Investor Protection and Optimal Contracts Under Risk Aversion and Costly State Verification*

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Abstract

We study financial contracting in a model encompassing costly state verification, risk aversion and imperfect investor protection. We characterize optimal contracts with special emphasis on repayment functions that are continuous on the firm's returns and provide a well specified condition for such contracts to take the form of standard debt. Moreover, we show that for some popular specifications of preferences, standard debt can be optimal only if investor protection is imperfect. Our comparative statics exercises demonstrate that, as long as the contract is continuous, the cost of funds and the probability of bankruptcy are decreasing in the level of investor protection, a result that can be extended to a dynamic setting. In a specific parametrization of the problem, we show that moderate changes in the level of investor protection can have substantial quantitative effects on the terms of the optimal contract and on the borrower's welfare. Finally, we study the relationship between investor protection and leverage and consider the consequences of implementing standard debt contracts when optimality conditions are not satisfied.

Keywords: Investor protection, risk aversion, financial contracts, standard debt.

JEL Classification Numbers: D86, E61, G10, G18, G38, K40

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1 Introduction

Scholars and policymakers seem to agree on the fact that good investor protection, broadly defined, bolsters financial development and economic growth. At the same time, considerable evidence tells us that rarely are investors well protected in financial contracts. The most widely used measure of investor protection -a creditor rights index that ranges from 0 ("weak") to 4 ("strong") introduced by La Porta et al. (1997)- suggests that protection to *financiers* is low on average, 1.8 in a sample of developed and developing countries, and varies widely across countries. Using an updated version of this index, panel (a) of Figure 1 shows that only 9 out of the 133 countries in the sample can be said to have "strong creditor rights", while 21 exhibit "weak creditor rights".[1]

Along with investor protection issues, recent research (e.g. Panousi and Papanikolaou (2012), Lewellen (2006)) has revisited the old working assumption of risk-neutral entrepreneurs and the consequences of risk-averse behavior for firm investment and financing decisions. For instance, if ownership is disperse or shareholders have well diversified portfolios, and if the manager's compensation scheme is independent of the firm's returns, the issues of firm finance and manager's insurance can be studied separately (i.e. we can model the firm as a relatively risk-neutral agent). While these may be plausible features of large firms, small and medium businesses are characterized precisely by the opposite: managers are typically owners and have limited access to hedging instruments. This inverse relationship between firm size and risk attitudes is formalized in the entrepreneurial decreasing absolute risk aversion (DARA) hypothesis of Cressy (2000) and supported by the empirical evidence collected by Fang and Nofsinger (2009) and Schmid et al. (2008). Panel (b) of Figure 1 uses some data from the former study to illustrate the DARA evidence.

In this paper we explore the roles of risk aversion and imperfect investor protection in jointly determining the terms of optimal financial contracts when there is asymmetric information. The starting point of our analysis is the celebrated costly state verification (CSV) model of Townsend (1979) in which an entrepreneur and an investor design a financial contract to fund a joint project with returns that are costlessly observed only by the entrepreneur.

This basic CSV model -which has been extended in several dimensions and is now the workhorse of dynamic macroeconomic models with financial frictions[2]-, is hereby extended to explicitly study the role of investor protection. In particular, we follow Krasa et al. (2008) and account for investor protection not only through verification costs, but also as the maximum fraction of the borrower's income that the investor can recover.[3]

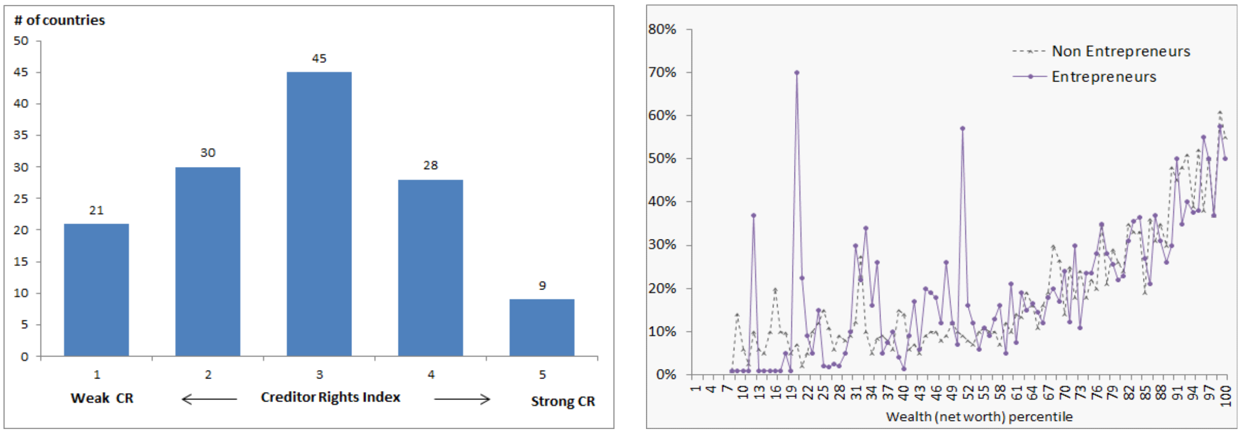
[1] A good survey on the finance-growth subject is Levine (2005). The creditors right problem is particularly acute in developing countries such as those in Latin American as Galindo and Micco (2004) point out.

[2] A partial list of extensions is: economies with production (Gale and Hellwig (1985)), multiple types of borrowers (Williamson (1987)), multiple investors (Winton (1995)), deviations from absolute priority (Bebchuk (2002)), limited commitment (Krasa and Villamil (2000)), and imperfect monitoring (Greenwood et al. (2010)). The use of the CSV model in macroeconomics was launched by Bernanke and Gertler (1989); for other macroeconomic applications see Carlstrom and Fuerst (1997) and Christiano et al. (2010).

[3] This is in contrats with Sevcik (2012) where investor protection is associated only with monitoring costs, and closer in spirit to investor protection in Castro et al. (2004) where borrowers can appropriate returns.

Figure 1: Investor protection and entrepreneurial risk aversion

(a) Creditor rights in 133 countries (b) Risky assets as % of net worth



Source: La Porta et al (1996, 1997) Source: Fang and Nofsinger (2009)

The motivation for introducing this additional source of imperfect investor protection comes from the legal codes and the extant evidence on bankruptcy settlements. Indeed, many countries have introduced various types of legislation aimed at limiting the fraction of income and property that creditors can extract from borrowers in the case of bankruptcy. For instance, in the U.S., Chapter 7 procedures specify that exempt assets (such as home equity and retirement accounts in some states) cannot be used to satisfy creditor claims. Likewise, in some Eastern European and Latin American countries, bankruptcy law requires (or used to require) that certain constituencies be repaid in full before any secured creditor, *de facto* reducing the fraction of assets that could be used to recover debts.[4]

In contrast to the model in Krasa et al. (2008), however, we retain the assumption that the entrepreneur is risk-averse as in Townsend's original model. Thus, we provide a more general framework that is consistent with the kind of empirical evidence presented in Figure 1 (panel (b)) and at the same time is amenable to comparisons with recent iterations of the CSV model (e.g. Winton (1995), Wang (2005)) and to other recent macro models of firm finance that include risk-aversion (e.g. DeMarzo et al. (2012), Smith and Wang (2006)).

We seek to answer two main sets of questions. First, how do optimal contracts look like in a CSV model that encompasses risk aversion and imperfect investor protection? Under which conditions is a standard debt contract (SDC) optimal (if at all)? Second how do the terms of the optimal contract change as the level of investor protection varies? what are the effects of such variations on the entrepreneur's welfare?

Our results can be summarized as follows. Optimal contracts may be catalogued in three main families: (i) standard debt contracts, (ii) debt-like contracts with continuous repay-

[4]In the Czech Republic, bankruptcy law requires that secured creditors be repaid preferentially from 70% of the value of the security, while the remaining 30% is to be added to the bankrupt's assets from which employees are then paid preferentially. Until early 2005, bankruptcy law in Barzil required that labor claims and tax liabilities were repaid before secured creditors. A similar provision in Mexico's bankruptcy procedures was removed in 2003, but countries like Ecuador and Colombia mantain the low priority of secured creditors.

ment functions and (iii) discountinuous debt-like contracts. This is in contrast with the risk-neutral borrower case where only SDCs are optimal and with the risk-averse case under perfect investor protection where the second family of contracts is not available.

We show that a SDC is optimal only if a well specified condition is satisfied, namely, if the marginal value of financing is relatively high. Such condition is more easily satisfied when the liquidation value of the project is high, when investor protection is relatively low and when the entrepreneur's degree of risk aversion is not too high; all of these predictions are supported by the empirical evidence on firm finance (see, e.g., Acharya et al. (2011), Williamson (1988)). Moreover, under some popular specifications of preferences, a SDC can be optimal only if investor protection is imperfect.

Our comparative statics results show that for any continuous contract, the cost of funds and the probability of bankruptcy are decreasing in the level of investor (i.e. creditor) protection, extending the main comparative statics result found in Krasa et al. (2008). We show how this result can be extended to a fully dynamic setting. When a SDC is optimal, our model implies that the borrower's welfare increases with the level of investor protection, but may increase or decrease with it if a SDC is implemented suboptimally.

To illustrate our analytical results, we parametrize the static model so as to reflect recent estimates of bankruptcy costs and recovery rates. We show that lowering investor protection can have considerable quantitative effects on the terms of the contract and on the entrepreneur's welfare. Intuitively, since debt contracts are optimal only when the marginal value of financing is high, it is not surprising that the borrower is willing to accept increasingly higher costs of funding (via higher interest rates and higher bankruptcy probabilities).

The paper is organized as follows. We set up the contracting problem in section 2, briefly describing the symmetric information contract first, and then proceeding to characterize optimal contracts under CSV with particular emphasis on continuous contracts. In section 3 we present the quantitative analysis and a discussion of the main results. Section 4 concludes.

2 The contracting problem

In this section we lay out a static CSV problem in which investor protection plays an explicit role. The model presented here is a blend of the ideas on CSV with risk aversion presented in Townsend (1979), and the notion of imperfect investor found in Krasa et al. (2008).

2.1 Physical environment

The prototype model we consider is one with a risk neutral investor and a risk averse entrepreneur. The entrepreneur owns a production technology which can only be operated by himself. Operating the technology (i.e. starting a project) requires investing one unit of input and we assume that the entrepreneur has only $0 \leq (1-b) < 1$ units. Thus to start the project, she must raise b from the investor who, for simplicity is assumed to have zero opportunity cost. The entrepreneur has no other investment alternatives.

After project returns are realized, the entrepreneur repays $R(\hat{s}, s)$ to the investor where s is the true state of nature and $\hat{s}$ is what the entrepreneur reports as the state. Under private information, only the entrepreneur can costlessly observe the state of nature so the investor would either rely on reports or verify them by paying a cost $0 \leq \gamma < 1$, and may penalize the entrepreneur if she misrepresents. Throughout the paper, we restrict the attention to full commitment environments and to pure strategy equilibria.[5] Since our model is static, we also abstract from the difference between default, liquidation or reorganization, and consider all these as bankruptcy situations.

