steady state values. Thus, this type of interest rate rule features deviations of each variable from its steady state value. The calibrated parameter values are such that the steady state value of inflation is set to zero in all the models; financial inefficiencies introduced in the model don't induce steady state inflation to be different from zero.

I assume that policymaker can commit to rule (1) and maximizes social welfare subject to equilibrium conditions of the models and a policy rule to find optimal parameter values  $\rho$ ,  $\alpha_{\pi}$ ,  $\alpha_{y}$ . Welfare maximization is not a common optimality criterion in the literature on monetary policy robustness. Most of the papers (Clarida et al., 1999, Cogley and Sargent, 2005, Levin and Williams, 2003, Cogley et al., 2011, etc.) use a criterion of quadratic loss function minimization. I have run optimization with respect to this criterion as well and have obtained similar results. The role of optimality criterion for the problem of monetary policy robustness could be an interesting direction of future research.

To make inferences about robustness of three optimal rules I run each of three model economies with all the policy rules and evaluate welfare costs of adopting suboptimal rules in these cases. First, I evaluate welfare in the BNK, FA and HCC models sequentially applying alternative specifications of (1), which are three simple policy rules identified as optimal for the models. Second, I compute welfare costs of adopting alternative rules relative to the equilibrium path associated with the optimal rule. In doing this I rely on a second-order approximation of the model's solution. The first-order approximation is not acceptable for the purpose of welfare comparison, because the implied expected values of variables coincide with their non-stochastic steady state; as a result, the volatility effect on variables is neglected (more on this is in Kim and Kim (2003) and Schmitt-Grohe and Uribe (2006)).

The welfare associated with the optimal policy rule conditional on a particular state of the economy in period 0 is:

$$\widetilde{W}_0 = E_0 \sum_{t=0}^{\infty} \beta^t U(\widetilde{C}_t, \widetilde{N}_t),$$
(2)

where  $E_0$  is conditional expectation over the initial state and  $\tilde{C}_t$  and  $\tilde{N}_t$  are contingent plans for consumption and hours worked under the optimal policy rule. Analogously, the welfare associated with the alternative policy rule conditional on a particular initial state of economy is an appropriate aggregation of contingent plans for consumption and hours under the alternative rule  $C_t^a$  and  $N_t^a$ :

$$W_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^a, N_t^a).$$
(3)

The use of conditionally expected discounted utility of the representative agent allows to account for transitional effects from non-stochastic steady state to an equilibrium path implied by alternative policy rules.

Welfare costs  $\lambda$  are measured as a fraction of consumption a representative household would agree to be compensated in order to gain the same level of welfare as under the optimal rule:

$$W_{0,\lambda} = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^a * (1+\lambda), N_t^a) = \widetilde{W}_0.$$
(4)

The level of  $\lambda$  for the HCC model is obtained by solving (4) for the given specification of utility function in HCC model (Appendix 3):

$$\lambda = \exp\left(\left(\widetilde{W_0} - W_0^a\right) * (1 - \beta)\right) - 1.$$
(5)

In calculating welfare cost for the HCC model I only account for welfare of patient (unconstrained) households; welfare of entrepreneurs and constrained households is disregarded as fractions of their consumption in the total welfare in negligibly small.

Because it is impossible to derive the level of  $\lambda$  analytically for the BNK and FA models given their utility specifications, I estimate welfare costs for these models by numerical search for  $\lambda$  over the grid so that condition (4) is satisfied.

In what follows in this section I set out my calculations and discuss the results.

For each model I search numerically for parameter values  $\rho$ ,  $\alpha_{\pi}$ ,  $\alpha_{y}$  that maximize households' welfare – they specify an optimal simple rule for every model. Parameter  $\rho$  is restricted to lie on the interval [0, 0.99],  $\alpha_{\pi}$  – on the interval [1, 3] (the values below 1 result in rational expectations equilibrium indeterminacy) and  $\alpha_{y}$  – on the interval [0, 3]. In this numerical search I solve the models to obtain second order approximation of the policy functions around non-stochastic steady state. The parameters of the optimal policy rules for all the models are given in Table 1.

One can see that policy rule parameters optimal for the HCC model are substantially different from those in the BNK and FA optimal rules. The reason for it is a stronger amplification mechanism in the collateral constraints model that produces sizable response in output (see Table 2 for output, inflation and interest rate variances). Thus, an optimal reaction to this amplification for a policymaker is to focus primarily on output stabilization, not on inflation stabilization; whereby the coefficient  $\alpha_y = 3$ , what is substantially higher than  $\alpha_y$  in BNK and FA optimal reactions, and  $\alpha_{\pi} = 1.5$  – the lowest inflation coefficient out of all obtained. HCC optimal response function features high interest smoothing motive:  $\rho = 0.99$ . By simulating HCC model with and without housing preference shock I find out that it is exactly due to presence of this shock in the HCC model why interest rate smoothing coefficient is high. Sizable exogenous disturbance to housing preferences affects

variability of savings of patient (unconstrained) households and borrowings of impatient households, their variances increase. This induces interest rate to vary as well, what entails welfare losses due to impaired ability of households to forecast future path of interest rates to smooth consumption intertemporaly. Thus, an optimal response to this mechanism calls for substantial interest rate smoothing, what implies a high value of  $\rho$  parameter in the HCC model.

