Electronic Purse *versus* Fiat Money: A Harsh Competition

Sébastien Lotz[1] & Françoise Vasselin[2]
LEMMA – Université Panthéon-Assas (Paris 2)

June, 2013

Working Paper in Progress

Abstract

We study the competition between fiat money and electronic money (such as e-purse or cash cards) in a search-theoretic model based on Lagos-Wright (2005). Indeed, both currency and electronic cash may be used to make payments in retail trade, and thus compete for low value transactions. However, our intent is to explain the reasons for the electronic purse failure in retail trade, and the necessary conditions for its large adoption in everyday transactions. To do so, we model the fact that fiat currency may be stolen but is universally accepted, contrary to electronic money which is safer but may not be always accepted as sellers have to pay a fixed investment cost in order to acquire a reading machine at the POS. We show that although buyers receive the storage device (smart cards) for free, or at negligible cost, they may decide not to carry and use e-money. Therefore, our framework captures the strategic complementarities between consumers and retailers, and studies the condition for the adoption of e-money and its replacement for cash.

Keywords: cash, electronic money, costly record keeping investment, network externalities
JEL Classification: D82, D83, E40, E50

[1] Address: LEMMA – Université de Paris 2, 92 rue d'Assas, 75006 Paris. E-mail: lotz@u-paris2.fr
[2] Address: LEMMA – Université de Paris 2, 92 rue d'Assas, 75006 Paris. E-mail: f.vasselin@orange.fr

Introduction

The two last decades are characterized by a growing use of new means of payment, meaning consumers can choose among a large variety of payment instruments, depending on the type of transaction they intent to conduct. The common point of these different payment instruments is that they are progressively replacing traditional ones, like cash. One of them, electronic money (like e-purse or cash-cards), appeared in recent years specifically to replace cash in low value transactions.

The European Central Bank defines e-money as "a claim on the issuer that is (a) stored on an electronic device (the first criterion), (b) issued on receipt of funds of an amount not less in value than the monetary value issued (the second criterion) and (c) accepted as means of payment by undertakings other than the issuer (the third criterion)" (ECB, 2000). The Bank of International Settlements defines electronic money as "a stored value or prepaid product in which a record of the funds or value available to the consumer for multipurpose use is stored on an electronic device in the consumer's possession" (BIS, 2004). Therefore, electronic money acts as a prepaid instrument that can be considered as an electronic substitute for coins and banknotes. In fact, electronic money is a bank liability to the issuer which circulates like paper money without bank authorization after emission, and which can be redeemed for one unit of fiat money.[3]

In order to reduce the social cost of cash, many initiatives have emerged around the world to provide individuals and merchants with an alternative to cash. Partisans of a cashless society state that technological innovations in payment instruments constitute a solution to reduce this social cost, estimated at around 250€ a year, per person, in Europe (McKinsey Report, 2008). One of them, the electronic purse, is the closest substitute for cash in everyday low-value transactions. However, in developed countries, electronic money does not constitute a credible substitute for cash despite the high number of storage devices available for free at Universities, Staff Canteens, and the development of mandatory uses for parking meters or in post offices. Indeed, the last international statistics on settlement systems state that electronic money constitutes 2.2% of the total number of transaction only (and 3% as a percentage of the total value transactions), although the number of transactions conducted with it increased by 21.4%. Moreover, between 2011 and 2012, the e-money loading transactions increased by 3.6% compared to the number of cash withdrawals that increased by 13.0% over the same period, which can explain the recent decreasing trend of e-money use in some developed countries such as Belgium, Germany, or Switzerland (BIS, 2013).

Despite the existence of alternatives to fiat money, cash remains the most widely means of payment held for everyday transactions in the retail sector. Consequently, our intent is to better understand, and explain such a situation. To do so, we study the

[3] Contrary to fiat money, traditionally referred as "outside money", electronic money is defined as "inside money" as it is a debt that is used as money.

conditions under which electronic money can replace cash for low value transactions in a framework where money is essential, like the Lagos and Wright (2005) search model.

In our model, we distinguish cash from electronic money in the following way. Cash is universally accepted by sellers at no cost but can be stolen, so there is always a risk of theft when carrying fiat money. Electronic money is only partially accepted in the sense that in order to be able to pay with it, some retailers must have invested in an electronic device at the POS to accept e-money payments. Such a device is costly for sellers (explaining why some retailers may not invest), but makes the payment safe: money collected is electronically recorded in the reading terminal before being sent by communication networks to sellers' bank account, to be credited[4]. For buyers, banks can also play this safekeeping role by guaranteeing the refund of e-money lost or stolen if agents have chosen a safe electronic means of storage which is personalized and associated with a bank account.

Carrying cash or e-money involves the same opportunity cost in terms of foregone interest earnings. However, these two means of payments are imperfect substitutes, and it is precisely why we analyze agents' decisions to adopt or not the new means of payment. On the one hand, an important feature is the lack of universal acceptability of electronic money which requires that buyers form expectations about the probability of being able to use their e-purse if they enter the market with it, given that e-money has the advantage of being safer to use than cash. On the other hand, sellers must decide whether or not to accept electronic money in parallel to cash, given the cost associated with the investment in an electronic reading terminal. The buyers and sellers joint decisions create strategic complementarities in the choice of the payment instrument, like in Nosal and Rocheteau (2011). Consequently, replacing cash with electronic money requires that the latter becomes a quasi-universal means of payment.

We provide a framework to explain the conditions under which electronic money can replace cash. The cost of investment is determinant to understand sellers' e-money adoption. At the same time, sellers' expectations about the buyers' portfolio composition are crucial to give them incentives to invest. Two ingredients also are important for buyers to decide to carry e-money or not: the level of risk of theft of cash, and the probability of being able to use e-money in future transactions. Our analysis shows that when the risk of theft increases, the quantity exchanged against cash decreases, and the monetary transfer increases, consisting of a risk premium paid to the seller for the risk of theft transferred with a cash payment. Moreover, when the investment cost is high, the e-money adoption requires a high probability of being stolen, although it may not be sufficient. Therefore, different payment patterns can emerge, where fiat money coexists with electronic money, or where one of the two monies dominates the other.

[4] The value spent by the buyer is reduced from his card balance and is added to the balance of the seller's reading machine. The seller redeems the monetary values at the end of the day from a clearing operation (like with credit cards) by sending to the network the stored values which are credited to his bank account (Lacker, 1996).

Related literature

The possibility for households to choose among a large variety of new electronic means of payment has involved an extended literature in modern monetary theory. However, the analysis of the buyer's choice of a means of payment is not a recent issue, and has been developed in different frameworks. For example, in a very different environment from ours, Folkertsma and Hebbink (1998) extended the Whitesell (1992) analysis of the payment instruments choice to the prepaid cards, and showed that the use of electronic money is efficient to settle recurrent low value transactions, compared to cash, checks or debit cards. Another analysis with cash and electronic money, conducted by Shy and Tarkka (2002), showed that the choice of a payment instrument depends on the size of the transactions, and that electronic purses are useful for low value transactions. More recently, the competition between different payment instruments has been studied in search based environments which explicitly model trade frictions. Based on the first generation of monetary search models (Kiyotaki and Wright, 1993), Lotz (2005) distinguishes fiat and electronic monies by considering that the use of these two monetary instruments is associated with various types of costs. Multiple equilibria then emerge. However, this framework assumes indivisible goods and money. Using the second generation of monetary search models (Trejos and Wright, 1995), Kim and Lee (2010) analyze the competition between debit card and cash when there is a fixed record-keeping cost to pay to being able to accept debit cards. We also introduce a fixed cost corresponding to an access price to the e-money technology paid by sellers who invest in the reading terminal. As in our study the fixed record keeping cost is only borne by sellers. However, Kim and Lee assume that the buyers support the cost of holding cash, and no cost if they use their credit card. By contrast, in our study, the opportunity cost associated with both competitive means of payment is exactly identical. As a result, the interest rate doesn't play a crucial role to explain the choice between the two media of exchange. Moreover, the present study introduces the role of strategic complementarities between the two sides of the market in the choice of a new medium of exchange. Based on the third generation of monetary search models (Lagos-Wright, 2005), several recent papers study situations where agents can pay for goods with two competitive medium of exchange. Closely related to our paper is He, Huang and Wright (2008) who model the concurrent circulation of cash and bank liabilities as media of exchange by considering a risk of theft for fiat money. They conclude that without theft, there is no role for banks. Li (2011) studies the role of a fixed record keeping cost to distinguish checking deposits from currency, and considers that cash holdings are costly because of the risk of loss or theft, and the forgone interest earnings. A key difference in these studies is the assumption of a buyer-take-all bargaining solution for the pricing mechanism. By contrast, we use the proportional bargaining solution in order to give sellers incentives to invest. In another approach, Nosal and Rocheteau (2011) also assume an investment cost in a record-keeping technology allowing sellers to accept credit, or assets, instead of cash. They show strategic complementarities between the buyers’ and sellers’ decisions. Lotz and Zhang (2013) investigate the substitution from cash to electronic payment such as credit cards in order to predict the demand for fiat money. By contrast with the two last studies, we can use neither the role of interest rate nor the role of inflation to explain the choice between the two monies since they have the same opportunity cost. The main difference in the present model is that we explain the choice between two currencies by introducing a second meeting in the decentralized market with a thief. This allows us to explain the adoption of a type of money from the risk of theft associated with cash, which provides multiple equilibria. Moreover, we explain the coexistence of both monies from the strategic complementarities between agents.