Both agents are expected utility maximizers. In particular, the risk averse agent values consumption according to the twice continuously differentiable, strictly increasing and strictly concave function $u(\cdot)$. Formally:

Assumption 1 *u is $\mathbb{C}^2$ with $u' > 0$ and $u'' < 0$.*

Project returns are stochastic and equal to the state of nature, which is itself a continuous random variable (r.v.) S with differentiable CDF $H(\cdot)$, $dH(\cdot) = h(\cdot)$. The support of S is assumed to be bounded, $\Sigma = [\underline{s}, \bar{s}] \subseteq \mathbb{R}_+$, such that if s is a particular realization of the state, $0 < \underline{s} \leq s \leq \bar{s}$ and $\infty > h(s) > 0$ if $s \in int(\Sigma)$.[6] The distribution and support of S are common knowledge.

Although in the traditional CSV literature the parameter γ is associated with protection of the investor, we now introduce an additional source of imperfect investor protection through the limited liability clauses of the contract, much in the spirit of Krasa et al. (2008). In particular, we will assume that:

Assumption 2 *The legal system is such that, in any contract, after realization s, the entrepreneur is bound to repay at most $(1-\eta)s$ with $\eta \in [0, 1]$.*

Notice that this limited liability clause also reflects the fact that the production technology is deemed useless to the investor without the entrepreneur. Next we describe the contracting problem under symmetric information and then we study the more interesting case of private information.

2.2 The case of symmetric information

In order to establish a benchmark, in this section we consider the problem under symmetric information (i.e. $\gamma = 0$). Naturally, under symmetric information the repayment function satisfies $R(\hat{s}, s) = R(s)$. The sequence of events in this setting is straightforward: the entrepreneur raises b from the investor, invests one unit of the input and when returns are realized, repays $R(s)$ to the investor according to the contract which we now define:

[5] These are perhaps substantive simplifications. Hvide and Leite (2010) show that the most important results of the CSV model hold even in settings without commitment and with stochastic verification.

[6] The assumption that $\underline{s} > 0$ is also in Townsend (1979) and allows us to better study reward functions that satisfy Inada conditions. Moreover, this assumption reflects the fact that salvage values are typically positive.

Definition 1 *A contract under symmetric information is a schedule $R(\cdot)$ where $R(s)$ is what the risk-averse entrepreneur repays the investor when the state of nature is s.*

There are a number of ways in which the contracting problem can be specified. Here we proceed in the tradition of Townsend (1979), Gale and Hellwig (1985), and Williamson (1987) and maximize the expected utility of the entrepreneur subject to the investor receiving at least her reservation utility (in this case, simply the amount funded, b). For an appropriately chosen weight vector, the solution to the program in (IRC)-(LLC) is also a solution to a problem that maximizes the weighted average of the payoffs of the match subject to IR constraints for both.[7] Accordingly, the optimal contract for the entrepreneur is given by the solution to:

$$\max_{R(\cdot)} \mathbb{E}u\left[S - R(S)\right]$$

$$s.t.: \mathbb{E}R(S) \geq b \tag{IRC}$$

$$(1-\eta)\, s \geq R(s) \geq 0 \;\; \forall\, s \in \Sigma \tag{LLC}$$

The first constraint is the individual rationality one for the investor (lender); given the risk-neutrality assumption, it specifies that she must at least break even in expectation. The second and third are limited liability constraints (LLCs). Under such an environment, the optimal repayment function satisfies a constrained-optimal risk-sharing rule:[8]

$$u'\left[s - R^*(s)\right] = \lambda^* - \frac{\mu_1^*(s) - \mu_2^*(s)}{h(s)}$$

where λ^*, and $\mu_i^*(\cdot)$ are the (optimal value) multipliers for (IRC) and (LLC), respectively. Notice that when $\mu_2^*(s) > 0$, we obviously have $\mu_1^*(s) = 0$ and $R^*(s) = 0$. On the other hand, if $\mu_1^*(s) > 0$ then $\mu_2^*(s) = 0$ and $R^*(s) = (1-\eta)\, s$. Finally, when the solution is interior ($\mu_2^*(s) = \mu_1^*(s) = 0 \; \forall\, s$), risk-neutrality implies that the investor provides full insurance to the entrepreneur whose consumption is constant. Having characterized the optimal benchmark contract, we now turn to the issue of costly state verification.

2.3 Costly state verification

The more interesting results arise when one introduces asymmetric information. Suppose now that the entrepreneur can costlessly observe project returns but the investor must pay a cost $0 < \gamma < 1$ for using the legal system to verify returns. Then a reporting strategy for the entrepreneur maps the state of nature into reports, $\{\hat{s}(s)\}_{s\in\Sigma}$, and a verification region, B, will now be part of the contract. The sequence of decisions and events is as follows:

[7] For a set up that maximizes the investor's payoff subject to the entrepreneur's IRC, see Krasa et al. (2008).

[8] Naturally, here and elsewhere, inequalities and equations involving random variables are assumed to hold almost everywhere, i.e., with probability 1. We also assume throughout the paper that it is legitimate to differentiate under the integral sign.

Entrepreneur obtains b, invests 1 unit of input	$\longrightarrow$	$s \in \Sigma$ is realized, entrepreneur reports $\hat{s}(s)$, investor verifies if $\hat{s} \in B$	$\longrightarrow$	Entrepreneur repays $R(\hat{s}, s)$ to investor

As is standard in the CSV literature (e.g. Freixas and Rochet (2008)), we assume that truthful reporting in the verification region is ensured by arbitrarily large misreporting penalties, i.e., $\hat{s}(s) = s \; \forall \, \hat{s} \in B$. We can now define a contract under private information:

Definition 2 *A contract under CSV is a pair $\{R(\cdot,\cdot), B\}$ where $R(\hat{s}, s)$ is what the entrepreneur repays when the state of nature is s and she reports state $\hat{s}$, and $B \subseteq \Sigma$ is the set of reports $\hat{s}$ for which the investor chooses to use the legal system to verify project returns.*

As usual in these type of contracting problems, the revelation principle allows us to focus only on direct revelation mechanisms that are incentive compatible, and to identify the reports set with the support of the r.v. S. Thus, the optimal contract for the case when verification is costly can be obtained by solving:

$$\max_{\{B, R(\cdot)\}} \mathbb{E}u\left[S - R(S,S)\right] \tag{$\mathcal{P}$.1}$$

$$s.t.\; \mathbb{E}R(S,S) - \gamma \int_{s \in B} dH(s) \geq b \tag{$\mathcal{P}$.2}$$

$$u\left[s - R(s,s)\right] \geq u\left[s - R(\hat{s},s)\right] \;\; \forall\, s, \hat{s} \in \Sigma \tag{$\mathcal{P}$.3}$$

$$(1-\eta)\, s \geq R(s,s) \geq 0 \;\; \forall\, s \in \Sigma \tag{$\mathcal{P}$.4}$$

$$R\left(s, s'\right) = R\left(s, s''\right) \;\; \forall\, s \notin B \text{ and } s'', s' \in \Sigma \tag{$\mathcal{P}$.5}$$

$$B \subseteq \Sigma \tag{$\mathcal{P}$.6}$$

The first constraint is the individual rationality one; it tells us that the investor should at least break even in expectation, after accounting for expected verification costs. The second constraint imposes incentive compatibility; it requires that the entrepreneur (or borrower) prefers to report truthfully in every state. The third and fourth constraints stand for limited liability and the fifth constraint requires that unverified payments depend only on the report (UPC). In order to have an interesting problem, we introduce:

Assumption 3 $\mathbb{E}(S) - \gamma > b > \underline{s}$.

This assumption is sufficient to ensure that, at least for some values of η, an optimal contract will give neither the manager nor the investor all of the firm's returns. Notice that when $\eta = 0$ the model effectively reduces to that in Townsend (1979) and when $u(c) = c$ (i.e. linear preferences) we have the CSV version of Krasa et al. (2008).[9] Thus, the model considered here contains in it some of the popular versions of the CSV framework while specifying an explicit role for investor protection.

[9] Krasa et al. (2008) consider a model that imposes sequential rationality in the players' strategies (the equilibrium contract must be a PBE) so the CSV is a special case of their model.

2.4 Optimal contracts

We now characterize optimal financial contracts under risk aversion, imperfect investor protection and costly state verification. We do so in steps, much in the spirit of Winton (1995): we first introduce a series of lemmas that partially characterize the optimal contract and then we rewrite the problem in a more convenient way that allows for an explicit solution (formal proofs can be found in the appendix).

Lemma 1 *In the optimal contract, for all $\hat{s} \notin B$, the repayment function is constant, i.e., $R(\hat{s}, s) = \bar{R}$ for some constant $\bar{R}$.*

Lemma 2 *Under the optimal contract, in the verification region the repayment function is given by $\hat{R}(s)$, with $\hat{R}(s) < \bar{R}$ almost everywhere (a.e.) and $\hat{R}(s) \leq \bar{R}$ everywhere.*

The first result above is a straightforward implication of the unverified payments constraint, while the second mainly follows from incentive compatibility (and the fact that the contract must be optimal). A more interesting result, which is a simple extension of Proposition 3.2 in Townsend (1979) to the case of imperfect investor protection is:

Lemma 3 *In the optimal contract, B is a lower interval.*

Corollary 1 *Whenever $B \neq \varnothing$, $\exists\, x \in \Sigma$ such that $\hat{s} \leq x \Rightarrow \hat{s} \in B$, and $\hat{s} > x \Rightarrow \hat{s} \notin B$.*

We can now summarize the findings of lemmas 1-3 by saying that the optimal repayment rule follows:

$$R(\hat{s}, s) = \begin{cases} \hat{R}(s) \leq \bar{R}, & \text{if } \hat{s} < x \\ \bar{R}, & \text{if } \hat{s} \geq x \end{cases}$$

Under such rule, incentive compatibility is always satisfied so that truthful reporting is ensured, $\hat{s}(s) = s \;\forall\, s$, and the constraints ($\mathcal{P}$.3) can be replaced. Moreover, since $s > x \Rightarrow s \notin B$, the limited liability constraints can also be replaced by the three constraints: $(1-\eta)\, x \geq \bar{R}$, $(1-\eta)\, s \geq \hat{R}(s)$ and $\hat{R}(s) \geq 0$. Hence, the contracting problem can be reformulated as:

$$\max_{\{\bar{R}, \hat{R}(\cdot), x\}} \int_{\underline{s}}^{x} u\left[s - \hat{R}(s)\right] dH(s) + \int_{x}^{\bar{s}} u\left[s - \bar{R}\right] dH(s) \qquad (\mathcal{PP}.1)$$

$$s.t. \int_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + \bar{R}\left[1 - H(x)\right] - \gamma H(x) \geq b \qquad (\mathcal{PP}.2)$$

$$(1-\eta)\, x \geq \bar{R}, \quad (1-\eta)\, s \geq \hat{R}(s) \geq 0 \;\; \forall\, s \leq x \qquad (\mathcal{PP}.3)$$

Necessary and sufficient conditions to find a solution for the program in ($\mathcal{PP}$.1)-($\mathcal{PP}$.3) are presented in the appendix as (2)-(8). These lead to the following classification of optimal contracts:

Theorem 1 (optimal contracts) *Suppose that the contract* $\Phi^* = \{\bar{R}^*, \hat{R}^*(\cdot), x^*\}$ *solves the program* ($\mathcal{PP}$.1)-($\mathcal{PP}$.3). *Let the constant* λ^* *be the optimal value of the multiplier on* ($\mathcal{PP}$.2). *Then:*

i) Either Φ^* *is the SDC of Gale and Hellwig (1985) with* $\hat{R}^*(x^*) = (1-\eta)x^* = \bar{R}^*$, *or*