Optimal rules for the models				Conditional welfare costs in the models, λ * 100, % (frictions on)			Conditional welfare costs in the models, $\lambda * 100$ , % (frictions off)	
<u>Frictions on</u>	ρ	$\alpha_{\pi}$	$\alpha_y$	BNK	FA	НСС	FA	HCC
BNK optimal rule	0.52	3	0	0	1.34	1.73	0.00	0.21
FA optimal rule	0.47	3	0.8	7.58	0	0.74	-	-
HCC optimal rule	0.99	1.5	3	23.59	5.84	0	-	-
<u>Frictions off</u>								
FA optimal rule	0.99	3	0	0.00	-	-	0	0.17
HCC optimal rule	0.99	1.1	0.54	0.002	-	-	4.24	0
HCC optimal rule (no shock to housing preferences)	0.99	3	0.67	0.0003	-	-	0.8	0.66

Table 1. Models'	optimal	rules performa	nce
	-	-	

The FA optimal rule is fairly similar to the one of the BNK model. Output coefficient different from zero is a mark of the financial accelerator at work. Financial accelerator amplifies the variables less than the HCC model and in the different way: there is a motive to stabilize inflation more ( $\alpha_{\pi} = 3$ ) than to stabilize output ( $\alpha_{y} = 0.8$ ). Apart from an amplified output in the FA model there are negligible differences between the BNK and the FA optimal policy rules.

Table 2. Standard deviation of variables under optimal rules

	Optimal rule								
	BNK				FA		НСС		
		Performance in the models							
Standard deviation:	BNK	FA	HCC	BNK	FA	HCC	BNK	FA	HCC
Interest rate - $\sigma_r$	0.004	0.003	0.2	0.004	0.006	0.2	0.01	0.011	0.73
Inflation - $\sigma_{\pi}$	0.001	0.0006	0.064	0.001	0.004	0.14	0.01	0.013	0.69
Output - $\sigma_y$	0.011	0.027	0.646	0.011	0.011	0.55	0.005	0.006	0.37

FA optimal rule induces poor performance in the BNK economy: welfare cost of adopting the FA rule is 7.58% of consumption. The HCC optimal rule performs even worse in the BNK model:  $\lambda = 23.59\%$ , what reflects an amplification mechanism in the HCC model being incompatible with the BNK optimal policy. The same is valid for the case of the HCC rule applied to FA economy: welfare cost in the FA model is unacceptably high: 5.84%. It is interesting that the FA optimal rule performs well in the HCC model economy; the welfare cost of adopting a suboptimal FA policy rule is only 0.74%. This reveals importance for the

HCC economy of output stabilization.  $\alpha_y$  in FA rule is small but greater than zero, which is enough to generate good performance of HCC model, due to the fact that output is stabilized to a sizable extent there, what is crucial for the HCC model economy.

The policy rule that performs the best across the set of three models is the BNK rule. The implied welfare losses in the FA and HCC models are respectively 1.34% and 1.73%. So, in the case of uncertainty about "true" type of financial frictions it is "safer" to resort to BNK model optimal rule – a rule of the model with no financial frictions in it - than to make a "mistake" by adopting a policy rule with a "wrong" type of frictions and incur damaging welfare losses.

Next I am to answer the question – are these exactly the frictions in the FA and HCC models that are the reason of non-robustness of FA and HCC optimal policy rules? Or the optimal policy rules are non-robust because of some other modeling features different in the models? To study this I simulate all the model economies with the frictions in them being inactive. To close the financial accelerator mechanism in the FA model I set elasticity of the external finance premium with respect to firm leverage ratio equal to zero. Thus, the marginal external finance cost does not depend on gross premium for external funds, what disables financial accelerator. To "switch off" the collateral constraints mechanism in the HCC model requires closing the asset price channel in the model. To do this I modify housing/consumption margin for entrepreneurs and constrained households. The resulting margins feature the borrowing limit being a constant independent from the asset value. Details of equilibrium conditions' modification are in Appendix 3.

As seen from the right panel of Table 1, closing the financial accelerator and collateral constraints mechanisms (and housing preference shock in HCC model) results in all the optimal rules being robust to model uncertainty. With the frictions closed in all the models the welfare consequences of adopting a suboptimal policy rule are small and acceptable.