The remainder of the paper is organized as follows. In section 1, we present the basic model, and the assumptions. In section 2, we define the activity on the centralized market. In section 3, we determine the terms of trade, depending on the buyers' portfolio composition, and we show how the risk of theft affects the value of fiat money compared to e-money. In section 4, we analyze the buyer's choice of his portfolio composition and the sellers' decision to invest or not in the e-money technology. In section 5, we study the different possible equilibria of the model and determine two critical values. The critical value for the risk of theft from which fiat money is no longer valued and the critical value for the fraction of sellers who adopted e-money technology and below which no buyers enter the market with e-money. Section 6 concludes.

1. Environment: the basic model and its assumptions

Our model is based on the monetary search model proposed by Lagos and Wright (2005), and developed by Nosal and Rocheteau (2011).

There is a [0,1] continuum of infinitely-lived agents in the economy, and time is discrete. Agents visit two markets at each period, which correspond to two sub-periods. In the first sub-period, agents visit a decentralized market called *DM*. Anonymous buyers and sellers, specialized in consumption or production, are matched bilaterally and randomly. Sellers produce an output $q \in R+$, but do not want to consume, while buyers want to consume but cannot produce. During this meeting, if an agreement is reached, agents trade a quantity $q = q(f, e)$ of a non storable specific good, called *search good*, in exchange for a means of payment, either fiat money (f) or electronic money (e) or both ($f + e$) depending of the buyer's portfolio. In this model, the lack of double coincidence of wants involves the exclusion of barter, and because of limited commitment and imperfect record keeping there is no role for credit. As a result, money is essential. Moreover, both consumption good and monetary balances are perfectly divisible. Finally, trading histories of agents are private information so trade must be quid pro quo. We introduce to the basic model a *second meeting* with a thief in the DM. Indeed, buyers or sellers meet a thief with probability $\gamma \in (0,1)$ at the end of the first sub-period. If an honest agent meets a thief, the latter robs his cash, and the honest agent enters the subsequent market without fiat money.

At the end of this first sub-period, agents enter a frictionless centralized market, called *CM* that corresponds to the second sub-period. In this market, thieves behave like honest people and all agents adjust their monetary balances depending of their portfolio composition. They can consume a *general good*, $x \in R+$ by supplying labor, h.

The utility functions for buyers and sellers for the entire period are supposed to be separable between the two sub-periods and linear in the CM.

$$U^b(q,x,h) = u(q) + x - h$$
$$U^s(q,x,h) = -c(q) + x - h$$

As in the basic model, we assume that $u'(q) > 0$, $u''(q) < 0$, $c'(q) > 0$, $c''(q) > 0$, $u(0) = c(0) = c'(0) = 0$ and $u'(0) = +\infty$. The optimal quantity produced and exchanged is denoted $q*$ and corresponds to the level of production and consumption of the search good that maximizes the trade surplus between agents, $u(q) - c(q)$. This quantity solves $u'(q*) = c'(q*)$ and is consequently defined such as $q* \equiv \{q : u'(q*) = c'(q*)\}$. All agents discount the future between all the periods but not between sub-periods. The discount factor is denoted $\beta \in (0,1)$.

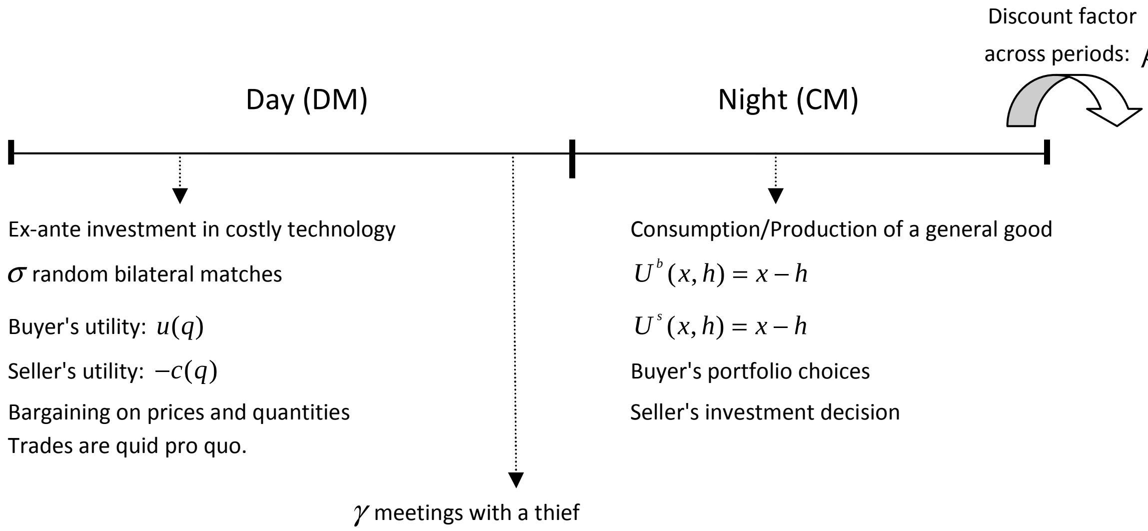
In this economy, there are two means of payment, fiat money which is legal tender, and electronic money which is the new means of payment that competes with fiat money, and which is intended to replace cash for small value transactions. Buyers enter the DM with a portfolio of real monetary balances ($z = f + e$) consisting with fiat money (f) and/or electronic money (e). Both monies have an identical opportunity cost measured by the interest earnings forgone. However, electronic money is safer than cash, which can be stolen with some probability at the end of the DM. Moreover, fiat money is costless while electronic money is costly for sellers and accessible for buyers at a negligible cost or for free.

Consequently, buyers can pay for goods with cash or with electronic money. However, only a fraction Λ of sellers have access to the e-money technology and hence only a measure Λ of sellers can accept both fiat money and electronic money as means of payment. The remaining fraction of sellers $1-\Lambda$ can only receive cash in exchange for goods. At the beginning, we consider an exogenous fraction of sellers who have access to the e-money technology. Then in the section 4, the probability Λ for a buyer to meet a seller who accepts e-money is made endogenous by considering the seller's decision problem to invest in the e-money technology at a fixed cost. Sellers can choose to invest or not to invest during the frictionless market (CM), before the realization of trades in the decentralized market (DM). As a result, when some sellers decide not to invest, electronic money is not a universal means of payment since a fraction $1-\Lambda$ of sellers will refuse it because they do not possess the reading terminal at the POS.

To summarize, we have added two supplementary frictions to the basic environment[5], (i) a risk of theft at the end of the day market (DM), before entering the night market (CM), and (ii) a limited acceptability of electronic money, meaning fiat money is always accepted as legal tender whereas e-money is not always accepted if sellers have not invested in the technology to accept e-money. The timing of events in a period is depicted in figure 1.

[5] The basic frictions defined in the monetary search models based on the Lagos-Wright framework are (i) the search frictions faced by agents in the decentralized market, (ii) the anonymity of agents, (iii) their limited commitment and (iv) the lack of record keeping of their decisions and actions.

Figure 1: Timing of events and preferences



2. The centralized market

We begin with the second subperiod. In the centralized market, all agents can produce and consume. They produce a general good with h units of labor, consume x units of general good and readjust their fiat and electronic balances. We consider at first the buyer's maximization problem. The buyer's maximization program holding the portfolio $z = f + e$ of real balances (consisting of fiat money and electronic money), and entering the centralized market, satisfies:

$$\begin{cases} W^b(z) = \max\limits_{x,h,z'\geq 0} \left\{ x - h + \beta V^b(z') \right\} & (1) \\ s.t. \quad x + z' = z + h & (2) \end{cases}$$

where V^b is the value function of a buyer at the beginning of the decentralized market (DM). Equation (1) indicates that the buyer finances his consumption of general good (x) in the CM, and his real balances of the next period $(z' = f' + e')$ from his current real balances $(z = f + e)$ and his income generated by labor (h). From equation (2), we obtain: $x - h = z - z'$. Then, by substituting $x - h$ in the value function (1), we obtain: $W^b(z) = \max\limits_{z'\geq 0} \left\{ z - z' + \beta V^b(z') \right\}$. After rearrangements, the buyer's value function in the CM with the portfolio $z = f + e$ is given by:

$$W^b(f,e) = f + e + \max_{f',e'\geq 0} \left\{ -f' - e' + \beta V^b(f',e') \right\} \quad (3)$$

The value function of the buyer at the beginning of the centralized market is linear with his real monetary balances $z = f + e$. From conditions (1) and (2), the buyer's portfolio, for the next period, is determined independently of x and z. It means that the choice of the optimal portfolio for the next decentralized market $z' = f' + e'$ is independent of the initial portfolio z that the buyer brought into the centralized market. The quasi linearity of the utility function eliminates the wealth effects between each period. From this linearity the buyer's current portfolio is: $W^b(f,e) = f + e + W^b(0,0)$.