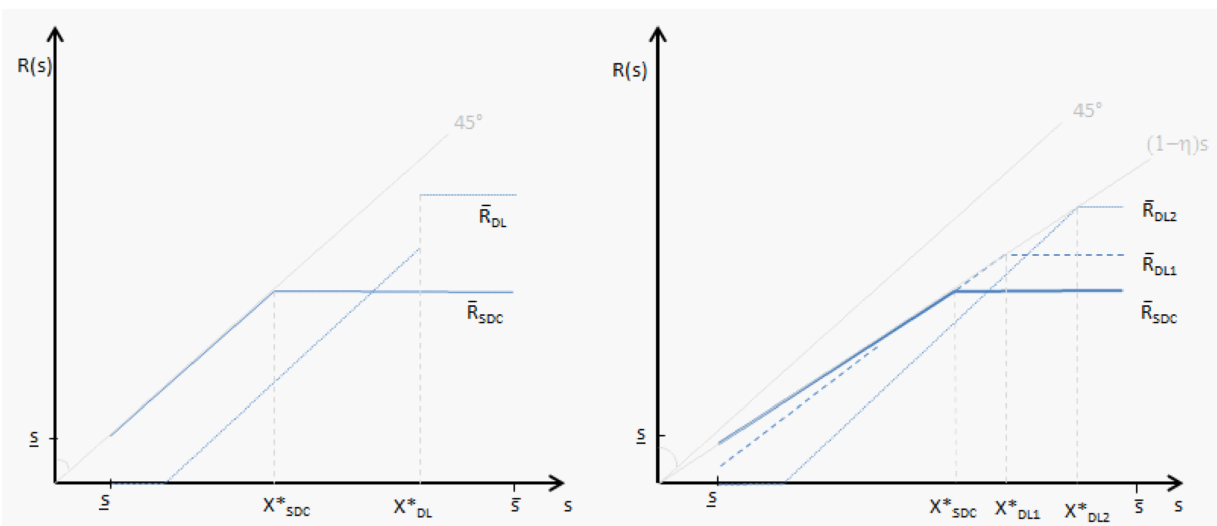
ii) Φ^* *is debt-like with* $s \mapsto \hat{R}^*(s)$ *continuous and* $\hat{R}^{*\prime}(s) = 1$ *for some* s, *or*

iii) Φ^* *is debt-like but discontinuous at* x^* *with* $\hat{R}^{*\prime}(s) = 1$ *for some* $s \leq x^*$, $\hat{R}^{*\prime}(s) = 0$ *for some* $s \leq x^*$, *and* $\bar{R}^* > \hat{R}^*(x^*)$.

The proof of Theorem 1 is simply an application of the Maximum Principle and Arrow's Sufficiency Theorem to the program in ($\mathcal{PP}$.1)-($\mathcal{PP}$.3). To gain some intuition about the general form of the optimal contract, first consider the case of $\eta = 0$. In such case, $\hat{R}^*(s) = \max\{0, s - \max[0, u'^{(-1)}(\lambda^*)]\}$ and, as in Winton (1995), the optimal contract can take one of two forms. Naturally, if $0 > u'^{(-1)}(\lambda^*)$ or equivalently if $u'(0) < \lambda^*$ the optimal contract is a SDC since $\hat{R}^*(x^*) = x^* = \bar{R}^*$. Otherwise the optimal contract is debt-like (DL) with $\hat{R}(s) = 0$ for some s and is discontinuous at $\bar{R}$ (see left panel of Figure 1).

Figure1. Alternative optimal contracts

$\eta = 0$ $\qquad\qquad$ $\eta > 0$



In the case of $\eta > 0$ the optimal contract specifies $\hat{R}^*(s) = \max\{0, s - \max[\eta s, u'^{(-1)}(\lambda^*)]\}$. Again, if $\eta s > u'^{(-1)}(\lambda^*)\ \forall\, s \leq x$, or, equivalently, if condition (1) is satisfied, we obtain the SDC with $\hat{R}^*(x^*) = (1-\eta)x^* = \bar{R}^*$. Yet, in contrast with the case of $\eta = 0$, the optimal contract need not be discontinuous when (1) is not satisfied. To see why, notice that when $\eta = 0$ and standard debt is not optimal, whenever $\hat{R}^*(s) > 0$, it must be that $\hat{R}^{*\prime}(s) = 1$. In $(s, R(s))$-space, this means that the schedule $\hat{R}^*(\cdot)$ never intersects the upper LLC, which in the case of $\eta = 0$ is simply the 45° line. On the other hand, when $\eta > 0$ and SDC is not optimal, for $\hat{R}(s) > 0$ and low values of s, we have again that $\hat{R}^{*\prime}(s) = 1$ but in this case $\hat{R}^*(\cdot)$

can intersect the upper LLC since its slope is now $(1-\eta) < 1$. In that case, we have that to the right of the intersection and for $s \leq x^*$ the repayment function is $\hat{R}^*(s) = (1-n)s$ as in the SDC (see right panel of Figure 1).

The following remark will be useful below:

Remark 1 *When the optimal contract is continuous, the cutoff value x^* precisely pins down the probability of verification, $H(x^*)$, and the implied cost of funds $(1-\eta)x^* = \bar{R}^*$.*

We are now ready to extend one of the main results of Krasa et al. (2008) to the case of a risk averse borrower and to general, continuous, optimal contracts:[10]

Proposition 1 *Whenever the optimal contract is continuous and $\eta \in (0,1)$, the probability of verification and the implied cost of funds are increasing in η and in γ.*

The proof of Proposition 1 is a straightforward exercise of comparative statics. This result unveils a tension between incentives and rewards for the borrower, in particular with respect to η. Given that the optimal contract is continuous, a lower level of investor protection will benefit the entrepreneur in (some or all of) the low states of nature, but will result in a higher probability of verification (legal bankruptcy) and a higher implicit cost of funds. When the optimal contract is discontinuous, the LLCs may not bind and therefore changes in η may leave the terms of the contract unaffected.

2.5 Optimality of a SDC and entrepreneur's welfare

Since the optimality of SDCs is a classic question in the CSV literature and the use of such contracts is pervasive in practice, in this section we explore the role that (γ, η) play in satisfying the conditions required for their optimality. Moreover, we consider the welfare effects of changes in η when debt contracts are optimal and when they are not.[11]

Notice that the proof of Theorem 1 provides a well specified condition for the optimality of a SDC. In particular, a SDC is optimal only if:

$$\lambda^* > u'(\eta \underline{s}) \tag{1}$$

For a given parametrization of $H(\cdot)$, including its support $[\underline{s}, \bar{s}]$, this condition is a restriction on γ, η and the curvature of $u(\cdot)$ since the proof of Theorem 1 implies that:

$$\frac{(1-\eta)\int_{x^*}^{\bar{s}} u'[s-(1-\eta)x^*]\,dH(s)}{(1-\eta)[1-H(x^*)] - \gamma h(x^*)} = \lambda^*$$

[10] In Krasa et al. (2008), risk neutrality (of the borrower) implies that a SDC is the only optimal contract so their comparative statics results naturally apply to SDCs only.

[11] The effect of increasing γ on the borrower's welfare is always negative and is independent of optimality considerations so we concentrate on η.

Intuitively, this condition is more likely to be satisfied when bankruptcy costs are relatively big, when the entrepreneur's degree of risk aversion is moderate and when investor protection is relatively low. Notice, that γ should not be "too big" either, for, otherwise, the constraint set given by the IRC can be empty for some values of η (see Assumption 3). Likewise, investor protection cannot be "too low" since $\lambda^* (\eta = 1) = 0$.

Precise comparative statics results are difficult to obtain from condition (1) because λ^* depends directly and indirectly on the parameters through x^* and some of these effects have opposite signs.[12] However, we are able to extract the following corollary from Theorem 1:

Corollary 2 *Suppose that $\eta = 0$. Then if u satisfies $\lim_{c \to 0} u'(c) = \infty$, a SDC is never optimal.*

In other words, under such specification of preferences, standard debt can be optimal only if $\eta > 0$. Functions satisfying this type of Inada condition include, for instance, the constant elasticity of substitution (CES) family with elasticity parameter greater than or equal to one (e.g. Cobb-Douglas), and the constant relative risk aversion (CRRA) function, one of the most widely used in macroeconomics.

The results recorded in Theorem 1, through condition (1) and Corollary 2, relate to the existing literature in at least two ways. First, they complement the capital structure-theoretical argument that higher liquidation values should favor debt as the choice contractual arrangement (e.g., Williamson (1988)). In our model, it is true that for a *given* level of η, a higher liquidation value $\underline{s}$ implies that condition (1) is more easily satisfied, facilitating the optimality of a SDC. However, as the fraction of the liquidation value that the investor receives decreases $(\eta \to 0)$, the conditions under which debt is optimal become harder to satisfy $\left(\lim_{\eta \to 0} \lambda^* (\eta) = 0 \text{ while } \lim_{\eta \to 0} u' (\eta \underline{s}) = \infty\right)$.

Secondly, our result can be seen as theoretical support for the findings of Acharya et al. (2011) who suggest a demand-side effect in the market for debt: if investors are well protected in debt contracts, risk-averse entrepreneurs are heavily damaged in the case of bankruptcy, and would therefore find debt less attractive.[13] We emphasize that this result has no bearing with the *aggregate* level of firm finance, but merely with the relative appeal of different financial contracts (i.e. the capital structure).

We now consider the welfare effects of changes in the level of investor protection. First, we assume that condition (1) indeed holds. Define the vector $\boldsymbol{\theta} = \{b, \gamma, \eta, \rho, \boldsymbol{\psi}\}$ where ρ captures the borrower's degree of risk aversion and $\boldsymbol{\psi}$ contains the parameters of the distribution $H(\cdot)$. Since (1) implies that a cutoff value x completely characterize the SDC, we can

[12] For instance, even knowing that $dx^* (\eta) / d\eta$ and $dx^* (\gamma) / d\gamma$ are positive, simple inspection of (1) tells us that we cannot easily sign $d\lambda^* (\eta) / d\eta$ or $d\lambda^* (\gamma) / d\gamma$. Moreover, if ρ captures the borrower's degree of risk aversion, it is not hard to see that $dx^* (\rho) / d\rho = 0$ but we still cannot easily sign $d\lambda^* (\rho) / d\rho$.