Thus, these are exactly amplification mechanisms of financial frictions that contribute into non-robustness of FA and HCC policy rules in the case when the frictions are active in the models. Therefore, "getting the frictions right" is important in models used for monetary policy analysis. Acknowledging present uncertainty about types of financial frictions one cannot disregard the consequences of making a mistake by adopting a policy rule that is optimal for either a model with financial accelerator or a model with collateral constraints. If there is no clear evidence about true amplification mechanisms at work to rely upon, the best option is to resort to the policy rule optimal for the model with no financial frictions at all.

### 4 Fault tolerance

The fault tolerance approach was initially proposed for analysis of monetary policy robustness by Levin and Williams (2003). This method is a test of how tolerant is a model economy to deviations from optimal policy. In this section I argue that the results of the original fault tolerance approach by Levin and Williams (2003) are not informative about whether a robust rule could be obtained across the set of models and about what the

parameters of the robust policy rule are. I propose to amend the original methodology so that these questions could be answered.

The idea of fault tolerance approach is to estimate how sensitive a model is to deviations from optimal monetary policy. Levin and Williams (2003) suggest that this is to be implemented by quantifying loss implied by changing a value of one policy rule parameter while holding all the other parameter values fixed at their optimal levels. Their original proposal is to fix the "non-fault" parameter values (for example,  $\alpha_{\pi}$  and  $\alpha_{y}$  if sensitivity to deviations in  $\rho$  is analysed) at the levels optimal for the model which is being analysed for fault tolerance. In short, in Levin and Williams' (2003) interpretation the values of "non-fault" parameters should coincide with their optimal values – as they are in the policy rule optimal for a model analysed. For instance, to estimate fault tolerance of HCC model to deviations in  $\rho$  parameters equal to their optimal values in the policy rule that is optimal for HCC model. Models' simulations with  $\alpha_{\pi}$  and  $\alpha_{y}$  at their BNK and FA optimal values are disregarded in the fault tolerance methodology suggested by Levin and Williams (2003). Thus, every model is tested for fault tolerance only to deviations from the parameter model's optimal policy.

This implies that in the original fault tolerance approach one should conduct as many experiments as there are parameters in the policy rule. First, one should estimate welfare losses in all the models by changing values of  $\rho$  while keeping  $\alpha_{\pi}$  and  $\alpha_{\nu}$  fixed: for the BNK model - at their optimal for BNK model simple rule values; for the FA model - at their optimal for FA model simple rule values; and for the HCC model – at their optimal for HCC model simple rule values. Then one is to see whether there are overlapping intervals of acceptable losses on the  $\rho$  scale for all three models implied by this change of  $\rho$  parameter. The second experiment should be in changing the value of  $\alpha_{\pi}$  parameter values while holding fixed the values of  $\rho$  and  $\alpha_{v}$ : for the BNK model - at their optimal BNK rule levels, for the FA model - at their optimal FA levels and for the HCC model - at their optimal HCC levels. The third experiment would consist in varying the value of  $\alpha_y$  in the similar fashion for all the models. Levin and Williams (2003) claim that in order for a robust policy rule to be attainable, there should be acceptable losses on overlapping intervals for three policy rule parameters. They argue that if loss function is relatively insensitive to changes in all three parameters then there exists a robust policy; and for that to be the case there should be overlapping regions of high fault tolerance of all the models with respect to changes in all the parameters of the policy rule.

However, this reasoning does not seem to be accurate. A robust rule across the set of models cannot have different parameter vales for different models. A robust rule a policymaker is interested in is a particular rule that works well enough for all the possible models. Thus, it is a rule for all the models; the welfare implications of following this rule are acceptable in all the model economies for these parameter values. Levin and Williams' method does not test for this because their approach is limited to fixing the "non-fault" parameter value only on the level optimal for a specific model optimal rule, and these levels are different for all the models. As a result there is no way to see whether the losses in all the models would be acceptable if different policy rules are adopted in them.

Thus, I suggest that the original fault tolerance methodology should be extended. This extension consists in testing the models' tolerance to deviations from the policy rules optimal for all the models. So, the models' performance for all the three sets of "non-fault" parameters fixed at their BNK, FA and HCC optimal values should be tested. Thus, nine experiments are to be conducted for the extended version of the fault tolerance approach in the context of this paper: for each of the model's optimal rules and for each of three deviating parameters' values. And then a robust rule is attainable if there is at least one out of nine experiments where the interval of overlapping acceptable losses is present. A robust rule is the one where two "non-fault" parameters are fixed on their optimal values of some optimal policy rule and the third parameter should lie somewhere on the overlapping interval of acceptable losses. This amendment of fault tolerance approach allows to see if modification of any of three optimal rules could result in the rule being robust. Besides, it allows to find out what are the parameters of the robust policy rule.