Identically, the value function of the seller who holds f units of fiat real balances and e units of electronic real balances at the beginning of the centralized market is given by:

$$W^s(f,e) = f + e + \beta V^s(0,0)$$

where $V^s(0,0)$ is the value function of a seller without money at the beginning of the subsequent decentralized market (DM). Indeed, on the DM sellers are specialized in production, and hence do not need money since they don't want to consume. Consequently, sellers have no incentive to enter the DM with real balances or to accumulate them as they are costly to hold.

3. The decentralized market and the terms of trade

We now describe the determination of the terms of trade in a bilateral match in the DM between a buyer holding the portfolio $z = f + e$ and a seller without money. They will bargain until achieving an agreement, i.e. a pair (q,d) where the buyer receives $q \geq 0$ units of the search good produced by the seller in exchange for $d \in [0,z]$ units of money. The monetary units transferred $(d \geq 0)$ from the buyer to the seller consist of an amount of fiat money d_f and an amount of electronic money d_e. We adopt the proportional bargaining solution to allow the seller to extract a fraction of the total match surplus, giving him incentives to invest. As a result, the buyer receives the share $\theta \in [0,1]$ of the total match surplus and the seller the share $(1-\theta) \in [0,1]$.
We analyze two situations depending of the composition of the buyer's portfolio. In a first case, the buyer enters the market with fiat money only ($z = f$), and in a second case he brings in addition to fiat money an amount of electronic monetary units ($z = f + e$).

3.1. Terms of trade when buyers enter the DM with fiat money ($z = f$)

We begin by the determination of the match surplus when the buyer enters the market with fiat money and then we solve the maximization problem to obtain the terms of trade. We begin by the situation where an agreement is reached between the buyer and the seller and then we will consider the case of an absence of agreement between the agents.
When an agreement is reached, the buyer’s and seller’s utilities are:

$$\begin{cases} u_f^b = u[q(f)] + (1-\gamma)W^b(f - d_f) + \gamma W^b(0) \\ u_f^s = -c[q(f)] + (1-\gamma)W^s(d_f) + \gamma W^s(0) \end{cases} \quad (4)$$

From (4), the buyer's utility u_f^b depends on the probability of the two types of meetings. At the beginning, the buyer meets a seller and the two agents decide to exchange goods for fiat money. After this first meeting, the buyer meets a thief with probability γ and no thief with the complementary probability $(1-\gamma)$. As a result, during the first meeting, when the buyer meets a seller, agents reach an agreement and the buyer transfers to the seller the amount (d_f) of fiat money in exchange for the quantity $q(f)$ of the search goods. If the buyer doesn't have a second meeting, event which occurs with the probability $(1-\gamma)$, he continues in the subsequent CM with his remaining quantity of fiat money ($f-d_f$). As opposite, if the buyer meets a thief with probability (γ) the thief steals his fiat balances and the buyer enters the subsequent market without fiat money ($f-d_f-(f-d_f)=0$). Similarly, the seller can have two meetings which determine his utility u_f^s. At the beginning, the seller meets a buyer and agents reach an agreement. The seller sells the quantity $q(f)$ of goods in exchange for an amount (d_f) of fiat money. If the seller doesn't meet a thief with probability $(1-\gamma)$, he enters the CM with the buyer's monetary transfer (d_f). If he meets a thief with probability (γ), he enters the market without fiat money.

From the linearity of the value function $W^b(f)$, we obtain:

$$\begin{cases} u_f^b = u\left[q(f)\right]+(1-\gamma)\left[-d_f + W^b(f)\right]+\gamma W^b(0) \\ u_f^s = -c\left[q(f)\right]+(1-\gamma)\left[d_f + W^s(0)\right]+\gamma W^s(0) \end{cases} \quad (5)$$

We now turn to the case where agents do not achieve an agreement. Without agreement, the buyer's and seller's utilities are:

$$\begin{cases} u_0^b = (1-\gamma)W^b(f)+\gamma W^b(0) \\ u_0^s = W^s(0) \end{cases} \quad (6)$$

Equations (6) indicates the buyer's utility (u_0^b) and the seller's utility (u_0^s) if agents didn't achieve any agreement in the DM. At the end of the DM, the buyer doesn't meet a thief, with probability $(1-\gamma)$, and then enters the subsequent market with his fiat monetary balances ($f>0$). At the opposite, with probability γ, the buyer meets a thief who steals his money and then enters the following market without money ($f=0$). A seller who does not hold money (if no transaction has occurred with a buyer) runs no risk of being theft, even if he meets a thief at the end of the DM. Hence, the seller enters the subsequent market without money.

We now determine the agent's trade surpluses when fiat money is the only means of payment. The buyer's trade surplus (S_f^b) and the seller's trade surplus (S_f^s) are written as the difference between the utility resulting from an agreement and that one without agreement: $S_f^b = u_f^b - u_0^b$ and $S_f^s = u_f^s - u_0^s$. By replacing the terms u_f^b, u_0^b, u_f^s and u_0^s with their expressions in (5) and (6), we obtain the following equations determining the agent's total trade surpluses:

$$\begin{cases} S_f^b = u\left[q(f)\right]-(1-\gamma)d_f \\ S_f^s = -c\left[q(f)\right]+(1-\gamma)d_f \end{cases} \quad (7)$$

The buyer's trade surplus (S_f^b) is measured by the difference between the utility incurred by the consumption of the quantity $q(f)$ and the probability of the monetary transfer from the buyer to the seller i.e. $(1-\gamma)d_f$. This last expression needs some explanation. Indeed, the expression $(1-\gamma)d_f$ means that the monetary transfer only occurs with the probability $(1-\gamma)$. Even if the buyer actually transfers the entire amount d_f of money to the seller, only the share $(1-\gamma)$ is likely to be transferred to the seller. Indeed, if after the market transactions the seller meets a thief who steals the money which was transferred by the buyer (with the probability γ), he enters the following market without money. So, the seller enters the CM with money only with the complementary probability $(1-\gamma)$, i.e. only if he didn't meet a thief. This situation is identical to the fact that the monetary transfer to the seller just occurs with the probability of no theft $(1-\gamma)$. As a result we notice that the *expected value of the monetary transfer* to the seller $(1-\gamma)d_f$ has two components: $(1-\gamma)d_f = d_f - \gamma d_f$:

- the effective transfer of an *amount of fiat money* in exchange for goods (d_f)
- the *risk of theft* γd_f i.e. the risk of being robbed his money that reduces the amount of money initially transferred.

The risk of theft γd_f intervenes as a reduction of the amount of money actually transferred d_f. The reason is that once the buyer has transferred money to the seller, the buyer no longer runs the risk of theft. Through the money transfer, the buyer also transfers the risk of being stolen. By transferring money, the buyer got rid of the risk and this is why this risk comes as a reduction of the amount of money given to the seller. After the transaction, it is the seller who runs the risk of being robbed and this risk occurs with the same probability γ for both agents. As a result, the buyer transfers to the seller an amount of money (d_f) plus a risk (γd_f) that reduces the total amount that he has transferred and the difference between the two components defines the *expected value* of the money moved between agents $(1-\gamma)d_f$.

From the previous definitions of the agent's trade surplus (7), we now turn to the bargaining problem. The *terms of trade* are determined by the following program:

$$(q,d) = \max_{d_f \le m_f} \left[u\left[q(f)\right] - (1-\gamma)d_f \right] \tag{8}$$

$$s.t. \begin{cases} q(f) \ge 0 \\ \left[u\left[q(f)\right] - (1-\gamma)d_f \right] = \dfrac{\theta}{1-\theta}\left[-c\left[q(f)\right] + (1-\gamma)d_f \right] & (9) \\ d_f \le f & (10) \end{cases}$$

According to the previous problem, the buyer maximizes (8) his utility of consuming the DM good net of the expected and likely value of the money he transfers to the seller: $(1-\gamma)d_f$; subject to the constraint (9) that the buyer's trading surplus is equal to $\theta/(1-\theta)$ times the seller's trading surplus and (10) the buyer cannot transfer to the seller more fiat money than he has ($d_f \leq f$). The term $\theta/(1-\theta)$ indicates the relative bargaining power of the two players, i.e.: their power ratio.

From (9) we obtain the expression of the *terms of trade* (see appendix 1) that can be written following two ways, either as the *expected value of the monetary transfer* from the buyer to the seller:

$$(1-\gamma)d_f = (1-\theta)u\left[q(f)\right] + \theta c\left[q(f)\right] \qquad (11)$$

or as the total amount of the money actually transferred:

$$d_f = \frac{(1-\theta)u\left[q(f)\right] + \theta c\left[q(f)\right]}{(1-\gamma)} \quad \text{for } \gamma \neq 1 \qquad (12)$$

Equation (11) precisely defines the terms of trade that establish a relationship between the amount of real balances available in a match that the buyer transfers to the seller in exchange for a amount of output. From (11) we notice that when the risk of being stolen becomes certain ($\gamma = 1$), the expected value of the monetary transfer is zero ($(1-\gamma)d_f = 0$). As a result, if $\gamma = 1$ there is no longer fiat money available for trades since agents no longer value it. Equation (12) shows that for $\gamma \neq 1$, an increase in the risk of theft increases the actual amount of money that must be transferred (d_f) to the seller and hence decreases the quantity of goods obtained in exchange. Moreover, from (12), we obtain the amount of money that must be transferred between agents:

$$d_f = (1-\theta)u\left[q(f)\right] + \theta c\left[q(f)\right] + \gamma d_f \qquad (13)$$

Equation (13) defines the total amount of cash that the buyer must give to the seller as the sum of the term $\omega(q_f) = (1-\theta)u(q_f) + \theta c(q_f)$ - which measures the value of the monetary transfer - and a *risk premium* defined by γd_f. The more the risk of theft is important and the more the probability of transferring money between honest agents is low. As a result, when the risk of being stolen increases (γ) the buyer must give the seller a higher amount of money (d_f) for the same quantity q_f of goods than that would be exchanged if there was no risk of theft. The additional quantity of money that the buyer transfers to the seller (γd_f) corresponds to a *risk premium* that compensates the seller for the risk he runs of being robbed of his money before entering the CM.