[13] Among the first to consider seriously the posibility that strong investor protection may discourage debt issuance are Rajan and Zingales (1995) although their analysis was far from conclusive. More recent studies provide sharper inference on the negative relationship between strong creditor rights and firm debt (e.g., Acharya et al. (2011), Ghoul et al. (2012)). This is in contrast with the supply-side hypothesis of credit provision bourne out of the strong association observed in the data between investor protection and measures such as credit-to-GDP (see, e.g., La Porta et al. (1998) and Djankov et al. (2007)).

reformulate the contracting problem as:

$$v(\boldsymbol{\theta}) = \max_{x \in \Sigma} \int_{\underline{s}}^{x} u[\eta s]\, dH(s) + \int_{x}^{\bar{s}} u[s - (1-\eta)x]\, dH(s) \tag{$\mathcal{SD}$.1}$$

such that:

$$b \leq \int_{\underline{s}}^{x} (1-\eta) s dH(s) - \gamma H(x) + (1-\eta) x [1 - H(x)] \tag{$\mathcal{SD}$.2}$$

Once again, notice that Assumption 3 ensures that at least for some values of $\eta \in [0,1]$, the constraint set given by ($\mathcal{SD}$.2) is nonempty. The following proposition analyzes the net effect on the borrower's welfare of an increase in η :

Proposition 2 *When a SDC is optimal, the borrower's welfare is decreasing in η.*

Debt contracts are frequently observed in practice even though the analysis presented here suggests that the conditions for their optimality can sometimes be fairly strict. Moreover, since SDCs are simple to understand and enforce, it is conceivable that policymakers or regulators may find these arrangements attractive even if they are suboptimal from the standpoint of the contracting parties. Thus, we now consider the welfare effects of implementing SDCs when (1) is not satisfied:

Proposition 3 *Suppose that a SDC is implemented but $\lambda^* < u'(\eta \underline{s})$. Then the borrower's welfare may increase or decrease with η.*

The intuition of Proposition 3 is simple: when condition (1) is not satisfied, a SDC gives the borrower too little consumption smoothing (across states) compared to what would be optimal (come back to Figure 1). Therefore, since the partial effect of η (on utility) is greater in the bankruptcy states, rising η gives the borrower more consumption in these states and reduces the gap between optimal and actual consumption smoothing.

2.6 A dynamic extension

Although a complete study of dynamic CSV is beyond the scope of this paper, in this section we show that in a simplified version of the repeated contracting problem, a result analogous to the first part of Proposition 1 exists. Our main purpose here is to show that the distortion generated by $\eta > 0$ remains even when the contract lasts more than one period.

Assume that time is discrete and the time horizon is infinite. For simplicity, suppose that S has finite support $\tilde{\Sigma} = [s_1, s_2, ... s_N]$ and that returns are i.i.d. across periods with $\Pr(S = s_i) = \pi_i \in (0,1)$ and $\sum_{i=1}^{N} \pi_i = 1$. Finally, we assume that agents discount the future at a common rate β.

Following Spear and Srivastava (1987) and Abreu et al. (1990), we note that there exists a recursive representation for the extensive form of the repeated game. That is, the problem can be reduced to a simple static variational problem where the entrepreneur's expected utility, v, is used as a state variable that summarizes all relevant past information. Define $\underline{v} = \frac{1}{1-\beta}\sum_{i=1}^{N}\pi_i u\left(\eta s_i\right)$ and $\bar{v} = \sup u\left(c\right) / \left(1-\beta\right)$, respectively as the minimum and maximum attainable expected (lifetime) utility by the borrower. Define also w as the one-period-ahead expected utility that a contract "promises" the entrepreneur.

Let $\mathcal{J} : [\underline{v}, \bar{v}] \rightarrow \mathbb{R}$ denote the value that the optimal contract delivers to the lender. Then $\mathcal{J}$ and the optimal contract $\left\{B^*\left(v\right), R^*\left(v, s_i\right), w^*\left(v, s_i\right), s_i \in \tilde{\Sigma}, v \in [\underline{v}, \bar{v}]\right\}$ can be characterized recursively by a functional equation that maximizes the lender's present value expected utility subject to incentive compatibility, limited liability and a promise keeping constraint (PKC). Since this problem has been studied elsewhere (e.g. in Wang (2005)) and the inclusion of imperfect investor protection poses no major technical challenges, we relegate the details of the current formulation to the appendix.

Instead of trying to fully characterize optimal contracts, which is difficult task without adding more structure to the problem, we focus on the verification set, and look for a result that is the dynamic analogue of Proposition 1. To do so, we first reproduce a key proposition found in Wang (2005) for the case of $\eta = 0$:

Proposition 4 (Wang (2005)) *Assume $\underline{v} > -\infty$. Then there exists a value of the borrower's expected utility, $\hat{v} \in [\underline{v}, \bar{v}]$, such that $\left\{s_1, s_2, ..., s_{N-1}\right\} \subseteq B^*\left(v\right) \; \forall \; v \leq \hat{v}$.*

Wang's result implies that after the entrepreneur's promised utility reaches a lower threshold, the optimal contract specifies that the she is monitored in all income levels except the highest. Such result is familiar in the firm financing literature and is analogous to e.g., that of Clementi and Hopenhayn (2006), in which the investment project is liquidated for sure after a sufficiently long sequence of low realizations of S. For our purposes, we let $\eta > 0$ and write the threshold found above as $\hat{v}\left(\eta\right)$. Now we can state our result:

Proposition 5 *Assume that $\underline{v} > -\infty$. Then $\eta_2 > \eta_1 \Rightarrow \hat{v}\left(\eta_2\right) > \hat{v}\left(\eta_1\right)$.*

Thus, a lower level of investor protection expands the set of states $[\underline{v}, \hat{v}\left(\eta\right)]$ under which the borrower is fully monitored (except for s_N). Additionally, since both agents have the same discount rate, $[\underline{v}, \hat{v}\left(\eta\right)]$ is a set of absorbing states so that $v \leq \hat{v}\left(\eta\right)$ resembles bankruptcy. That is, once $v \leq \hat{v}\left(\eta\right)$, the borrower is fully monitored forever.

Notice that propositions 4 and 5 depend crucially upon the assumption that $\underline{v} > -\infty$. In particular, Wang's result does not admit reward functions that are unbounded from below (e.g., CRRA). However, in our model, the fact that $\eta > 0$ implies that $\underline{v} > -\infty$ even for the case of $\lim_{c\rightarrow 0} u\left(c\right) = -\infty$.

3 Quantitative analysis

We now study the quantitative implications of the static model described in sections 2.3 - 2.5. Our main goal is to illustrate proposition 1 and the results of section 2.5, so we focus on SDCs without loss of generality. That is, we solve the problem ($\mathcal{SD}$.1)-($\mathcal{SD}$.2) and then test if condition (1) holds under various parametrizations, with special attention given to the relationship between the degree of risk aversion and the remaining parameters of the model. We also consider the relationship between investor protection and leverage, and the quantitative effects of implementing SDCs when they are not optimal.

3.1 Benchmark parametrization

3.1.1 Functional forms and non-bankruptcy parameters

Throughout the quantitative analysis we use $u(c) = (c^{1-\rho} - 1)/1 - \rho$ for the entrepreneur's payoff function, and for our benchmark parametrization we choose a risk aversion coefficient $\rho = 0.5$. This specification of preferences satisfies Assumption 1 and can deliver SDCs as optimal contracts over a range of values for η (see section 3.2.1). Moreover, this function is consistent with the DRRA hypothesis referred to in the introduction, and satisfies the Inada condition $\lim_{c \to 0} u'(c) = \infty$ implying that a SDC would never be optimal if investor protection were perfect ($\eta = 0$). For the distribution function we choose to follow Krasa et al. (2008) and assume $S \sim N(\mu, \sigma^2)$. Table 1 summarizes our choices:

Table 1: Functional forms

Function	Form
preferences	$u(c) = \begin{cases} c, \text{ investor} \\ \frac{c^{1-\rho}-1}{1-\rho}, \text{ entrepreneur} \end{cases}$
distribution	$h(s) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{s-\mu}{\sigma}\right)^2\right\}$

As for the parameters of the density function $h(\cdot)$, we set $\mu = 1.1$ and $\sigma = 0.18$ which are slightly higher than the average and standard deviation of the real return on the S&P500.[14] As in Krasa et al. (2008) we choose these slightly higher figures to account for the fact that we are considering an individual firm rather than an index. Finally, we set $b = 0.57$ which is in between the asset:equity ratio of 2:1 required by loans from the Small Business Administration (SBA) and the 2.45:1 mean leverage ratio reported by Kalemli-Ozcan et al. (2011) for non-financial firms in the U.S. in the 2004-2009 period.

3.1.2 Bankruptcy related parameters

The two key parameters for our quantitative exercise are γ which captures bankruptcy costs and η which measures the level of debtor protection. We build our baseline parametrization

[14] One more comment is in place regarding the distribution of returns. Recall from section 2.1 that we assume that H has a bounded support and $\underline{s} > 0$, which is obviously not the case for our choice of empirical distribution. In order for the normal to be a good approximation to our H, we choose $\underline{s} = 0.2$ and $\bar{s} = 2$ which, along with $\mu = 1.1$ and $\sigma = 0.18$ preserve the symmetry of the distribution and yield $\int_{\underline{s}}^{\bar{s}} h(s)\, ds = 0.999998$.

following the findings of a recent and influential paper by Bris et al. (2006) who report on costs of bankruptcy and recovery rates for what their consider "the largest and most comprehensive sample of (U.S.) corporate bankruptcies assembled for an academic paper".

Importantly for our purposes, Bris et al. (2006) present lower and upper bounds for both parameters, under what they label, respectively, as an "optimistic" and a "reported-only" basis. For the average η, the authors report a range between 0.2 and 0.83 while for the average γ, they report a range between 38% and 80% of distributed post-bankruptcy assets.[15,16] Interestingly enough, the parameter values for the benchmark specification in Krasa et al. (2008) lie outside these two ranges since they choose $\eta = 0.1$ and $\gamma = 0.1$, resulting in bankruptcy costs that are 25% of distributed assets. We settle for $\gamma = 0.35$ and $\eta = 0.3$ which lie inside the ranges found by Bris et al. (2006) and, along with the parameter values and functional forms described in the previous section, also deliver reasonable implied default probabilities and interest rates. Our baseline parametrization is summarized in Table 2:

Table 2: Parameter values

Parameter	Value	Source
γ	0.35	Bris et al. (2006)
b	0.573	Kalemli-Ozcan et al. (2011)
η	0.3	Bris et al. (2006)
μ	1.1	S&P500 and Krasa et al. (2008)
σ	0.18	S&P500 and Krasa et al. (2008)

3.2 Results of the benchmark parametrization

Here we present some numerical results from the solution to the contracting problem under the baseline parametrization. The first task is to check that under this benchmark the condition for optimality of standard debt is satisfied; that is, we check that, at the optimum, $\lambda^* = 4.09 > u'(\eta \underline{s}) = (\eta \underline{s})^{-1/2} = 4.08$.

Our baseline scenario yields a cutoff value $x^* = 0.89$ which in turn results in a bankruptcy probability of 9.9%. Such probability is relatively high but is by no means unheard of: Altman and Bana (2004) estimate default probabilities on bonds that are close to 15% for some quarters in 1991 and 2002 in the U.S. This also suggests that our models is perhaps

[15] Given the heterogeneity of bankruptcy cases, we would very much like to work with the median rather than the average. However, averages allow us to recover some figures from the original data and ensure internal consistency which the median does not. For instance, in Table III of Bris et al. (2006) the average optimistic recovery rate *before* expenses is 80% of total assets. This is consistent with the average recovery rate of 51% *after* expenses reported on their table XIII, and with the average costs being 38% and 8% of bankrupcty assets and total assets, respectively. On the other hand their reported median recovery rate before expenses is 38%, and after expenses is 70%; something altogether problematic for the purposes of our exercise.

[16] An alternative source of estimates for η is Blazy et al. (2010) who estimate that recovery rates (η) range between 76% (senior creditors) and 10% (junior creditors) in Germany and between 31% and 6% in the U.K.

better suited for the study of bank loans to small or relatively new firms since bond issuance may require to establish a reputation of creditworthiness (low default) first.