The results of adopting the fault tolerance methodology with this extension are presented graphically in the Appendix 5, Figures 1-9. Deviations from optimal policies are implemented in order to search for the possible cases where changing one parameter of an optimal policy improves performance, i.e. reduces welfare costs, in the alternative models. Analysing the graphs with the results of fault tolerance experiment one can see the following. Changing the value of  $\rho$  parameter in the BNK optimal rule does not improve welfare in the FA and HCC models – welfare costs are not acceptable in the FA and HCC models when the BNK model optimal simple rule is applied when the values of interest rate smoothing parameter is modified (Appendix 5, Figure 1).

The same applied to deviations of  $\alpha_{\pi}$  parameter in BNK policy rule (Appendix 5, Figure 2). The situation is different with output coefficient in BNK policy rule being changed (Appendix 5, Figure 3): increasing  $\alpha_y$  to 0.8-1.4 reduces welfare losses in the alternative models down to acceptable levels. However, the graph illustrates that this increase is harmful for the BNK model: the welfare loss of this change costs 7% of consumption and is larger for the higher values of  $\alpha_y$ .

This result for BNK rule could be interpreted as follows. If there is certainty that either of two mechanisms – financial accelerator or collateral constraints – is the "true" one, and thus, a BNK model is disregarded as a possible representation of economy, then the robust policy rule could be obtained from the BNK rule by increase in  $\alpha_y$  coefficient up to 0.8-1.4. But when BNK is conceived as a plausible model economy and one cannot ignore negative welfare consequences in BNK model of increasing output coefficient, then there is no way to improve the performance of BNK policy rule comparing with its optimal form, which implies welfare costs of 1.34 and 1.73% in the FA and HCC models. Whether this level of welfare costs is acceptable and then the optimal form of BNK rule could be considered as robust across the set of BNK, FA and HCC models is a matter of defining the threshold of acceptable welfare costs.

Changing interest rate and inflation coefficients in the FA optimal rule does not result in welfare costs decrease (Appendix 5, Figures 4 and 5). Reducing output coefficient in the FA model has positive welfare implications for the BNK model. Setting  $\alpha_y$  to 0 results in welfare costs for the BNK model reduction from 7.5% to 0%. However, this reduction is not good for FA and HCC models – the costs rise to about 2% in these models when  $\alpha_y = 0$ . A compromise change of output coefficient is to set its value to 0.2. This results in welfare cost in BNK model being equal to 1.2%, in FA model – 0.69% and 1.38% in the HCC model. This is the best result obtainable throughout the set of models and could be regarded as a robust to model uncertainty.

Changing parameter values in the HCC optimal rule does not lead to sizable reduction in the welfare costs (Appendix 5, Figures 7, 8 and 9). Thus, there is no way to improve this rule's performance by adhering to suboptimal policy in search of robust rule.

In sum, applying the fault tolerance methodology enables to obtain a robust monetary policy rule. Though, this robust rule specification is not the optimal one for any of the models in the set, applying it to all the model economies entails acceptable losses. So, in the case of model uncertainty about type of financial frictions (financial accelerator, collateral constraints or no frictions at all) the best rule is obtained by adopting a modified policy rule optimal for the FA model with output coefficient appropriately reduced (to 0.2 in the current calibration).

### **5** Conclusion

This paper demonstrates that uncertainty about financial frictions in DSGE models is not a trivial issue. I show that when there is no certainty about what financial amplification mechanism should be at work – financial accelerator or collateral constraints - neither of them could be used in a DSGE model to infer a policy rule robust to model uncertainty. Policy rules that are optimal for the financial accelerator model and the model of collateral constraints entail unacceptably high losses in the alternative model economies and thus are not robust. I establish that these are exactly financial frictions that drive this result. A better strategy one can use in the case of uncertainty about frictions is to follow a policy rule which is optimal for the model with no financial frictions at all – for example, a rule optimal for the basic New Keynesian model.

I show how by adopting the fault tolerance approach (Levin and Williams approach, 2003) and extending it one can find a policy robust across the set of models. Sizable reduction of output coefficient in the policy rule optimal for the financial accelerator model results in this rule being robust as it delivers satisfactory welfare performance in all the models.