The fact that the buyer accepts a bargaining solution which requires spending more money than the amount only justified by the exchange of goods is interpreted as a *preference for goods*. This means that the buyer prefers to purchase goods and so to spend more fiat money instead of running the risk of being stolen if he didn't spend enough. The risk of theft increases the preference for the goods. In other words, the buyer prefers to get rid of his cash in order to hold goods. For its part, the seller requires a larger amount of money because after the exchange he runs the risk of being stolen and a larger quantity of money received compensates this risk.

By substituting d_f by its expression in (12) into (8) and (10), the problem is simplified as below (see appendix 2):

$$\begin{cases} q = \arg\max_{q(f)} \theta\left[u\left[q(f)\right] - c\left[q(f)\right]\right] & (14) \\ \text{subject to} \quad d_f = \dfrac{(1-\theta)u\left[q(f)\right] + \theta c\left[q(f)\right]}{(1-\gamma)} \leq f & (15) \end{cases}$$

Equation (14) indicates the buyer's maximization problem. The buyer chooses the quantity consumed $q(f)$ so that he obtains with it the share (θ) of the total match surplus with fiat money: $S_f^T = u\left[q(f)\right] - c\left[q(f)\right]$. When the risk of theft increases, the quantity of money transferred increases and the quantity of goods exchanged decreases. If the buyer carries enough money he consumes the optimal quantity; otherwise, he purchases the quantity which corresponds to his cash in hand. Thus, for $\gamma \neq 1$:

1. If $d_f \leq f$ the constraint (10) does not bind, that is if $f \geq (1-\theta)u\left[q(f)\right] + \theta c\left[q(f)\right] / (1-\gamma)$ the buyer has sufficient monetary balances to purchase the efficient level of output and the solution to (14) is $q(f) = q*$.

2. Otherwise, if $f < d_f$, the constraint $d_f \leq f$ binds, meaning $d_f = f$ and the production and consumption level of the DM output $q(f)$ which will be produced and consumed in exchange for fiat money is solution to $f = \left\{(1-\theta)u\left[q(f)\right] + \theta c\left[q(f)\right]\right\} / (1-\gamma)$, where $q(f) = q\left[d(f)\right] < q*$.

As a result the terms of trade are defined such as:

$$\text{If:} \quad \begin{cases} f \geq d_f \\ f < d_f \end{cases} \Rightarrow \begin{cases} f* = \dfrac{(1-\theta)u\left[q*(f)\right] + \theta c\left[q*(f)\right]}{(1-\gamma)} \\ f = \dfrac{(1-\theta)u\left[q(f)\right] + \theta c\left[q(f)\right]}{(1-\gamma)} \end{cases} \tag{16}$$

$$\text{Or, if:} \quad \begin{cases} (1-\gamma) f \geq (1-\gamma) d_f \\ (1-\gamma) f < (1-\gamma) d_f \end{cases} \Rightarrow \begin{cases} (1-\gamma) f* = (1-\theta)u\left[q*(f)\right] + \theta c\left[q*(f)\right] \\ (1-\gamma) f = (1-\theta)u\left[q(f)\right] + \theta c\left[q(f)\right] \end{cases} \tag{17}$$

We define $\omega(q_f) = (1-\theta)u(q_f) + \theta c(q_f)$ as the value of the money transferred from the buyer to the seller, events which incurs with the probability $(1-\gamma)$. Finally, the DM output q_f solves:

$$\omega(q_f) = \min\left\{\omega(q_f*), (1-\gamma) f\right\} \tag{18}$$

3.2. Terms of trade when buyers enter the DM with fiat and electronic monies ($z = f + e$)

The quantity of goods exchanged depends on the total amount of monetary units in the buyer's portfolio that consists of fiat and electronic monies. Thief may only steal fiat money but not electronic money. Indeed, if the thief steals the smart card or another electronic means of storage (such as the mobile phone), buyer will immediately report the issuer that he has been stolen. In this case, the buyer will be redeemed of the total amount stolen and the funds recorded on the electronic device can no longer be used[6].

As in the previous section we consider two different situations in order to define the trade surplus of agents. At first, we consider the case of an agreement between a buyer and a seller and then the case of a lack of agreement. When an agreement is reached, the buyer's and seller's utilities are defined as follows:

$$\begin{cases} u_{f,e}^{b} = u\left[q(f,e)\right] + (1-\gamma)W^{b}\left[f-d_{f}, e-d_{e}\right] + \gamma W^{b}\left[0, e-d_{e}\right] \\ u_{f,e}^{s} = -c\left[q(f,e)\right] + (1-\gamma)W^{s}(+d_{f}, +d_{e}) + \gamma W^{s}(0, +d_{e}) \end{cases} \tag{19}$$

The buyer's utility $u_{f,e}^{b}$ resulting from an agreement with a seller is measured as the sum of three terms. The first term corresponds to the consumption utility of an amount $q(f,e)$ of search good in the DM depending of the buyer's portfolio composition i.e. both types of balances. The two following terms describe two events that occur with a complementary probability. The previous utility is increased by the probability $(1-\gamma)$ for the buyer to enter the CM with all his monetary units that he has not spent if he didn't meet a thief plus the probability γ of entering the CM only with its electronic money if he met a thief who has taken all the fiat money he had in hand.

Similarly, the seller's utility $u_{f,e}^{s}$ of an agreement with a buyer is measured as the difference between the production costs of an amount $q(f,e)$ of search good on one hand and the probability for the seller to continue in the next CM with the buyer's monetary transfer if he doesn't meet a thief $(1-\gamma)W^{s}(+d_{f}, +d_{e})$ plus the probability to enter the CM only with the electronic money if he has met a thief who stole the day's takings: $\gamma W^{s}(0, +d_{e})$ on the other hand.

From the linearity of the value functions W^{b} and W^{s} equations (19) can be rewritten as follows:

$$\begin{cases} u_{f,e}^{b} = u\left[q(f,e)\right] + (1-\gamma)\left[-d_{f} + W^{b}(f)\right] + \gamma W^{b}(0) - d_{e} + W^{b}(e) \\ u_{f,e}^{s} = -c\left[q(f,e)\right] + (1-\gamma)\left[+d_{f} + W^{s}(0)\right] + \gamma W^{s}(0) + d_{e} + W^{s}(0) \end{cases} \tag{20}$$

We now consider the case of a lack of agreement between agents i.e. when no monetary transfer occurs. As a result, the buyers and sellers utility functions satisfy:

$$\begin{cases} u_{0}^{b} = (1-\gamma)W^{b}(f) + \gamma W^{b}(0) + W^{b}(e) \\ u_{0}^{s} = W^{s}(0) = 0 \end{cases} \tag{21}$$

[6] The three main international issuers of electronic money guarantee the replacement of e-cash funds lost or stolen. American Express, Mastercard and Visa offer this guarantee for their reloadable cards and state in their advertising presentation that reloadable cards are "safer than cash". When the owner registers his card, in the case of loss or theft, the individual can recover his money and "*you can't say that about cash*" (mastercard.us/prepaid-card).

The buyer's utility function without agreement u_0^b is measured by the sum of the probability $(1-\gamma)$ to enter the next market with his fiat balances if he didn't meet a thief plus the probability γ to enter the CM without fiat balances if he met a thief plus the continuity value of entering the CM with electronic money which cannot be stolen. For his part, the seller doesn't carry money and hence he runs no risk of being theft. Consequently, his utility function without agreement corresponds to his continuity value without money is $u_0^s = W^s(0)$. We can now determine the trade surplus for each agent as the difference between (20) and (21) which implies: $S_{f,e}^b = u_{f,e}^b - u_0^b$ and $S_{f,e}^s = u_{f,e}^s - u_0^s$.