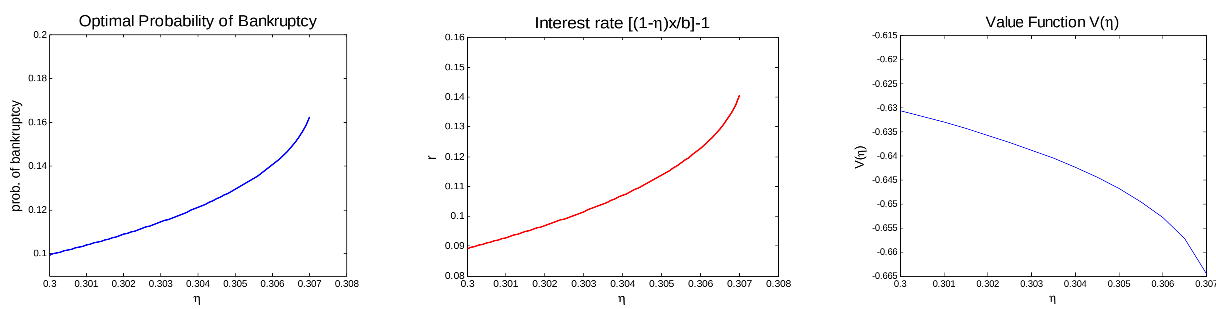
Next, from the value of x^* we can compute expected *distributed* bankruptcy assets:

$$(1-\eta)\,\mathbb{E}\left[s \mid s \leq x^*\right] = (1-\eta)\,\frac{\int_{\underline{s}}^{x^*} sh\,(s)\,ds}{\int_{\underline{s}}^{x^*} h\,(s)\,ds} = 0.57$$

and bankruptcy costs as a fraction of such assets: $\gamma\left\{(1-\eta)\,\mathbb{E}\left[s \mid s \leq x^*\right]\right\}^{-1} = 0.61$, which stands around the midpoint of the range reported by Bris et al. (2006). Finally, the benchmark exercise yields a real rate of interest of $(1-\eta)\,x^* b^{-1} - 1 = 8.8\%$ which is some 1.5% above the average interest rate on small loans reported by the SBA.

We now carry out some simple comparative statics in a neighborhood of the baseline scenario. In particular we study how the terms of the contract and the borrower's welfare change as our parameter of interest, η, varies. The comparative statics exercise is limited, however, by the fact that optimality of SDC requires a tight IRC (see section 3.2.1). That is, we are able to vary the level of investor protection only up to about 3% before the constraint set given by ($\mathcal{SD}$.2) becomes empty.

Figure 2. Comparative statics when a SDC is optimal



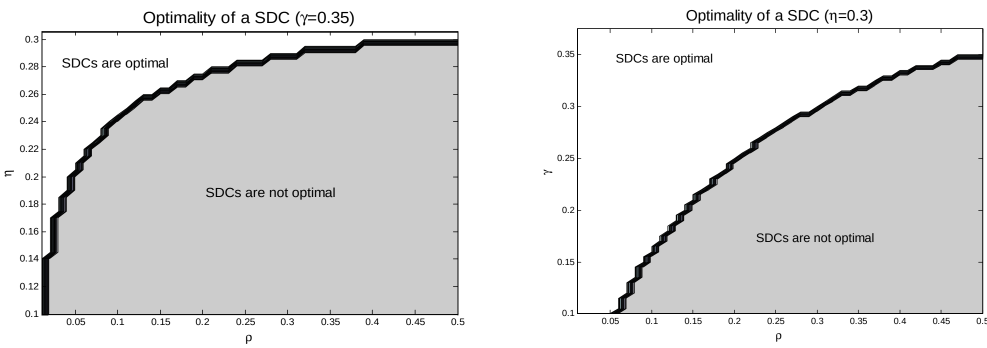
The three panels of Figure 2 are simply an illustration of propositions 1 and 2. They show, for instance, that a decrease of 2% in the level of investor protection (i.e. from $\eta = 0.3$ to $\eta = 0.306$) increases the probability of bankruptcy by about 5 percentage points (from around 10% to 15%), increases the interest rate by about 320 basis points and lowers the borrower's value function by almost 4%.

Thus, moderate decreases in creditor protection have substantial quantitative effects on the terms of the contract and the welfare of the borrower. One must keep in mind, however, that such dramatic responses to modest changes in investor protection are largely driven by the tightness of the constraint ($\mathcal{SD}$.2), which in turn is required by condition (1). In other words, given that a SDC is only optimal when the marginal value of financing is very high, it is not surprising that the borrower is willing to incur in increasingly higher costs of funding (via higher interest rates and higher bankruptcy probabilities).

3.3 Risk aversion and optimality of standard debt

In the previous subsection, we chose a parametrization that guaranteed the optimality of a SDC. We now test if this condition holds for alternative combinations of the parameters and explore the quantitative consequences of implementing such contracts when they fail to be optimal. In particular, we study the role of γ, η and ρ in satisfying (1), while keeping μ, σ, b as in the baseline exercise:

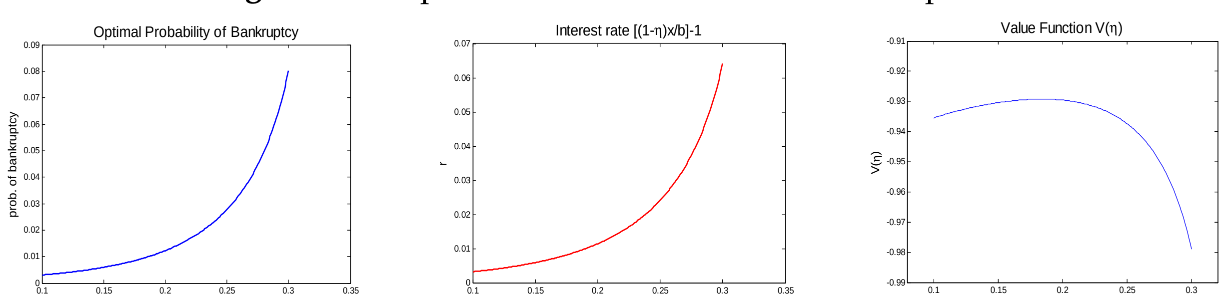
Figure 3. When is a SDC optimal?



These results illustrate the highly non-linear relationships between η and ρ, on the one hand, and γ and ρ on the other, in delivering conditions for the optimality of SDCs. The effect is particularly dramatic in the case of (η, ρ) : for condition (1) to be satisfied even at a relatively low value $\rho = 0.2$, we already require that the borrower keeps at least 25% of the assets in the event of bankruptcy. In fact, given $\gamma = 0.35$ and $b = 0.57$, any value of $\rho > 0.4$ requires that $\eta > 0.3$ for a SDC to be optimal.[17] On the other hand, with these values of γ, b, the constraint set given by IRC ($\mathcal{SD}.2$) is empty for any $\eta \geq 0.31$ (see left panel of Figure 4). This suggests that as the entrepreneur becomes more risk averse, it becomes increasingly hard to satisfy (1) without varying any of the remaining parameters.

What if a SDC is implemented but condition (1) is not satisfied? To explore such possibility, we parametrize the model with σ, μ, b as in the baseline and solve the contracting problem for $\gamma = 0.3, \rho = 1.5$ and a grid of $\eta \in [0.1, 0.3]$ all of which ensure that $\lambda^* < u'(\eta \underline{s})$. Figure 5 summarizes the comparative statics results:

Figure 5. Comparative statics when a SDC is not optimal



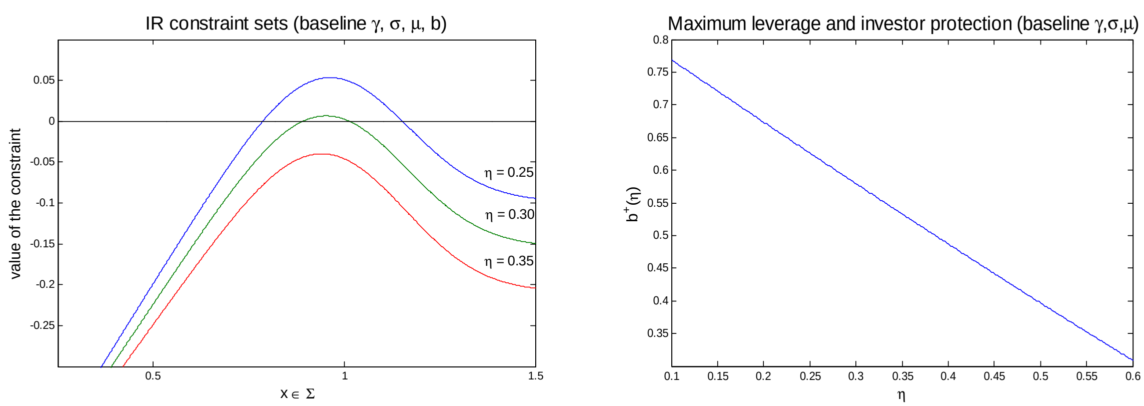
[17]For instance, $u(c) = \ln c$ requires $\eta \geq 0.3064$, which results in a vey tight constraint, with $\lambda^* \geq 18.3$.

It is perhaps not surprising that the implications of Proposition 1 remain valid. The more provocative result, though, is the fact that, using values for (η, γ) that lie inside the intervals reported by Bris et al. (2006), and a value for ρ that is used in many macroeconomic studies, the borrower's welfare is not strictly decreasing in the level of debtor protection, illustrating the content of Proposition 3.

3.4 Investor protection and leverage

What if we want to consider the case of $\eta \geq 0.31$? This requires alternative values for the remaining parameters. Here we analyze in particular the relationship between leverage and investor protection. Although this exercise is independent of risk aversion, the issue has not been addressed by the CSV literature and therefore we think it is relevant to pursue it here. We want to find out how b, which measures the fraction of debt, needs to be varied if we want to consider σ, γ, μ as in the baseline parametrization and $\eta \geq 0.31$.

Figure 4. The IR-constraint set, maximum leverage and investor protection



In the right panel of Figure 4 we have defined $b^{+}(\eta)$ as the maximum fraction of debt (inverse of the leverage ratio) consistent with η, such that the constraint set given by ($\mathcal{SD}.2$) is non-empty. Our model, thus, gives some analytical background to the recent empirical evidence provided by Pereira and Ferreira (2011) and Cheng and Shiu (2007) who look at panel data regressions and conclude that firms in countries with better creditor protection have higher leverage.

4 Concluding remarks

Building upon the existing literature, in this paper we have presented a simple theory of debt when there is costly verification, imperfect investor protection and a risk averse entrepreneur. These features make our model more amenable to informationally opaque, small and medium sized firms with concentrated ownership or contingent compensations.

Since much of the theoretical literature on investor protection focuses on monitoring costs (γ), a natural avenue for future research is the study of our main parameter of interest, η. Two extensions that come to mind are allowing for η to be either uncertain when financial contracts are signed (i.e. stochastic) and/or endogenously determined by the contracting parties. This last extension would clearly have political economy ramifications, another promising area of research.