This paper is not a comprehensive study of the model uncertainty problem. A number of questions should be answered in order to obtain a full strategy of dealing with model uncertainty for the purposes of policymaking. First, as noted above, there is a marked increase in the number of models with divergent financial factors and amplification mechanisms aiming to capture important regularities: the models of financial intermediation (Adrian and Shin, 2010, Brunnermeier and Pedersen 2009, Gertler and Karadi, 2011), the transmission of contagion (Mendoza and Quadrini, 2010), model economies with asset price bubbles (Farhi and Tirole, 2011, Martin and Ventura, 2011), credit shocks (Christiano et al., 2008, Del Negro et al., 2010) and other financial factors. In principle, not all of the models could and would be reasonable to be considered in search of robust monetary policy rule as possible true representations of economy. Thus, one direction of future research on model uncertainty is to develop methodology on model selection. A strategy is to be developed on how to select models for the set of possible economy representations so that only relevant models are taken into account when searching for a robust policy rule. Second, Bayesian updating could be used in the current context so that prior beliefs about the probabilities about each model being a "true" one and their updating are incorporated in the analysis (as in Cogley et al., 2011 or Brock et al., 2007). This could possibly lead to a more realistic setup comparing to the equal weights methodology of the model averaging approach used here.

## Appendix 1

Equilibrium conditions of the basic New Keynesian model: Walsh (2010). Variables without time subscripts denote steady state values of these variables.

$$c_t^{-\sigma} = \beta * r_t * c_{t+1}^{-\sigma} / \pi_{t+1}$$

$$\chi * \frac{n_t^{\eta}}{c_t^{-\sigma}} = w_t$$

$$mc_t = \frac{w_t}{z_t}$$

$$1 = \omega * \pi_t^{\theta-1} + (1-\omega) * p_t^{1-\theta}$$

$$x 1_t = c_t^{1-\sigma} * mc_t + \omega * \beta * x 1_{t+1} * \pi_{t+1}^{\theta}$$

$$x 2_t = c_t^{1-\sigma} + \omega * \beta * x 2_{t+1} * \pi_{t+1}^{\theta-1}$$

$$p_t = \frac{\theta}{\theta-1} * \frac{x 1_t}{x 2_t}$$

$$y_t = z_t * n_t$$

$$y_t = c_t + g_t$$

$$\log(z_t) = \rho_z * \log(z_{t-1}) + \epsilon_{zt}$$

$$\log\left(\frac{g_t}{g}\right) = \rho_g * \log\left(\frac{g_{t-1}}{g}\right) + \epsilon_{gt}$$

$$\ln\left(\frac{r_t}{r}\right) = \rho * \ln\left(\frac{r_{t-1}}{r}\right) + \alpha_\pi * \ln\left(\frac{\pi_t}{\pi}\right) + \alpha_y * \ln\left(\frac{y_t}{y}\right) + \epsilon_{rt}$$

Representative agent utility function:

$$U(C_t, N_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi * \frac{n_t^{1+\eta}}{1+\eta}$$

Variables and parameters

Description	Notation
Household consumption	Ct
Household labour supply	$n_t$
Marginal costs	$mc_t$
Government spending	$g_t$
Output	y <sub>t</sub>
Productivity	$Z_t$
Real aggregate price level	$p_t$
Inflation	$\pi_t$
Gross nominal interest rate	$r_t$
Real wage	w <sub>t</sub>
Productivity shock innovation	$\epsilon_{zt}$
Shock to government spending innovation	$\epsilon_{gt}$
Monetary policy shock innovation	$\epsilon_{rt}$
Auxiliary variables	$x1_t, x2_t$
Discount rate	β
Relative risk aversion	σ
Weight of labour in the utility function	χ
Labour aversion	η
Calvo parameter	ω
Price elasticity of demand for each good variety	θ
Coefficients on lagged interest rate, inflation and output in the interest rate policy rule	$\rho, \alpha_{\pi}, \alpha_{\nu}$

## Appendix 2

Equilibrium conditions of the financial accelerator model: Christensen and Dib (2008).

Hatted variables denote log-deviations of these variables from their steady state values. Variables without time subscripts denote steady state values of these variables.