From the linearity of the value function, we obtain the agent's trade surpluses with both monies:

$$\begin{cases} S_{f,e}^b = u\left[q(f,e)\right] - (1-\gamma)d_f - d_e \\ S_{f,e}^s = -c\left[q(f,e)\right] + (1-\gamma)d_f + d_e \end{cases} \quad (22)$$

The determination of the terms of trade when buyers enter the market with the two competitive means of payment results from the following program:

$$(q,d) = \max_{d_f + d_e \leq f+e} \left\{ u\left[q(f,e)\right] - (1-\gamma)d_f - d_e \right\} \quad (23)$$

$$s.t. \begin{cases} \left[u\left[q(f,e)\right] - (1-\gamma)d_f - d_e\right] = \frac{\theta}{1-\theta}\left[-c\left[q(f,e)\right] + (1-\gamma)d_f + d_e\right] & (24) \\ d_f \leq f, d_e \leq e & (25) \end{cases}$$

From (24) we obtain the following expression of the *terms of trade* (see appendix 3):

$$(1-\gamma)d_f + d_e = (1-\theta)u\left[q(f,e)\right] + \theta c\left[q(f,e)\right] \quad (26)$$

Equation (26) implies: $d_f + d_e = (1-\theta)u\left[q(f,e)\right] + \theta c\left[q(f,e)\right] + \gamma d_f$ and as in the previous section, the more the risk of theft (γ) is important the more the risk premium (γd_f) is high. However, unlike of cash, the use of electronic money is no risky and no risk premium is associated with this means of payment. Consequently, if the risk of theft increases, buyers prefer to use e-money because they obtain more goods for the same amount of money than if they have used cash. And sellers get more money in exchange for their production.

By substituting the terms of trade $(1-\gamma)d_f + d_e$ from (26) into the buyer's objective function (23) and in the respect of the constraint (25), the buyer's problem is simplified as below (see appendix 4):

$$\begin{cases} q = \arg\max_{q_f, q_e} \theta\left[u\left[q(f,e)\right] - c\left[q(f,e)\right]\right] & (27) \\ \text{subject to } \quad d_f + d_e \leq f + e & (28) \end{cases}$$

Equation (27) indicates the buyer's maximization problem. The buyer chooses the quantities consumed $q(f,e)$ with his portfolio composed with fiat and electronic monies so that he obtains with it the share (θ) of the total match surplus with both monies: $S^T_{f,e} = u[q(f,e)] - c[q(f,e)]$. If the buyer carries enough monetary balances he consumes the optimal quantity $q*$; otherwise, he purchases the quantity $q(f,e)$ which corresponds to his cash in hand. Thus:

1. If $f + e \geq d_f + d_e$, or if $(1-\gamma)f + e \geq (1-\gamma)d_f + d_e$, the buyer has sufficient monetary balances to purchase the efficient level of output and $q(f,e) = q*$.
2. Otherwise, if $f + e < d_f + d_e$, or if $(1-\gamma)f + e < (1-\gamma)d_f + d_e$, the production and consumption level of output on the DM $q(f,e)$ will satisfy: $(1-\gamma)f + e = (1-\theta)u[q(f,e)] + \theta c[q(f,e)]$, where: $q(f,e) = q[d(f,e)] < q*$.

As a result, given the amount of money held by the buyer in his portfolio comparatively to the terms of trade bargained with the seller we conclude that if:

$$\begin{cases}(1-\gamma)f + e \geq (1-\gamma)d_f + d_e \\ (1-\gamma)f + e < (1-\gamma)d_f + d_e\end{cases} \Rightarrow \begin{cases}(1-\gamma)f^* + e^* = (1-\theta)u[q^*(f,e)] + \theta c[q^*(f,e)] \\ (1-\gamma)f + e = (1-\theta)u[q(f,e)] + \theta c[q(f,e)]\end{cases} \quad (29)$$

Equation (29) defines the expected amount of balances moved between agents as a function of the output level. The term $\omega(q_{f,e}) = (1-\theta)u(q_{f,e}) + \theta c(q_{f,e})$ corresponds to the available or expected value of the money transferred from the buyer to the seller that consists with an amount of fiat money transferred with the probability $(1-\gamma)$ and an amount of electronic money transferred with the probability one. The output traded in the DM $q_{f,e}$ solves:

$$\omega(q_{f,e}) = \min\{\omega(q^*_{f,e}), (1-\gamma)f + e\} \quad (30)$$

4. Buyer's and seller's decisions

In this section, we study the buyer's portfolio choice from the maximization of his value function on the DM as well as the seller's decision to invest or not in the reading terminal allowing the reception of electronic funds.

4.1. Buyer's problem when he enters the DM with fiat and electronic monies ($z = f + e$)

Before entering the DM, the buyer decides the type and the quantity of money to hold in his portfolio. This choice results from the maximization of his DM value function. The DM buyer's value function satisfies:

$$V^b(f,e) = \sigma(1-\Lambda)\theta\left(u[q(f)] - c[q(f)]\right) + \sigma\Lambda\theta\left(u[q(f,e)] - c[q(f,e)]\right) \\ + (f+e) + W^b(0,0) \qquad (31)$$

Where: $q(f)$ is the quantity of search good which is exchanged against fiat money, $q(f,e)$ the quantity exchanged against cash and e-money and σ is the probability of a single coincidence in a meeting between two agents. The buyer receives a constant share θ of the total match surplus in all trades.

According to the equation (31), when a buyer is matched with a seller who doesn't accept e-money with the probability $\sigma(1-\Lambda)$ he receives the share θ of the total match surplus $u[q(f)] - c[q(f)]$ and can only exchange goods for fiat money. With the probability $\sigma\Lambda$, a buyer who is matched with a seller who accepts both fiat money and e-money receives the share θ of the total match surplus $u[q(f,e)] - c[q(f,e)]$ and can pay with both monies. Since fiat money is the legal tender, sellers have to accept it even if they have invested in the e-money reading terminal. The last two terms result from the linearity of the value function W^b and correspond to the value of continuing in the CM with the entire portfolio.

Given the linearity of W^b and by moving $V^b(f,e)$ in (12) of period and by substituting its value into $W^b(f,e)$ defined in (3), (see appendix 5) we obtain the buyer's objective function $\Psi(f,e)$ which determines the quantity of real monetary balances ($z = f + e$) the buyer decides to hold in his portfolio before entering the following DM in order to consume the quantity $q(f)$ or $q(f,e)$ which maximizes his expected surplus net of the cost of money holdings.

$$\Psi(f,e) = \max_{f,e\geq 0}\left\{-i(f+e) + \sigma\theta\left[(1-\Lambda)\left(u[q(f)] - c[q(f)]\right) + \Lambda\left(u[q(f,e)] - c[q(f,e)]\right)\right]\right\} \qquad (32)$$

where $i = (1-\beta)/\beta$ represents the cost of holding real balances. As a result, the first term represents the cost of carrying fiat money and electronic money into the subsequent DM. The cost of holding an additional unit of each real balances is identical for both monies and is equal to the opportunity cost of balances measured by the interest rate: i. The second term measures the buyer's share of the expected trade surplus when (1) he enters the market with fiat money only and (2) when he enters with both monies. Both surpluses depend on the fraction of sellers who invested in e-money technology.

When $i > 0$, and for each type of money the buyer shall never accumulate more monetary balances on the CM than he can spend on the DM ($z \leq z*$). He chooses his money balances ($z \geq 0$) to maximize his share (θ) of the total expected trade surplus in the DM net of the cost i of holding a portfolio z in real value terms. The quantity of goods exchanged in equilibrium corresponds to the value of the buyer's monetary balances.

Equation (32) describe the buyer's objective function $\Psi(f,e)$ which is a concave function[7], this guarantees that a solution exists. The buyer's choice of the balances to carry into the DM in steady state and hence the value of the real portfolio ($z = f + e$) is the solution to the maximization problem (32) given the terms of trade defined in (27).

From now, to simplify notations $q(f)$ is replaced by q_f and $q(f,e)$ by $q_{f,e}$.

The first order conditions associated with the problem (32) when $z = f + e > 0$ are:

$$\begin{cases} \dfrac{\partial \max}{\partial f} = -i + \sigma\theta(1-\Lambda)\left(u'(q_f) - c'(q_f)\right)\dfrac{dq_f}{df} + \sigma\theta\Lambda\left(u'(q_{f,e}) - c'(q_{f,e})\right)\dfrac{dq_{f,e}}{df} = 0 & (33) \\ \dfrac{\partial \max}{\partial e} = -i + \sigma\theta\Lambda\left(u'(q_{f,e}) - c'(q_{f,e})\right)\dfrac{dq_{f,e}}{de} = 0 & (34) \end{cases}$$

By replacing the terms $\frac{dq_f}{df}$, $\frac{dq_{f,e}}{df}$ and $\frac{dq_{f,e}}{de}$ by their expression in (33) and (34) we obtain the following first order conditions (see appendix 6):

$$\begin{cases} \dfrac{-i}{\sigma\theta} + (1-\Lambda)\left[\dfrac{(1-\gamma)\left(u'(q_f) - c'(q_f)\right)}{(1-\theta)u'(q_f) + \theta c'(q_f)}\right] + \Lambda\left[\dfrac{(1-\gamma)\left(u'(q_{f,e}) - c'(q_{f,e})\right)}{(1-\theta)u'(q_{f,e}) + \theta c'(q_{f,e})}\right] \le 0 & (35) \\ \dfrac{-i}{\sigma\theta} + \Lambda\left[\dfrac{u'(q_{f,e}) - c'(q_{f,e})}{(1-\theta)u'(q_{f,e}) + \theta c'(q_{f,e})}\right] \le 0 & (36) \end{cases}$$

Condition (35) is satisfied with an equality if $f > 0$ and condition (36) if $e > 0$. Condition (35) shows that the buyer can spend his marginal unit of fiat money in matches with sellers who accept both monies but a marginal unit of e-money can only be transferred in an match with a seller who accepts e-money. We focus on symmetric equilibria where all buyers bring the same portfolio in the DM. The next section is devoted to the issue of the seller's decision to invest in the electronic money technology. This choice makes endogenous the fraction Λ of sellers who have invested to receive electronic money as means of payment.