A Appendix

A.1 Proof of Theorem 1

Proof of Lemma 1. The UPC implies that unverified payments must only depend on the message, that is, $R(\hat{s}, s) = R(\hat{s})$ for any $\hat{s} \notin B$. Therefore, the entrepreneur will choose $\tilde{s} = \arg\min_{\hat{s} \notin B} R(\hat{s})$ so the contract may as well set $R(\tilde{s}) = \bar{R}$. ■

Proof of Lemma 2. First, notice that $\hat{s}(s) = s \; \forall \; s \in B$ so that in the verification region $R(\hat{s}, s) = \hat{R}(s)$ for some $\hat{R}(\cdot)$. Now, for $s \in B$, $\hat{R}(s) > \bar{R}$ can never be optimal since in this case the entrepreneur will prefer to misreport and pay $\bar{R}$ (the ICC is not satisfied). Next, if $\hat{R}(s) = \bar{R}$ *on a set of positive measure*, then the investor will inefficiently pay verification costs when she does not need to so the contract cannot be optimal. This implies that $\hat{R}(s) = \bar{R}$ can hold only for a zero-measure event (i.e. a single point). Therefore, $\hat{R}(s) < \bar{R}$ a.e. and $\hat{R}(s) \leq \bar{R}$ everywhere. ■

Proof of Lemma 3. We first show that B is a connected set. This part of the proof is constructive and is a special case of item (iii) of Proposition 1 in Winton (1995). Without loss of generality, suppose that the contract has as verification set a disjoint interval $B = [\underline{s}, x] \cup [s_1, s_2]$ for some $\bar{s} > s_2 > s_1 > x > \underline{s}$, and repayment function $\hat{R}(s)$ for $\hat{s} \in B$ and $R(\hat{s}, s) = \bar{R}$ for $\hat{s} \notin B$. The investor's payoff from this contract is then given by:

$$
\begin{aligned}
V = & \int_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + \bar{R}\left[H(s_1) - H(x)\right] \\
& + \int_{s_1}^{s_2} \hat{R}(s)\, dH(s) + \bar{R}\left[1 - H(s_2)\right] - \gamma\left[H(s_2) - H(s_1) + H(x)\right]
\end{aligned}
$$

and the entrepreneur's payoff from the contract is given by:

$$
\begin{aligned}
U = & \int_{\underline{s}}^{x} u\left[s - \hat{R}(s)\right] dH(s) + \int_{x}^{s_1} u\left[s - \bar{R}\right] dH(s) \\
& + \int_{s_1}^{s_2} u\left[s - \hat{R}(s)\right] dH(s) + \int_{s_2}^{\bar{s}} u\left[s - \bar{R}\right] dH(s)
\end{aligned}
$$

Now, incentive compatibility then requires that for $s \in [s_1, s_2]$, $\bar{R} \geq \hat{R}(s)$. If $\bar{R} = \hat{R}(s)$ there is nothing to prove so suppose that $\bar{R} > \hat{R}(s)$. Now construct a new contract $(\triangle)$. To do so, notice that $\bar{R} > \hat{R}(s)$ implies:

$$\int_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + \bar{R}[1 - H(x)] - \gamma H(x) > V$$

Therefore, there exists a contract with $B^{\triangle} = [\underline{s}, x]$, $R^{\triangle}(s,s) = \hat{R}(s)$ for $\hat{s} \in B^{\triangle}$ and $R^{\triangle} \in [\hat{R}(s), \bar{R})$ satisfying:

$$V^{\triangle} = \int_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + R^{\triangle}[1 - H(x)] - \gamma H(x) = V$$

Such a contract is feasible since the initial contract implied that for $s \in [s_1, s_2]$, $(1-\eta)s \geq \hat{R}(s) > \bar{R} > R^{\triangle}$. It is also incentive compatible since the repayment function is constant for all $\hat{s} \notin B$ and satisfies $R^{\triangle} \geq \hat{R}(s)$. Under such a contract, the concavity of u guarantees that:

$$U^{\triangle} = \int_{\underline{s}}^{x} u\left[s - \hat{R}(s)\right] dH(s) + \int_{x}^{\bar{s}} u\left[s - R^{\triangle}\right] dH(s) \geq U$$

Thus, we have found a contract that is feasible, incentive compatible and that weakly improves the entrepreneur's welfare, while leaving the investor as well off. Summarizing, when the contract specifies B as a disjoint interval, the contract fails to be optimal.

We now show that B is in fact a *lower* interval. It suffices to show that $B \neq \varnothing \Rightarrow \underline{s} \in B$ and we proceed by contradiction. Suppose that $B \neq \varnothing$ but $\underline{s} \notin B$. Since $\underline{s} \notin B$, we have $R(\underline{s}, s) = \bar{R}$, while incentive compatibility requires $\bar{R} \geq \hat{R}(s)$. On the other hand, limited liability requires $(1-\eta)\underline{s} \geq \bar{R}$. Since $(1-\eta)\underline{s} \leq (1-\eta)s \;\forall s \in \Sigma$, it follows that $\bar{R} = \hat{R}(s) = (1-\eta)\underline{s}$ which in turn implies that $B = \varnothing$, a contradiction. ■

Proof of Theorem 1. In the reformulated problem ($\mathcal{PP}$.1)-($\mathcal{PP}$.3) the IRC will bind along the optimal path. Moreover, the first and second LLCs will also bind for some s. It is easy to see that whenever these constraints bind, the rank of their Jacobian matrix equals its number of rows so that the rank constraint qualification is satisfied. Thus, the problem is equivalent to problem (43) on page 102 of Caputo (2005) with no differential constraints. The Maximum Principle (e.g. Theorem 4.4 in of Caputo (2005)) then implies that there exist constants $\lambda > 0$, $\phi \geq 0$ and nonnegative, continuous functions $\mu_1(\cdot)$, $\mu_2(\cdot)$ such that the following conditions hold:

$$-\mu_1(s) = \left\{u'\left[s - \hat{R}(s)\right] - \lambda\right\} h(s) - \mu_2(s) \quad \forall s \leq x \tag{2}$$

$$-\phi = \int_{x}^{\bar{s}} u'\left[s - \bar{R}\right] dH(s) - \lambda\left[1 - H(x)\right] \tag{3}$$

$$\frac{-\phi(1-\eta)}{h(x)} = u\left[x - \hat{R}(x)\right] - u\left[x - \bar{R}\right] + \lambda\left[\hat{R}(x) - \bar{R} - \gamma\right] \tag{4}$$

along with complementary slackness conditions:

$$0 = \lambda \left\{ \int_{\underline{s}}^{x} \hat{R}(s)\, dH(s) + \bar{R}\left[1 - H(x)\right] - \gamma H(x) - b \right\} \tag{5}$$

$$0 = \phi\left[(1-\eta)x - \bar{R}\right] \tag{6}$$

$$0 = \mu_1(s)\left[(1-\eta)s - \hat{R}(s)\right] \tag{7}$$

$$0 = \mu_2(s)\hat{R}(s) \tag{8}$$

where $\lambda, \phi, \mu_1(\cdot), \mu_2(\cdot)$ are, respectively, the multipliers on the IRC and LLCs. Now suppose that $\{\bar{R}^*, \hat{R}^*(\cdot), x^*, \lambda^*, \phi^*, \mu_1^*(\cdot), \mu_2^*(\cdot)\}$ is a solution to the system comprising (2)-(8) and complementary slackness. Then the triplet $\{\bar{R}^*, \hat{R}^*(\cdot), x^*\}$ achieves the unique maximum of ($\mathcal{PP}$.1). To see this, notice that the constraint set is convex and the "maximized Hamiltonian" of the problem above $H(s, \hat{R}(s), \bar{R}^*)$ is strictly concave in $\hat{R}$ for every $s \in \Sigma$. Thus, Arrow's Sufficiency Theorem (see, e.g. Theorem 6.4 in Caputo (2005)) immediately applies. We now classify optimal contracts into families:

i) If $\mu_1^*(s) > 0\ \forall\, s \leq x^*$, which clearly implies $\mu_2^*(s) = 0$, then (7) tells us that $(1-\eta)s = \hat{R}^*(s)\ \ \forall\ s \leq x^*$. Since Lemma 2 proves $\hat{R}^*(s) \leq \bar{R}^*$ everywhere and LLC requires $(1-\eta)x^* \geq \bar{R}^*$ we have that $\hat{R}^*(x^*) \geq \bar{R}^*$ and $\hat{R}^*(x^*) \leq \bar{R}^*$ from which it follows that $\hat{R}^*(x^*) = (1-\eta)x^* = \bar{R}^*$ (i.e. $\phi > 0$). It is easy to see from (2) that $\mu_1^*(s) > 0 \Leftrightarrow u'(\eta s) < \lambda^*$. Thus, only if $\lambda^* > u'(\eta \underline{s})$ holds, a SDC solves the problem ($\mathcal{PP}$.1)-($\mathcal{PP}$.3) which is equivalent to problem ($\mathcal{P}$.1)-($\mathcal{P}$.5).

ii) Now suppose that $\mu_1^*(s) = 0$ for some $s < x^*$. As long as $\mu_1^*(x^*) > 0$ the contract is continuous since $\hat{R}^*(x^*) = (1-\eta)x^* = \bar{R}^*$. Using again condition (2) we know $\mu_1^*(s) = 0 \Leftrightarrow u'(\eta s) \geq \lambda^*$. There are two cases to consider. First suppose that $\mu_2^*(s) = 0\ \forall\ s \leq x^*$ which holds iff $\lambda^* > u'(s)$. Then the optimal contract specifies that $\hat{R}^*(s) > 0\ \forall\ s$, $\hat{R}^*(s) = s - u'^{(-1)}(\lambda^*)$ whenever $u'(\eta s) \geq \lambda^* > u'(s)$ and $\hat{R}^*(s) = (1-\eta)s$ when $\lambda^* > u'(\eta s)$. Next suppose that $\mu_2^*(s) > 0$ for some s which implies that $s < u'^{(-1)}(\lambda^*)$. Then the optimal contract specifies $\hat{R}^*(s) = 0$ whenever $u'(s) > \lambda^*$, $\hat{R}^*(s) = s - u'^{(-1)}(\lambda^*)$ whenever $u'(\eta s) \geq \lambda^* > u'(s)$ and $\hat{R}^*(s) = (1-\eta)s$ as long as $\lambda^* > u'(\eta s)$. That $\hat{R}^{*\prime}(s) = 1$ for some s follows immediately from $\hat{R}^*(s) = s - u'^{(-1)}(\lambda^*)$ since λ^* is unique and independent of s.

iii) Finally, suppose that $\mu_1^*(s) = 0\ \forall\ s \leq x^*$ and $\mu_2^*(s) = 0$ for some $s \leq x^*$. Then (2) implies that $(1-\eta)x^* > \hat{R}^*(x^*) = x^* - u'^{(-1)}(\lambda^*)$. Thus, LLC and Lemma 2 imply that the optimal contract is discontinuous, i.e., $\bar{R}^* > \hat{R}^*(x^*)$. To see this, suppose that it is continuous and find a contradiction. Continuity implies $\hat{R}^*(x^*) = \bar{R}^* < (1-\eta)x^*$ which in turn implies that $\phi = 0$. But then (4) then implies that $0 = -\lambda\gamma$. This can only hold if $\lambda = 0$ but then (3) would require $\int_{x^*}^{\bar{s}} u'\left[s - \bar{R}^*\right] dH(s) = 0$, a contradiction since $\bar{R}^* < (1-\eta)x^*$.