$$\begin{split} \left((1-\gamma)*\lambda*c-1\right)*\hat{c}_{t} &=\lambda*m*\frac{r_{t}-1}{r_{t}}*\left(\hat{b}_{t}+(\gamma-1)*\hat{m}_{t}\right)-\gamma*\hat{c}_{t}\\ &\gamma*\hat{r}_{t}/(r-1)=\hat{b}_{t}+\hat{c}_{t}-\hat{m}_{t}\\ &h*\hat{h}_{t}=(1-h)*\left(\hat{w}_{t}+\hat{\lambda}_{t}\right)\\ \hat{y}_{t} &=\alpha*\hat{k}_{t}+(1-\alpha)*\hat{h}_{t}+(1-\alpha)*\hat{A}_{t}\\ &\hat{y}_{t}*y=c*\hat{c}_{t}+i*\hat{\iota}_{t}+g*\hat{g}_{t}\\ &\hat{w}_{t}=\hat{y}_{t}+\hat{c}_{t}-\hat{h}_{t}\\ &\hat{z}_{t}=\hat{y}_{t}+\hat{c}_{t}-\hat{h}_{t}\\ &\hat{f}_{t}=\hat{m}_{t}-\hat{m}_{t-1}+\hat{m}_{t}\\ &\hat{f}_{t}=\hat{f}_{t}*\hat{z}_{t}+\frac{1-\delta}{f}*\hat{q}_{t}-\hat{q}_{t-1}\\ &\hat{q}_{t}=\chi*\left(\hat{\iota}-\hat{k}_{t}\right)-\hat{x}_{t}\\ &\hat{\pi}_{t}=\beta*\hat{\pi}_{t+1}+(1-\beta*\phi)*\frac{1-\phi}{\phi}*\hat{c}_{t}\\ &\hat{\pi}_{t+1}=\hat{\lambda}_{t}-\hat{r}_{t}+\hat{\pi}_{t+1}\\ &\hat{k}_{t+1}=\hat{s}+\hat{\iota}_{t}+\delta*\hat{x}_{t}+(1-\delta)*\hat{k}_{t}\\ &\hat{\pi}_{t+1}=\hat{\kappa}+\hat{\tau}_{t}+\hat{w}+\hat{q}_{t}+\hat{\kappa}_{t+1}-\hat{\pi}_{t+1})\\ &\hat{\pi}_{t+1}=\hat{r}_{t}-\hat{\pi}_{t+1}+\psi*\left(\hat{q}_{t}+\hat{k}_{t+1}-\hat{\pi}_{t+1}\right)\\ &\hat{n}_{t}(\hat{r}_{t})&=\rho*\ln\left(\frac{r_{t-1}}{r}\right)-\psi*\left(\frac{k}{n}-1\right)*\left(k_{t}+\hat{q}_{t-1}\right)+\left(\psi*\left(\frac{k}{n}-1\right)+1\right)*\hat{\pi}_{t}\\ &\hat{u}&=\rho_{0}*\hat{b}_{t-1}+\hat{c}_{bt}\\ &\hat{\lambda}_{t}&=\rho_{0}*\hat{b}_{t-1}+\hat{c}_{bt}\\ &\hat{\lambda}_{t}&=\rho_{0}*\hat{\delta}_{t-1}+\hat{c}_{st}\\ &\hat{y}_{t}&=\rho_{0}*\hat{x}_{t-1}+\hat{c}_{st} \end{split}$$

To make financial accelerator mechanism inactive I set elasticity of external financial premium to firm leverage ratio equal to zero:  $\psi = 0$ .

Representative agent utility function:

$$u(.) = \gamma * \frac{e_t}{\gamma - 1} * \log\left(c_t^{\frac{\gamma - 1}{\gamma}} + b_t^{\left(\frac{1}{\gamma}\right)} * m_t^{\frac{\gamma - 1}{\gamma}}\right) + \eta * \log(1 - h_t)$$

## Variables and parameters

Description	Notation
Household consumption	Ct
Household labour supply	$h_t$
Net worth	$n_t$
Government spending	$g_t$
Output	$y_t$
Productivity	$A_t$
Gross nominal interest rate	r <sub>t</sub>
Real wage	w <sub>t</sub>
Lagrange multiplier	$\lambda_t$
Real money balances	$m_t$
Aggregate capital	$k_t$
Aggregate investment	i <sub>t</sub>
Lagrange multiplier associated with production function	$\epsilon_t$
Real marginal productivity of capital	Zt
Money growth	$\mu_t$
Inflation	$\pi_t$
Real interest rate on external borrowed funds	$f_t$
Price of capital	$q_t$
Weight of preference for consumption	et
Money demand	$b_t$
Investment specific productivity	x <sub>t</sub>
Preference shock innovation	$\epsilon_{et}$
Money demand shock innovation	$\epsilon_{bt}$
Investment specific shock innovation	$\epsilon_{xt}$
Productivity shock innovation	$\epsilon_{At}$
Shock to government spending innovation	$\epsilon_{at}$
Monetary policy shock innovation	$\epsilon_{rt}$
Constant elasticity of substitution between consumption	γ
and real money balances	-
Weight of leisure in the utility function	η
Price elasticity of demand for each good variety	θ
Capital adjustment costs	χ
Capital share	α
Depreciation rate	δ
Probability of survival of entrepreneurs	υ
Discount factor	β
Elasticity of external finance premium to firm leverage ratio	$\psi$
Calvo parameter	$\phi$
Coefficients on lagged interest rate, inflation and output in the interest rate policy rule	$\rho, \alpha_{\pi}, \alpha_{y}$

## Appendix 3

Equilibrium conditions of collateral constraints model: Iacoviello (2005).