4.2. Seller's problem: investment choice:

At the beginning of each period during the CM period, sellers must decide if they invest or not in the e-money technology. This choice results from the following maximization program:

$$\max\left\{\sigma(1-\theta)\left(u[q(f)] - c[q(f)]\right), -I + \sigma(1-\theta)\left(u[q(f,e)] - c[q(f,e)]\right)\right\} \qquad (37)$$

[7] For all (f, e), such that: $(1-\gamma)f + e < \theta c(q^*) + (1-\theta)u(q^*)$, the objective function is strictly jointly concave. For a proof of the concavity of the buyer's objective function in the case of the choice between cash and a risk-free real asset, see Nosal and Rocheteau (2011) chapter 11, p315.

According to (37), if the seller doesn't invest in the e-money reading terminal, he can only trade goods for fiat money, the quantity exchanged is $q(f)$. In this case, he obtains the share $(1-\theta)$ of the total match surplus with fiat money. As opposite, if the seller chooses to invest in the e-money technology, he incurs the cost of investment ($I > 0$). In this case, the quantity traded is that with both monies: $q(f,e)$ and the seller obtains the share $(1-\theta)$ of the total match surplus with two monies. Consequently, from (37) the fraction (Λ) of sellers who decide to invest in the reading terminal will satisfy:

$$\Lambda \begin{cases} =1 \\ \in [0,1] \\ =0 \end{cases} \text{if} \quad -I+\sigma(1-\theta)\left(u(q_{f,e})-c(q_{f,e})\right) \begin{cases} > \\ = \\ < \end{cases} \sigma(1-\theta)\left(u(q_f)-c(q_f)\right) \tag{38}$$

A stationary symmetric equilibrium is a list $(q_f, q_{f,e}, f, e, \Lambda)$ that satisfies conditions on the terms of trade defining the quantity traded in both types of matches (18) and (30), the FOC (35), (36), and the seller's problem (37). From the previous sections, we can now study the different equilibria which can emerge from the interaction between the buyer's and the seller's maximization problems. This interaction highlights the strategic complementarities between the two sides of the market.

5. Determination of the different monetary equilibriums

In this section, we determine multiple equilibria that result from previous decisions.

5.1 Equilibrium with electronic money ($\Lambda = 1$): all sellers accept electronic money

We consider first equilibrium where all sellers have invested in the electronic money technology and hence where all *sellers accept electronic money* ($\Lambda = 1$). In this case two means of payment are available for the transactions: fiat money and electronic money. From (35) and (36) with $\Lambda = 1$, the equilibrium output traded $q^1_{f,e}$ is the solution to:

$$\begin{cases} \dfrac{i}{\sigma\theta} = \left[\dfrac{(1-\gamma)\left(u'(q^1_{f,e})-c'(q^1_{f,e})\right)}{(1-\theta)u'(q^1_{f,e})+\theta c'(q^1_{f,e})} \right] & (39) \\ \dfrac{i}{\sigma\theta} = \left[\dfrac{u'(q^1_{f,e})-c'(q^1_{f,e})}{(1-\theta)u'(q^1_{f,e})+\theta c'(q^1_{f,e})} \right] & (40) \end{cases}$$

Where the exponent "1" refers to equilibrium with $\Lambda = 1$ where all sellers accept e-money. Equations (39) and (40) state conditions for fiat money and electronic money to be valued. Agents are indifferent between the two means of payment when those equations are equal that is if $(1-\gamma) = 1$ i.e. if $\gamma = 0$. Hence, when there is no risk of theft, the buyer is indifferent between holding cash or electronic money. An additional unit of one or of the other money involves the same marginal increase of his match surplus.

In electronic-money equilibrium (when $\Lambda = 1$) electronic money is universally accepted and is a legal tender. As a result, electronic money equilibrium with fiat money is impossible since it is not rational for a buyer to hold cash when e-money, which is safer, is always accepted. When $\Lambda = 1$, the only equilibrium is a "**pure electronic-money equilibrium**" which corresponds to a situation where the buyer's portfolio consists only with e-money: $z^1 = e^1$, $f^1 = 0$. Buyers don't hold cash because they would obtain with their fiat money a lower quantity of goods ($q_f < q_e$) due to the risk premium associated with the payment by cash. In e-money equilibrium since fiat money is no longer used nor valued, the risk of theft associated with this means of payment has disappeared ($\gamma^1 = 0$) and the goods are only traded against e-money ($q^1_{f,e} = q^1_e$) that involves a higher quantity traded than if cash was used. However, the quantity of goods traded on the market is lower than the optimal level ($q^1_e < q*$) except at the Friedman rule[8]. Hence, when $\Lambda = 1$ we obtain the following conclusions:

$$\Lambda = 1 \Rightarrow \begin{cases} f^1 = 0 \\ e^1 > 0 \\ q^1_{f,e} = q^1_e < q* \end{cases} \tag{41}$$

From the terms of trade defined in (27) the value of the monetary transfer from the buyer to the seller is: $e^1 = \omega(q^1_e)$. A "**pure electronic money equilibrium**" (without fiat money) when all sellers accept electronic-money ($\Lambda = 1$) and for a positive risk of theft[9] is defined as follows:

$$\text{if } \Lambda=1, \forall \gamma > 0 \;\Rightarrow\; \begin{cases} z^1 = e^1 \\ e^1 = \omega(q^1_e) \\ q^1_e < q* \end{cases} \tag{42}$$

We now check whether it is optimal for a seller to invest in the e-money reading terminal in electronic money equilibrium. From (38), when $\Lambda = 1$, it is optimal for a seller to invest in electronic technology if:

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[8] If the interest rate is positive ($i > 0$), there is a cost of holding money whatever the type of money the buyer has in his portfolio. Indeed, the use of money is associated with prior credit balances in a bank account. These credit balances are essential for withdrawing cash, or for loading a smart card, and do not involve interest earnings, creating an opportunity cost. The optimal quantity ($q*$) may be traded only at the Friedman rule, when the interest rate is zero, that is from (35) and (36) $\frac{i}{\sigma\theta} = \Lambda\left[\frac{u'(q_{f,e}) - c'(q_{f,e})}{(1-\theta)u'(q_{f,e}) + \theta c'(q_{f,e})}\right]$, $i = 0$ requires: $u'(q_{f,e}) = c'(q_{f,e})$ or $u'(q_{f,e})/c'(q_{f,e}) = 1$. The quantity q_e approaches $q*$ as i tends to zero, meaning output is inefficiently low when $i > 0$. When $i \to 0$, money becomes less costly to hold.

[9] Without risk of theft ($\gamma = 0$) it is not rational for sellers to invest in the e-money technology which is costly. An equilibrium with $\Lambda = 1$ and $\gamma = 0$ is impossible.

$$I < G^1_{f,e} \equiv \sigma(1-\theta)\left\{\left(u(q^1_{f,e}) - c(q^1_{f,e})\right) - \left(u(q^1_f) - c(q^1_f)\right)\right\} \tag{43}$$

where $G^1_{f,e}$ is the net gain obtained by a seller when he accepts both monies (fiat and electronic monies) instead of cash only in an equilibrium where all others sellers accept electronic money ($\Lambda = 1$). According to (43) if the expected increase in the total match surplus associated with the acceptation of both monies instead of cash only is higher than the flow cost of investment ($I < G^1_{f,e}$), seller decides to invest in the new means of payment. In (43), the quantity q^1_f represents the DM output if a seller chooses not to invest since all other sellers invested to receive e-money and have replaced the use of cash and then trade the quantity q^1_e. The quantity q^1_f is the solution to $(1-\gamma) f^1 = \omega(q^1_f)$ and since $f^1 = 0$ when $\Lambda = 1$, the quantity traded against cash is zero: $q^1_f = 0$. With $q^1_{f,e} = q^1_e$ and $q^1_f = 0$ in (43) we obtain $I < G^1_{f,e} \equiv \sigma(1-\theta)\left\{\left(u(q^1_e) - c(q^1_e)\right) - \left(u(0) - c(0)\right)\right\}$. Consequently, it is optimal for a seller to invest if: $I < G^1_{f,e}$ that is if the investment cost is lower than the net gain resulting from the adoption of e-money as means of payment.

$$\text{if } \Lambda = 1, \forall \gamma > 0 \;\Rightarrow \begin{cases} q^1_{f,e} = q^1_e \\ G^1_{f,e} > 0 \\ \text{invest if: } I < G^1_{f,e} \end{cases} \tag{44}$$

If the amount of investment is less than the seller's share of the electronic money surplus: $I < \sigma(1-\theta)\left(u(q^1_e) - c(q^1_e)\right)$: it is a strictly dominant strategy for *a* seller to invest in e-money technology when $\Lambda = 1$ (when all *other sellers* have invested). In this situation, there exists an e-money equilibrium where all sellers accept e-money if the cost of e-money technology is sufficiently low, lower than $G^1_{f,e}$. In the opposite case, it is rational for a seller not to invest and to continue to be paid with cash as long as it will be valued by buyers. Indeed, buyers will continue to use fiat money if the risk of meeting a thief is not too high as we shall see in the next section.