■

A.2 Proof of propositions 1, 2 and 3

Proof of Proposition 1. First, notice that continuity of the contract implies that $u\left[x-\hat{R}(x)\right]-u\left[x-\bar{R}\right]=\hat{R}(x)-\bar{R}$ so that condition (4) reduces to $-\phi(1-\eta)=-\lambda\gamma h(x)$. Combining this result with (3) and writing $x^*(\eta), \lambda^*(\eta)$ to explicitly account for the dependence on η we arrive at:

$$\frac{(1-\eta)\int_{x^*(\eta)}^{\bar{s}} u'\left[s-(1-\eta)x^*(\eta)\right]dH(s)}{(1-\eta)\left[1-H(x^*(\eta))\right]-\gamma h(x^*(\eta))}=\lambda^*(\eta) \tag{9}$$

which implies that $\lambda^*(\eta)>0$ given $\eta\in(0,1)$. Thus, condition (5) requires:

$$(1-\eta)\int_{\underline{s}}^{x^*(\eta)} sdH(s)-\gamma H(x^*(\eta))+(1-\eta)x^*(\eta)\left[1-H(x^*(\eta))\right]=b \tag{10}$$

Next, totally differentiate (10) w.r.t. η:

$$\begin{aligned} 0 &= -\int_{\underline{s}}^{x^*(\eta)} sdH(s)-\gamma h(x^*(\eta))\frac{dx^*}{d\eta}-x^*(\eta)\left[1-H(x^*(\eta))\right]+(1-\eta)\frac{dx^*}{d\eta}\left[1-H(x^*(\eta))\right] \\ &= -\int_{\underline{s}}^{x^*(\eta)} sdH(s)+\frac{dx^*}{d\eta}\left\{(1-\eta)\left[1-H(x^*(\eta))\right]-\gamma h(x^*(\eta))\right\} \end{aligned}$$

and using (9) and the fact that $\lambda^*(\eta)>0$, we conclude that:

$$\frac{dx^*}{d\eta}=\frac{\int_{\underline{s}}^{x^*(\eta)} sdH(s)}{(1-\eta)\left[1-H(x^*(\eta))\right]-\gamma h(x^*(\eta))}>0$$

since the probability of bankruptcy, given by $H(x^*)$, and the repayment function, $(1-\eta)x^*$, are both strictly increasing in x^*, the first part of the proposition follows. Now write $x^*(\gamma)$ to explicitly account for the dependence on γ and (10) as:

$$(1-\eta)\int_{\underline{s}}^{x^*(\gamma)} sdH(s)-\gamma H(x^*(\gamma))+(1-\eta)x^*(\gamma)\left[1-H(x^*(\gamma))\right]=b$$

differentiating w.r.t. γ yields:

$$\frac{dx^*}{d\gamma}=\frac{H(x^*(\gamma))}{(1-\eta)\left[1-H(x^*(\gamma))\right]-\gamma h(x^*(\gamma))}>0$$

and since $\lambda^*(\gamma)>0$, the second part of the proposition trivially follows. ■

Proof of Proposition 2. Let $\boldsymbol{\theta}_{-\eta}=\{b,\gamma,\rho,\boldsymbol{\psi}\}$. Applying the Envelope theorem to ($\mathcal{SD}$.1)-($\mathcal{SD}$.2) we have:

$$\begin{aligned} \frac{dv\left(\boldsymbol{\theta}_{-\eta};\eta\right)}{d\eta} &= \int_{\underline{s}}^{x^*} u'\left[\eta s\right]sdH(s)+x^*\int_{x^*}^{\bar{s}} u'\left[s-(1-\eta)x^*\right]dH(s) \\ &\quad -\lambda^*\left\{\int_{\underline{s}}^{x^*} sdH(s)+x^*\left[1-H(x^*)\right]\right\} \\ &= \int_{\underline{s}}^{x^*}\left[u'(\eta s)-\lambda^*\right]sdH(s)+x^*\left\{\int_{x^*}^{\bar{s}} u'\left[s-(1-\eta)x^*\right]dH(s)-\lambda^*\left[1-H(x^*)\right]\right\} \end{aligned}$$

now using the expression for λ^* and after a minor algebraic manipulation we get:

$$\frac{dv\left(\boldsymbol{\theta}_{-\eta};\eta\right)}{d\eta}=\int_{\underline{s}}^{x^*}\left[u'\left(\eta s\right)-\lambda^*\right]sdH\left(s\right)-x^*\frac{\gamma h\left(x^*\right)\int_{x^*}^{\bar{s}}u'\left[s-\left(1-\eta\right)x^*\right]dH\left(s\right)}{\left[1-H\left(x^*\right)\right]\left(1-\eta\right)-\gamma h\left(x^*\right)}$$

now, the last term of this expression is clearly negative since $\lambda^*>0\Rightarrow\left[1-H\left(x^*\right)\right]\left(1-\eta\right)-\gamma h\left(x^*\right)>0$. Moreover, the first term is also negative since the optimality of SDCs implies that $\forall\ s\leq x^*$, $u'\left(\eta s\right)<\lambda$. Thus, we conclude that $dv\left(\boldsymbol{\theta}_{-\eta};\eta\right)/d\eta<0$. ■

Proof of Proposition 3. It suffices to show that:

$$\int_{\underline{s}}^{x^*}\left[u'\left(\eta s\right)-\lambda\right]sdH\left(s\right)>x^*\frac{\gamma h\left(x^*\right)\lambda}{\left(1-\eta\right)}$$

for some parametrization that satisfies assumptions 1-3 and violates condition (1). Consider the case used in Corollary 2, that is, suppose that $\lim_{c\to0}u'\left(c\right)=\infty$ and $\eta=0$. Then obviously $\lambda^*<u'\left(\eta\underline{s}\right)$ and assumptions 1-3 are satisfied but $\int_{\underline{s}}^{x^*}\left[u'\left(\eta s\right)\right]sdH\left(s\right)>\lambda^*\left[H\left(x^*\right)+x^*\gamma h\left(x^*\right)\right]$. ■

A.3 The dynamic contracting problem and propositions 4 - 5

In this section we spell out the details of the dynamic extension of the contracting problem. The first modification we introduce is that, because the contract now lasts more than one period, we allow for the transfers between agents to be positive or negative and assume that the lender has instant access to a (limited) credit market. Thus, the lower LLC is relaxed to a negative number, $-s_0$ with $s_0>0$, while the upper LLCs remain unchanged.

At time $t=0$ the lender makes a take-it-or-leave-it offer to the entrepreneur with the terms of a financial contract. Each t is associated with a history of events $h^t=\{h_1,h_2,...,h_t\}\in\mathcal{H}^t$ where $\mathcal{H}^t$ is the set of all possible such histories and without loss of generality $h^t=\emptyset\ \forall\ t\leq0$. Under CSV, histories include all past announcements by the entrepreneur and the list of previous periods in which verification took place. That is, the typical component of a particular history is a pair $h_t=\{\hat{s}_t,q_t\}$ where $\hat{s}_t$ is what the entrepreneur reports as the state and $q_t=1$, if monitoring occurred in period t, and $q_t=0$ otherwise. Moreover, under CSV the contract also includes $B\left(h^{t-1}\right)$, a set of states in which the lender verifies after observing history h^{t-1}. In the symmetric information case ($\gamma=0$), $h^t=s^t=\{s_1,s_2,...,s_t\}$.

In order to proceed to the formulation of the problem, we need some definitions:

Definition 3 *A reporting strategy for the entrepreneur, $\hat{\mathfrak{Z}}$, is a sequence of functions that maps histories up to t into reports of the state, i.e.:* $\hat{\mathfrak{Z}}=\{\hat{s}_t\left(h^t\right)\}_{t=1}^{\infty}=\{\hat{s}_t\left(h^{t-1},s_t\right)\}_{t=1}^{\infty}$.

We note that in the current environment the revelation principle still holds so $\hat{\mathfrak{Z}}\subseteq\tilde{\Sigma}$.[18] As in the static problem, large penalties prevent the entrepreneur from misrepresenting in the verification region. Thus, $q_t=1\Rightarrow h_t=\{s_t,1\}$ and $q_t=0\Rightarrow h_t=\{\hat{s}_t,0\}$.

[18]A formal proof of this assertion can be found in Monnet and Quintin (2005).

Definition 4 *A verification strategy for the lender is a sequence of set-valued mappings* $\{B_t(h^{t-1})\}_{t=1}^{\infty}$ *assigning to each history* h^{t-1} *a verification region, i.e., a set of states of nature for which verification occurs.*

The set B_t is the natural, time-varying extension of the set B in section 2.3. We can now define a contract under dynamic CSV:

Definition 5 *A dynamic contract under CSV is a sequence of mappings* $\boldsymbol{\sigma} = \{B_t(h^{t-1}), R_t(h^t)\}_{t=1}^{\infty}$*, assigning current period verification strategies and repayments to each history.*

Notice that $B_t(h^{t-1}) \subseteq \tilde{\Sigma}$ depends upon the history of events up to $t-1$ as verification decisions are independent of the current period realization of the state. On the other hand, even though the form of $R_t(h^t)$ may be known ex-ante, it is contingent on the current realization of the state and therefore depends upon the history up to t.

Next, define the payoffs from the strategies in the subgame starting from h^t given a contract $\boldsymbol{\sigma}$ and a reporting strategy $\hat{\mathfrak{Z}} = \{\hat{s}_t\left(h^t\right)\}_{t=1}^{\infty}$:

$$\mathcal{Q}\left(h^t, \hat{\mathfrak{Z}}, \boldsymbol{\sigma}\right) = \sum_{j=1}^{\infty} \beta^{j-1} \left\{ \sum_{i=1}^{N} \pi_i \left[R_{t+j}(h^{t+j-1}, \hat{s}_{t+j}\left(h^{t-1}, s_{i,t+j}\right)) - b \right] - \sum_{i \mid B_{t+j}(h^{t+j-1})} \pi_i \gamma \right\}$$

for the lender, and:

$$V\left(h^t, \hat{\mathfrak{Z}}, \boldsymbol{\sigma}\right) = \sum_{j=1}^{\infty} \beta^{j-1} \sum_{i=1}^{N} \pi_i \left[u(s_{i,t+j} - R_{t+j}(h^{t+j-1}, \hat{s}_{t+j}\left(h^{t-1}, s_{i,t+j}\right))) \right] dH\left(s\right)$$

for the entrepreneur. We can now define incentive compatible contracts.