Variables with time subscripts denote log-deviations of the steady state values of these variables. Variables without time subscripts denote steady state values of these variables.

$$\begin{split} Y_t &= \frac{c}{Y} * c_t + \frac{c'}{Y} * c_t' + \frac{c''}{Y} * c_t'' + \frac{l}{Y} * l_t + \frac{g}{Y} * g_t \\ & c_t' = c_{t+1}' - rr_t \\ \\ l_t - K_{t-1} &= \gamma * (l_{t+1} - K_t) + \frac{1 - \gamma * (1 - \delta)}{\psi} * (Y_{t+1} - X_{t+1} - K_t) + \frac{1}{\psi} * (c_t - c_{t+1}) \\ q_t &= \gamma_e * q_{t+1} + (1 - \gamma_e) * (Y_{t+1} - X_{t+1} - h_t) - m * \beta * rr_t - (1 - m * \beta) * \Delta c_{t+1} - \phi_e \\ &* (\Delta h_t - \gamma \Delta h_{t+1}) \\ q_t &= \gamma_h * q_{t+1} + (1 - \gamma_h) * (j_t - h''_t) - m'' * \beta * rr_t - (1 - m'' * \beta) * (c_t'' - \omega * c_{t+1}'') - \phi_h \\ &* (\Delta h_t'' - \beta'' \Delta h''_{t+1}) \\ q_t &= \beta * q_{t+1} + (1 - \beta) * j_t + i * h_t + i'' * h_t'' + c_t' - \beta * c_{t+1}' + \phi_h / h' * (h\Delta h_t + h''\Delta h_t'' - \beta \\ &* h\Delta h_{t+1} - \beta'' h'' \Delta h''_{t+1}) \\ b_t &= q_{t+1} + h_t - rr_t \\ b_t'' &= q_{t+1} + h_t' - rr_t \\ Y_t &= \frac{\eta}{(\eta - (1 - \upsilon - \mu))} * (A_t + \upsilon * h_{t-1} + \mu * K_{t-1}) - \frac{1 - \upsilon - \mu}{\eta - (1 - \upsilon - \mu)} * (X_t + \alpha * c_t' \\ &+ (1 - \alpha) * c_t'') \\ \pi_t &= \beta * \pi_{t+1} - \kappa * X_t + u_t \\ K_t &= \delta * l_t + (1 - \delta) * K_{t-1} \\ \frac{b}{Y} * b_t &= \frac{c''}{Y} * c_t + q * \frac{h}{Y} * \Delta h_t + \frac{l}{Y} * l_t + r * \frac{b'}{Y} * (r_{t-1} + b_{t-1} - \pi_t) - (1 - s' - s'')(Y_t - X_t) \\ \frac{b'''}{Y} &= \frac{\eta''}{Y} * b_t'' = \frac{c''}{Y} * c_t''_t + q * \frac{h''}{Y} * \Delta h''_t + r * \frac{b''}{Y} * (r_{t-1} + b''_{t-1} - \pi_t) - s''(Y_t - X_t) \\ \ln \left(\frac{r_t}{r}\right) &= \rho * \ln \left(\frac{r_{t-1}}{r}\right) + \alpha_\pi * \ln \left(\frac{\pi_t}{\pi}\right) + \alpha_y * \ln \left(\frac{y_t}{Y}\right) + \epsilon_{rt} \\ \frac{\tilde{A}_t}{\theta} = \rho_d * \tilde{A}_{t-1} + \epsilon_{dt} \\ \tilde{\beta}_t &= \rho_d * \tilde{A}_{t-1} + \epsilon_{dt} \\ \end{array}$$

$$\omega = (\beta'' - m''\beta'')/(1 - m''\beta)$$

$$i = (1 - \beta)h/h'$$

$$i'' = (1 - \beta)h''/h'$$

$$\gamma_h = \beta'' + m''(\beta - \beta'')$$

$$rr_t = r_t - E_t \pi_{t+1}$$

$$\gamma_e = m * \beta + (1 - m) * \gamma$$

$$\gamma_h = \beta'' + m'' * (\beta - \beta'')$$

To close the effects of collateral constraints I modify housing/consumption margin conditions of entrepreneurs and impatient households so that the asset price channel is inactive:

$$\begin{split} \widehat{q_{t}} &= \gamma * \widehat{q_{t+1}} + (1 - \gamma_{e}) * \left( \widehat{Y_{t+1}} - \widehat{X_{t+1}} - \widehat{h_{t}} \right) - \widehat{c_{t+1}'} * (\gamma + 1 - \gamma_{e}) + \widehat{c_{t}} - \phi_{e} * (\widehat{h_{t}} - \widehat{h_{t-1}}) \\ &- \gamma * (\widehat{h_{t+1}} - \widehat{h_{t}})) \end{split} \\ \widehat{q_{t}} &= \beta'' * \widehat{q_{t+1}} + (1 - \gamma_{h}) * (\widehat{j_{t}} - \widehat{h''_{t}}) - \widehat{c_{t+1}''} * \beta'' + \widehat{c''_{t}} - \phi_{h} * (\widehat{h''_{t}} - \widehat{h''_{t-1}} - \beta'' \\ &+ (\widehat{h''_{t+1}} - \widehat{h''_{t}})) \end{split}$$