5.2 Equilibrium with fiat money ($\Lambda = 0$): all sellers refuse e-money and accept only cash

We consider now equilibrium where all sellers have decided not to invest in the e-money reading terminal ($\Lambda = 0$). Consequently, they can only trade goods for cash since fiat money is the only means of payment. The equilibrium output traded against fiat money q^0_f corresponds to that one of a pure fiat money economy. From (35) with $\Lambda = 0$, the output q^0_f is the solution to:

$$\frac{i}{\sigma\theta} = \left[\frac{(1-\gamma)\left(u'(q^0_f) - c'(q^0_f)\right)}{(1-\theta)u'(q^0_f) + \theta c'(q^0_f)}\right] \tag{45}$$

where the exponent "0" refers to an equilibrium with $\Lambda = 0$, where the fraction Λ of sellers who accept e-money is zero. After rearrangements, equation (45) implies:

$$\frac{u'(q_f^0)}{c'(q_f^0)} = \frac{[i+\sigma(1-\gamma)]\theta}{[i+\sigma(1-\gamma)]\theta - i} \quad (46)$$

Equation (46) gives the threshold value for γ from which no more quantity is traded against fiat money: $q_f^0 = 0$. Indeed, the right side of (46) is increasing in γ which takes away the DM output q_f^0 from its optimal value. An increase in the probability to meet a thief (γ) decreases the quantity of the search good produced and consumed in exchange for a given amount of cash. As a result, the buyer's choice of real cash balances is decreasing in γ. If it is more likely to meet a thief, it is optimal to hold fewer fiat money balances. There is a critical value γ_c for γ above which buyers no longer hold cash[10] and hence from which no good is exchanged for cash ($q_f^0 = 0$). This is the case when the denominator of equation (46) is equal to zero or when $\gamma_c = [\sigma\theta - (1-\theta)i]/\sigma\theta$[11]. As a result, if $\gamma \geq \gamma_c$ in equilibrium where all sellers refuse e-money $\Lambda = 0$ fiat money is no longer valued[12], agents would rationally prefer an autarky economy than a monetary economy as long as there is no substitute for cash.

Consequently, the study of the competition between cash and e-money requires taking into account the strategic complementarities between buyers and sellers for the choice of the new means of payment. Indeed, the buyers' decision to hold e-money depends on their anticipations about the fraction of sellers (Λ) who accept it. When no seller accepts e-money ($\Lambda = 0$) and even if the risk of theft is high, electronic money cannot be used as a medium of exchange. As a result no buyers shall hold electronic money because the probability to meet a seller who accepts it is zero ($\sigma\Lambda = 0$). Even if they obtain the electronic device for free, no buyer will load his support of storage with electronic monetary units because electronic funds constitute useless and worthless funds when no sellers can receive them as means of payment.

As a result, in an "*equilibrium where no seller accepts e-money*" ($\Lambda = 0$), the buyer's portfolio may only contain fiat money, the amount of electronic units is zero ($z^0 = f^0 + e^0, e^0 = 0$) and goods

[10] The risk of theft is so important that agents prefer autarky to an unsafe monetary economy.

[11] By equating the denominator of (46) to zero, we determine the condition for which $u'(q_f^0) = 0$, i.e. the value of γ from which a supplementary unit of fiat money involves no increase in the marginal utility. The risk of theft increases the cost of holding cash, usually measured by the interest rate i. Above some critical value ($\gamma \geq \gamma_c$), an additional unit of fiat money doesn't increase the marginal gain of holding cash because the advantage of holding cash (the possibility to trade goods) does not compensate its holding cost. Consequently, when $\gamma \geq \gamma_c$, buyers decide not to hold cash and no goods are exchanged against money since there is no longer means of payment ($q_f^0 = 0$). Moreover, $\gamma_c > 0$ is associated with a low interest rate: $i < \sigma\theta/(1-\theta)$.

[12] Fiat money is no longer valued because a higher risk of theft increases too much the risk premium and hence decreases too much the purchasing power of fiat money.

are only traded for fiat money. Notice that the quantity exchanged is lower than the optimal quantity ($q_f^0 < q*$) since the interest rate is positive[13].

$$\Lambda = 0 \Rightarrow \begin{cases} e^0 = 0 \\ f^0 \geq 0 \\ q_{f,e}^0 = q_f^0 \\ q_f^0 < q* \end{cases} \quad (47)$$

We shall distinguish two cases depending on the risk of theft level which determine whether there is or not a role for fiat money as media of exchange. From (47), those situations are summarized as follows:

$$when\ \Lambda{=}0 \Rightarrow e^0 = 0\ and \begin{cases} if\ \ \gamma < \gamma_c \Rightarrow f^0 > 0\ and\ q_f^0 = q_{f,e}^0 > 0 \\ if\ \ \gamma \geq \gamma_c \Rightarrow f^0 = 0\ and\ q_f^0 = q_{f,e}^0 = 0 \end{cases} \quad (48)$$

<u>CASE 1</u>: Pure fiat money equilibrium

In equilibrium where no sellers accept electronic money ($\Lambda = 0$) and when economic environment is safe ($\gamma < \gamma_c$) there is a role for fiat money. Hence, pure fiat money equilibrium is defined by:

$$\text{If } \gamma < \gamma_c \quad \Rightarrow \begin{cases} z^0 = f^0 > 0 \\ (1-\gamma) f^0 = \omega(q_f^0) \\ 0 < q_f^0 < q* \end{cases} \quad (49)$$

Condition (38) implies that it is optimal for sellers to continue to be paid with cash only and not to invest in the e-money technology if:

$$I > G_{f,e}^0 \equiv \sigma(1-\theta)\left\{\left[u(q_{f,e}^0) - c(q_{f,e}^0)\right] - \left[u(q_f^0) - c(q_f^0)\right]\right\} \quad (50)$$

where $G_{f,e}^0$ is the net gain for a seller who accepts both monies (fiat and electronic monies) instead of cash only in equilibrium where all other sellers refuse electronic money and accept only cash ($\Lambda = 0$).

[13] As in the previous section, the optimal quantity of goods may be traded in the market only at the Friedman rule which corresponds to an interest rate equal to zero ($i = 0$). In our model, we don't take into account the role of monetary policy since the opportunity cost of both monies is identical. Hence, without monetary policy, that is with a constant quantity of money ($\gamma = 1$) the interest rate $i = (\gamma - \beta)/\beta$ is equal to $i = (1-\beta)/\beta$ and is strictly positive ($i > 0$), which corresponds to the rate at which agents depreciate the future. The interest rate is equal to zero only at the Friedman rule, when $\gamma = \beta$. When the cost of holding real balances is zero, agents can trade the optimal quantity of goods.

According to (50) if the expected increase in the total match surplus associated with the acceptation of both monies instead of cash only is lower than the cost of investment ($G^0_{f,e} < I$), seller rationally decides not to invest in the new means of payment. In (50), the quantity $q^0_{f,e}$ corresponds to the DM output if a seller has invested in the e-money technology while all other sellers didn't invest and is given implicitly by (27) with $e^0 = 0$ and $(1-\gamma) f + e = (1-\gamma) f^0$. The quantity q^0_f corresponds to the DM output when all sellers can only receive cash as means of payment. The net gain depends on the realization of two DM outputs ($q^0_f, q^0_{f,e}$) that determine different trade surplus in each case. However, because electronic money is not valued in an equilibrium with $\Lambda = 0$ ($e^0 = 0$), no exchange takes place against e-money and $q^0_f = q^0_{f,e}$, as a result both surpluses are equal and there is no gain. From (44) we conclude that it is a strictly dominant strategy for sellers no to invest even if the amount of investment I is very low because when electronic money is not valued ($e^0 = 0$) they obtain no gain by accepting e-money $G^0_{f,e} = 0$. As long as the investment is costly (not accessible for free) $I > G^0_{f,e} = 0$; no sellers shall incur the cost of the e-money technology.

$$when\ \gamma < \gamma_c \ \Rightarrow \begin{cases} q^0_{f,e} = q^0_f \\ G^0_{f,e} = 0 \\ \text{no investment if: } I > G^0_{f,e} = 0 \end{cases} \tag{51}$$

From (51) we conclude that in a safe economic environment ($\gamma < \gamma_c$) and when no seller can receive e-money , there exists an equilibrium where agents trade a amount of goods lower than the optimal quantity in all matches and fiat money is the only means of payment. If the risk of theft is very rare to the point that it can be considered inexistent, this equilibrium is socially efficient since sellers don't need to realize investment. If fiat money is a safe means of payment; the society saves technological costs giving information about the probability of the monetary transfer between agents and offsetting the risk premium. However, when the risk of theft really exists even being less than the critical value ($\gamma < \gamma_c$), there is a need for electronic money because the level of output is inefficiently low. Nevertheless (51) shows that no seller decides rationally to invest in the new means of payment if the investment is costly: $I > 0$ since no buyer holds e-money. If the e-money technology is not free, no seller will decide rationally to accept e-money ($\Lambda = 0$). Fiat money is the only means of payment and this equilibrium is socially inefficient although it is rational.