Definition 6 *A contract* $\boldsymbol{\sigma}$ *is incentive compatible if* $V(h^t, \mathfrak{Z}, \boldsymbol{\sigma}) \geq V(h^t, \hat{\mathfrak{Z}}, \boldsymbol{\sigma})\ \forall\ \hat{\mathfrak{Z}}\ \forall\ h^t$

In an optimal contract the lender maximizes her time 0 expected utility, the entrepreneur is assured a continuation value v at time 1, repayment functions are feasible, and incentive compatibility holds at each history. Formally:

Definition 7 *An optimal contract,* $\boldsymbol{\sigma}$*, maximizes* $\mathcal{Q}(h^0, \mathfrak{Z}, \cdot)$ *subject to* $V\left(h^0, \mathfrak{Z}, \boldsymbol{\sigma}\right) = v$*, feasibility and* $V(h^t, \mathfrak{Z}, \boldsymbol{\sigma}) \geq V(h^t, \hat{\mathfrak{Z}}, \boldsymbol{\sigma})$ *for all* h^t *and all* $\hat{\mathfrak{Z}}$

Define $\underline{v} = \frac{1}{1-\beta} \sum_{i=1}^{N} \pi_i u\left(\eta s_i\right)$ and $\bar{v} = \sup u\left(c\right) / \left(1-\beta\right) = \frac{1}{1-\beta} \sum_{i=1}^{N} \pi_i u\left[s_0 - (1-\eta) s_i\right]$, respectively as the minimum and maximum attainable expected (lifetime) utility by the borrower. For each $v \in [\underline{v}, \bar{v}]$ consider the problem: maximize the lender's time 0 payoff subject to the entrepreneur getting minimum payoff v, and subject to feasibility and incentive compatibility constraints. Denote the solutions to this problem:

$$\mathcal{J}\left(v\right) = \max \left\{ \mathcal{J} \mid \exists\ \boldsymbol{\sigma} \text{ such that } V\left(h^0, \mathfrak{Z}, \boldsymbol{\sigma}\right) = v \text{ and } \mathcal{Q}\left(h^0, \mathfrak{Z}, \boldsymbol{\sigma}\right) = \mathcal{J} \right\}$$

then all the points $(v, \mathcal{J}(v))$ constitute the Pareto frontier. Notice that $v \mapsto \mathcal{J}(v)$ must be non-increasing, for, otherwise, the lender could simply offer the entrepreneur more and be better off. The recursive form of the contract then becomes $\sigma = \{B(v), R(v, s_i), w(v, s_i), s_i \in \tilde{\Sigma}, v \in [\underline{v}, \bar{v}]\}$ where $R(v, s_i)$ is the repayment schedule and $w(v, s_i)$ is next period promised utility when the current expected payoff for the borrower is v and the state of nature is s_i. The dynamic contracting problem can now be written in recursive form:

$$\mathcal{J}(v) = \max_{R(v,\cdot),w(v,\cdot),B(v)} \left\{ \sum_{i=1}^{N} \pi_i \left[R(v, s_i) - b + \beta \mathcal{J}(w(v, s_i))\right] - \sum_{i|s_i \in B(v)} \pi_i \gamma \right\} \tag{11}$$

$$\text{subject to :}$$

$$u(s_i - R(v, s_i)) + \beta w(v, s_i) \geq u(s_i - R(v, s_j)) + \beta w(v, s_j) \tag{12}$$

$$\forall\, s \in \tilde{\Sigma} \text{ and } \forall\, s_j \notin B(v)$$

$$v = \sum_{i=1}^{N} \pi_i \left[u(s_i - R(v, s_i)) + \beta w(v, s_i)\right] \tag{13}$$

$$-s_0 \leq R(v, s_i) \leq (1-\eta)\, s_i \;\; \forall\, s_i \in \tilde{\Sigma} \tag{14}$$

$$B(v) \subseteq \tilde{\Sigma} \tag{15}$$

where (12) are the dynamic analogues to the set of incentive constraints ($\mathcal{P}.3$), (14) are the modified LLCs and (13) is the so-called promise-keeping constraint (PKC). From this formulation of the problem it is easy to obtain the results of propositions 4 and 5 in the main text.

Proof of Proposition 5. Suppose that the borrower has been promised $\underline{v}$. It is enough to show that if $s_{N-1} \notin B^*(\underline{v})$ the contract fails to be optimal. Suppose that $s_{N-1} \notin B^*(\underline{v})$; and suppose that the borrower uses a strategy that calls her to report s_{N-1} when when she observes s_N. Then the PKC is necessarily violated, for, the minimum expected utility that the borrower can obtain is $\sum_{i=1}^{N-2} \pi_i u(\eta s_i) + \sum_{i=N-1}^{N} \pi_i u\left[s_i - R^*(\underline{v}, s_{N-1})\right] + \beta w^*(\underline{v}, s_{N-1})$. But, since $w^*(\underline{v}, s_{N-1}) \geq \underline{v}$ and $R^*(\underline{v}, s_{N-1}) \leq (1-\eta)\, s_{N-1}$ (by (14)), this lower bound cannot be less than:

$$\sum_{i=1}^{N-1} \pi_i u(\eta s_i) + \pi_N u\left[s_N - (1-\eta)\, s_{N-1}\right] + \beta \underline{v} > \sum_{i=1}^{N} \pi_i u(\eta s_i) + \beta \underline{v} = \underline{v}$$

So define the left hand side of the first inequality to be the threshold $\hat{v}(\eta)$. Clearly then, for any $v \leq \hat{v}(\eta)$, $\{s_1, s_2, ..., s_{N-1}\} \nsubseteq B^*(v)$ is not incentive compatible. It is now easy to see that $\eta \mapsto \hat{v}(\eta)$ is increasing so that the statement of the proposition follows. ∎

References

D. Abreu, D. Pearce, and E. Stacchetti. Toward a theory of discounted repeated games with imperfect monitoring. *Econometrica*, 58:1041–1063, 1990.

V.V. Acharya, Y. Amihud, and L. Litov. Creditor rights and corporate risk-taking. *Journal of Financial Economics*, 102 (1):150–166, 2011.

E. Altman and G Bana. Defaults and returns on high-yield bonds. *The Journal of Portfolio Management*, 30(2):58–73, 2004.

L. Bebchuk. Ex ante costs of violating absolute priority in bankruptcy. *Journal of Finance*, 57(1):445–460, 2002.

B. Bernanke and M. Gertler. Agency costs, net worth, and business fluctuations. *American Economic Review*, 79(1):14–31, 1989.

R. Blazy, J. Petey, and L. Weil. Can bankruptcy codes create value? evidence from creditorsâĂŹ recoveries in france, germany, and the uk. *Working Paper, Laboratoire de Recherche en Gestion et Economie (LaRGE), UniversitÃl' de Strasbourg (France).*, 2010.

A. Bris, I. Welch, and N. Zhu. The costs of bankruptcy: Chapter 7 liquidation versus chapter 11 reorganization. *Journal of Finance*, 61(3):1253–1303, 2006.

M. Caputo. *Foundations of Dynamic Economic Analysis: Optimal Control Theory and Applications*. Cambridge University Press, 2005.

C. Carlstrom and T. Fuerst. Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *American Economic Review*, 87:893–910, 1997.

R. Castro, G. Clementi, and G. MacDonald. Investor protection, optimal incentives, and economic growth. *Quarterly Journal of Economics*, 119(3):1131–1175, 2004.

S. Cheng and C. Shiu. Investor protection and capital structure: International evidence. *Journal of Multinational Financial Management*, 17:30–44, 2007.

L. Christiano, R. Motto, and M. Rostagno. Financial factors in economic fluctuations. *Working Paper Series 1192, European Central Bank.*, 2010.

G. Clementi and H. Hopenhayn. A a theory of financing constraint and firm dynamics. *Quarterly Journal of Economics*, 121:229–265, 2006.

R. Cressy. Credit rationing or entrepreneurial risk aversion? an alternative explanation for the evans and jovanovic finding. *Economic Letters*, 66:235–240, 2000.

P. DeMarzo, M. Fishman, Z. He, and N. Wang. Dynamic agency and the q theory of investment with. *Journal of Finance*, 67:2295–2341, 2012.

S. Djankov, C. McLiesh, and A. Shleifer. Private credit in 129 countries. *Journal of Financial Economics*, 84:299–329, 2007.

H. Fang and J. Nofsinger. Risk aversion, entrepreneurial risk, and portfolio selection. *Journal of Entrepreneurial Finance*, 13:25–55, 2009.

X. Freixas and J.C. Rochet. *Microeconomics of Banking*. The MIT Press, 2008.

D. Gale and M. Hellwig. Incentive compatible debt contracts: the one period problem. *Review of Economic Studies*, 52(4):647–663, 1985.

A. Galindo and A. Micco. Creditor protection and financial markets: Empirical evidence from latin america. *Economic Review, Federal Reserve Bank of Atlanta*, Second Quarter:29–37, 2004.

S.E. Ghoul, Guedhami O., S. Cho, and J. Suh. Creditor rights and capital structure: Evidence from international data. In *Preceedings of the Annual Meetings, European Financial Management Association*, 2012.

J. Greenwood, J.M. Sanchez, and C. Wang. Financing development: The role of information costs. *American Economic Review*, 100:1875–1891, 2010.

H.K. Hvide and T.E. Leite. Optimal debt contracts under costly enforcement. *Economic Theory*, 44(1):149–165, 2010.

S. Kalemli-Ozcan, B. Sorensen, and S. Yesiltas. Leverage across firms, banks and countries. *NBER Working Paper No. 17354*, 2011.

S. Krasa and A.P. Villamil. Optimal contracts when enforcement is a decision variable. *Econometrica*, 68(1):119–134, 2000.

S. Krasa, T. Sharma, and A.P. Villamil. Bankruptcy and firm finance. *Economic Theory*, 36: 239–266, 2008.

R. La Porta, F. Lopez-de Silanes, A. Shleifer, and R. W. Vishny. Legal determinants of external finance. *Journal of Finance*, 52(30):1131–50, 1997.

R. La Porta, F. Lopez-de Silanes, A. Shleifer, and R. W. Vishny. Law and finance. *Journal of Political Economy*, 106(6):1113–1155, 1998.

R. Levine. *Handbook of Economic Growth. Vol. 1a*, chapter Finance and growth: Theory and evidence, pages 865–923. North-Holland, 2005.

K. Lewellen. Financing decisions when managers are risk averse. *Journal of Financial Economics*, 82:551–589, 2006.

C. Monnet and E. Quintin. Optimal contracts in a dynamic costly state verification model. *Economic Theory*, 26:867–885, 2005.

V. Panousi and D. Papanikolaou. Investment, idiosyncratic risk, and ownership. *Journal of Finance*, 67:1113–1148, 2012.

P.F. Pereira and M.A. Ferreira. Capital structure and law around the world. *Journal of Multinational Financial Management*, 21:119–150, 2011.

R. Rajan and L. Zingales. What do we know about capital structure? some evidence from international data. *Journal of Finance*, 50:1421–1460, 1995.

T. Schmid, M. Ampenberger, C. Kaserer, and A-K. Achleitner. Family firms, agency costs and risk aversion: empirical evidence from diversification and hedging decisions. *CEFS Working Paper No. 2008-13*, 2008.

P. Sevcik. Financial contracts and the political economy of investor protection. *American Economic Journal: Macroeconomics*, 4:163–197, 2012.

A. Smith and C. Wang. Dynamic credit relationships in general equilibrium. *Jorunal of Monetary Economics*, 53:847–877, 2006.

S. Spear and S. Srivastava. On repeated moral hazard with discounting. *Review of Economic Studies*, 54:599–617, 1987.

R. Townsend. Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21(2):265–293, 1979.

C. Wang. Dynamic costly state verification. *Economic Theory*, 25:887–916, 2005.

O. Williamson. Corporate finance and corporate governance. *Journal of Finance*, 43:567–591, 1988.

S. Williamson. Costly monitoring, loan contracts, and equilibrium credit rationing. *Quarterly Journal of Economics*, 102:135–146, 1987.

A. Winton. Costly state verification and multiple investors: The role of seniority. *Review of Financial Studies*, 8(1):91–123, 1995.