Representative agent utility function:

$$u(.) = \log(c'_t) + j * \log(h'_t) - (L'_t)^{\eta} / \eta$$

Variables and parameters

Description	Notation
Output	$y_t$
Entrepreneurs', patient and impatient households' consumption	$c_t, c_t', c_t''$
Patient and impatient households' labour supply	$L_t', L_t''$
Entrepreneurs', patient and impatient households' holding of housing	$h_t, h'_t, h''_t$
Aggregate investment	i <sub>t</sub>
Aggregate capital	$k_t$
Markup	$X_t$
Price of housing	$q_t$
Real borrowing, lending	$b_t$
Inflation	$\pi_t$
Gross nominal interest rate	$r_t$
Government spending	$g_t$
Preference for housing	j <sub>t</sub>
Productivity	$A_t$
Inflation shock	$u_t$
Preference for housing shock innovation	$\epsilon_{jt}$
Cost-push shock innovation	$\epsilon_{ut}$
Productivity shock innovation	$\epsilon_{At}$
Shock to government spending innovation	$\epsilon_{gt}$
Monetary policy shock innovation	$\epsilon_{rt}$

Auxiliary variables	$\omega, i, i'', \gamma_h$
Ex ante real rate	$rr_t$
Discount rate of patient, impatient households and entrepreneurs	β,β'',γ
Income shares of patient and impatient households	s', s"
Labour aversion	η
Capital share	μ
Elasticity of output to housing	υ
Capital adjustment costs	$\psi$
Depreciation rate	δ
Calvo parameter	θ
Relative size of the group of impatient households	α
Slope of Phillips curve	к
Coefficients on lagged interest rate, inflation and output in the interest rate policy rule	$\rho, \alpha_{\pi}, \alpha_{y}$

# Appendix 4

Calibrated parameters values

Description	Value
Steady state share of government consumption	0.17
Discount factor	0.9902
Price elasticity of demand for each good variety	6
Labour supply aversion	3
Relative risk aversion	2
Share of capital	0.3
Capital adjustment cost parameter	0.5882
Depreciation rate	0.03
Weight of labour in the utility function	1
Steady state markup	1.2
Calvo parameter	0.75
Persistence of housing preference shock	0.85
Standard deviation of housing preference shock	24.89
Persistence of money demand shock	0.7206
Standard deviation of money demand shock	0.0103
Persistence of preference shock	0.947
Standard deviation of preference shock	0.0405
Persistence of investment specific shock	0.6562
Standard deviation of investment specific shock	0.0331
Persistence of productivity shock	0.8556
Standard deviation of productivity shock	0.0064
Standard deviation of monetary policy shock	0.0031
Persistence of cost push shock	0.9625
Standard deviation of cost push shock	0.0012
Persistence of government spending shock	0.87
Standard deviation of government spending shock	0.016
Constant elasticity of substitution between consumption and real balances	0.0598
Elasticity of external finance premium to firm leverage ratio	0.042
Probability of survival of entrepreneurs	0.9728
Discount factor of impatient households	0.95
Discount factor of entrepreneurs	0.98
Relative size of the group of impatient households	0.64
Elasticity of output to housing	0.03

The parameter values are borrowed from Iacoviello (2005), Christensen and Dib (2008), Ireland (2004) and Schmitt-Grohe and Uribe (2006).

## Appendix 5

Fault tolerance graphical analysis



Figure 1. Fault tolerance to deviations of parameter  $\rho$  in BNK optimal rule

Figure 2. Fault tolerance to deviations of parameter  $\alpha_{\pi}$  in BNK optimal rule





Figure 3. Fault tolerance to deviations of parameter  $\alpha_y$  in BNK optimal rule

Figure 4. Fault tolerance to deviations of parameter  $\rho$  in FA optimal rule





Figure 5. Fault tolerance to deviations in parameter  $\alpha_{\pi}$  in FA optimal rule

Figure 6. Fault tolerance to deviations in parameter  $\alpha_y$  in FA optimal rule





Figure 7. Fault tolerance to deviations in parameter  $\rho$  in the HCC optimal rule

Figure 8. Fault tolerance to deviations in parameter  $\alpha_{\pi}$  in the HCC optimal rule





Figure 9. Fault tolerance to deviations in parameter  $\alpha_y$  in the HCC optimal rule

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