<u>CASE 2</u>: Pure autarky equilibrium requiring a social planner intervention

In equilibrium where no sellers accept electronic money ($\Lambda = 0$) and when economic environment is unsafe ($\gamma \geq \gamma_c$) there is no role for fiat money and pure autarky equilibrium is defined by:

$$\text{If } \gamma \geq \gamma_c \quad \Rightarrow \begin{cases} z^0 = f^0 = 0 \\ (1-\gamma) f^0 = \omega(q_f^0) = 0 \\ q_f^0 = 0 \end{cases} \tag{52}$$

We consider now the seller's decision. Conditions (38) and (44) with $q_{f,e}^0 = q_f^0 = 0$ imply that it is optimal for a seller to continue to be paid only with cash if:

$$I > G_{f,e}^0 \equiv \sigma(1-\theta)\left\{\left[u(q_f^0) - c(q_f^0)\right] - \left[u(q_f^0) - c(q_f^0)\right]\right\} \tag{53}$$

As in the first case, because there is no gain by accepting the new money ($G_{f,e}^0 \equiv 0$) if investment is costly: $I > 0$ it is not rational for a seller to invest in e-money technology as long as: $\Lambda = 0$. However, unlike the previous case, when $\gamma \geq \gamma_c$: there are no longer any fiat monetary exchanges.

$$\textit{when } \gamma \geq \gamma_c \ \Rightarrow \begin{cases} q_f^0 = q_{f,e}^0 = 0 \\ G_{f,e}^0 = 0 \\ \text{no investment if: } I > G_{f,e}^0 = 0 \end{cases} \tag{54}$$

From (54) we conclude there is no longer any equilibrium with media of exchange. When $\gamma \geq \gamma_c$, fiat money is no valued. As a result, in an unsafe economic environment, there is no media of exchange as long as e-money hasn't replaced the cash. Two solutions can emerge from this situation where fiat money is no longer valued and doesn't have any substitute.

First solution: Autarky economy

Because agents are specialized, *pure barter equilibrium* is impossible. As a result, without media of exchange, the only possibility is to return to autarky that existed before barter economy and monetary exchange economy. Agents must become self-sufficient and consume only what they produce. Without money, the labor division and the specialization which resulted from the monetary exchange economy should disappear.

Second solution: Social planer

Since there is no longer any means of payment, sellers cannot sell their production and they should all make the choice to invest in the money technology to reach a new equilibrium with a media of exchange, where all sellers accept e-money ($\Lambda = 1$). However, only a social planer could organize the passage from equilibrium to another. When $\Lambda = 0$ and $\gamma \geq \gamma_c$ equilibrium corresponds to an autarky economy and (54) indicates there is no gain from accepting e-money $G_{f,e}^0 = 0$[14]

[14] (50): $I > G_{f,e}^0 \equiv \sigma(1-\theta)\left\{\left[u(q_{f,e}^0) - c(q_{f,e}^0)\right] - \left[u(q_f^0) - c(q_f^0)\right]\right\}$ with $q_f^0 = q_{f,e}^0$ we obtain: $I > G_{f,e}^0 = 0$.

because no buyer hold e-money. Then sellers have no incentives to invest in the e-money reading terminal because they believe that no buyer hold e-money. Symmetrically, buyers will not hold e-money since they believe that no seller will accept e-money. In this situation it is optimal for seller not to invest and for buyer not bring e-money into the market. Hence the decision of each agent depends on the decision of the other. This shows the strategic complementarities between them.

However, because autarky is less efficient than an exchange economy, there is a need for a substitute for fiat money, electronic money responds to such a need. As a result, a social planner can intervene to subsidize e-money technology. If so, e-money becomes a legal tender valued by buyers and such as : $e^1 > 0$. In this case: $\Lambda = 1 \; and \; G^1_{f,e} > 0$. Consequently, buyers may decide to load their cash-cards ($e^1 > 0$) if they expect that all sellers can accept e-money. When a social planner subsidizes the e-money technology to permanently replace the cash it allows the realization of pure e-money equilibrium.

$$\text{If} \quad \gamma \geq \gamma_c \quad \text{and } \Lambda = 1 \;\Rightarrow \begin{cases} f^1 = 0 \\ e^1 > 0 \\ e^1 = \omega(q^1_e) \\ q^1_e < q^* \end{cases} \tag{55}$$

5.3 Mixed money equilibrium: coexistence of sellers refusing e-money and sellers accepting e-money ($\Lambda \in [0,1]$)

In "mixed equilibrium", the fraction of sellers who accept electronic money is $\Lambda \in [0,1]$ which means that some sellers accept e-money and some others don't accept it. Even if electronic money is not a universal medium of exchange it may be valued by buyers because they anticipate that some sellers accept it as means of payment[15]. As a result, buyers may decide to enter the market with electronic units ($e^{0,1} \geq 0$) in addition or not with cash because they can obtain with e-money a higher quantity of goods than with cash which requires the payment of a risk premium. The portfolio with both monies and the DM output traded are described as follows:

$$\Lambda \in [0,1] \Rightarrow \begin{cases} e^{0,1} \geq 0 \\ f^{0,1} \geq 0 \\ (1-\gamma) f^{0,1} + e^{0,1} = \omega(q^{0,1}_{f,e}) \\ q^0_f \leq q^{0,1}_{f,e} \leq q^1_e \\ q^{0,1}_{f,e} < q^* \end{cases} \tag{56}$$

[15] The parameter Λ denotes the fraction of sellers who invested in the e-money technology and it is common knowledge in the match whether the seller invested in this technology or not. This parameter also indicates the probability that e-money is accepted in payment by a random seller. As in the previous section, to determine the buyer's portfolio, we first assume that this fraction is exogenous. Then, we make it endogenous by analyzing whether it is optimal for sellers to invest.

where the exponent "0,1" refers to an equilibrium with $\Lambda \in [0,1]$, where a fraction Λ of sellers accepts e-money and another fraction doesn't accept it. Condition (38) implies that it is optimal for a seller to invest in the e-money technology and then to trade the quantity $q_{f,e}^{0,1}$ instead of the quantity $q_f^{0,1}$ if:

$$I = G_{f,e}^{0,1} \equiv \sigma(1-\theta)\left\{\left[u(q_{f,e}^{0,1}) - c(q_{f,e}^{0,1})\right] - \left[u(q_f^{0,1}) - c(q_f^{0,1})\right]\right\} \qquad (57)$$

where $G_{f,e}^{0,1}$ is the net gain for a seller who accepts both monies instead of cash only in equilibrium where some sellers refuse electronic money and some of them accept it ($\Lambda \in [0,1]$). The quantity $q_f^{0,1}$ represents the DM output if a seller chooses not to invest in the e-money technology in equilibrium where the fraction of sellers who have invested can trade the quantity $q_{f,e}^{0,1}$

If there exists a gain $G_{f,e}^{0,1}$ when $\Lambda \in [0,1]$ such as $G_{f,e}^{0} < G_{f,e}^{0,1} < G_{f,e}^{1}$ there will exist a mixed equilibrium, an equilibrium where some sellers invest in the e-money technology and one where some sellers do not for any $I \in \left[G_{f,e}^{0}, G_{f,e}^{1}\right]$ since conditions (43), (50) and (57) can be simultaneously satisfied. As a result, we must demonstrate the possibility of this equilibrium or that: $0 < G_{f,e}^{0,1} < G_{f,e}^{1}$ i.e. the possible coexistence of the fiat money and the e-money equilibriums.

Let $l = (1-\gamma)f + e$ be the maximum of monetary balances that the buyer can transfer to the seller in a match. From (19) when $\Lambda = 0$, $l^0 = (1-\gamma)f^0$; from (31) when $\Lambda = 1$, $l^1 = e^1$ and from (57) when $\Lambda \in [0,1]$ we obtain: $l^{0,1} = (1-\gamma)f^{0,1} + e^{0,1}$. Since buyers enter the market with the same portfolio and since $\gamma \in [0,1]$ we have: $l^0 < l^{0,1} < l^1$. Those values are reported on the horizontal axis in the figure 2. Moreover, the total match surplus: $S(l) \equiv u[q(l)] - c[q(l)]$ as a function of the buyer's monetary balances l is concave and strictly concave if $l < \theta c(q^*) + (1-\theta)u(q^*) = \omega(q^*)$ [16]. This condition is always respected in our model since the positive interest rate prevents agents to trade the optimal quantity q^*.

The concavity of the surplus function ensures that the trade surplus increases less than proportionally than the increase in the monetary balances available in a match. Moving along the x-axis from the origin the buyer holds less fiat money and more e-money in his portfolio, this evolution of the buyer's portfolio composition increases the monetary balances available for a match (see figure 2). To demonstrate that this evolution involves an increase in the seller's gain ($0 < G_{f,e}^{0,1} < G_{f,e}^{1}$) we use the second concavity theorem [17] (see appendix 7).

We consider at first pure e-money equilibrium. From (43) and since $(1-\gamma)f^1 < (1-\gamma)f^1 + e^1$ we obtain:

$$\sigma(1-\theta)S'(e^1)(e^1) \le \sigma(1-\theta)\left\{\left(u(q_{f,e}^{1}) - c(q_{f,e}^{1})\right) - \left(u(q_f^{1}) - c(q_f^{1})\right)\right\} = G_{f,e}^{1} \qquad (58)$$

[16] For a proof of the concavity of the total match surplus function, see Nosal and Rocheteau (2011) chapter 11, p315.

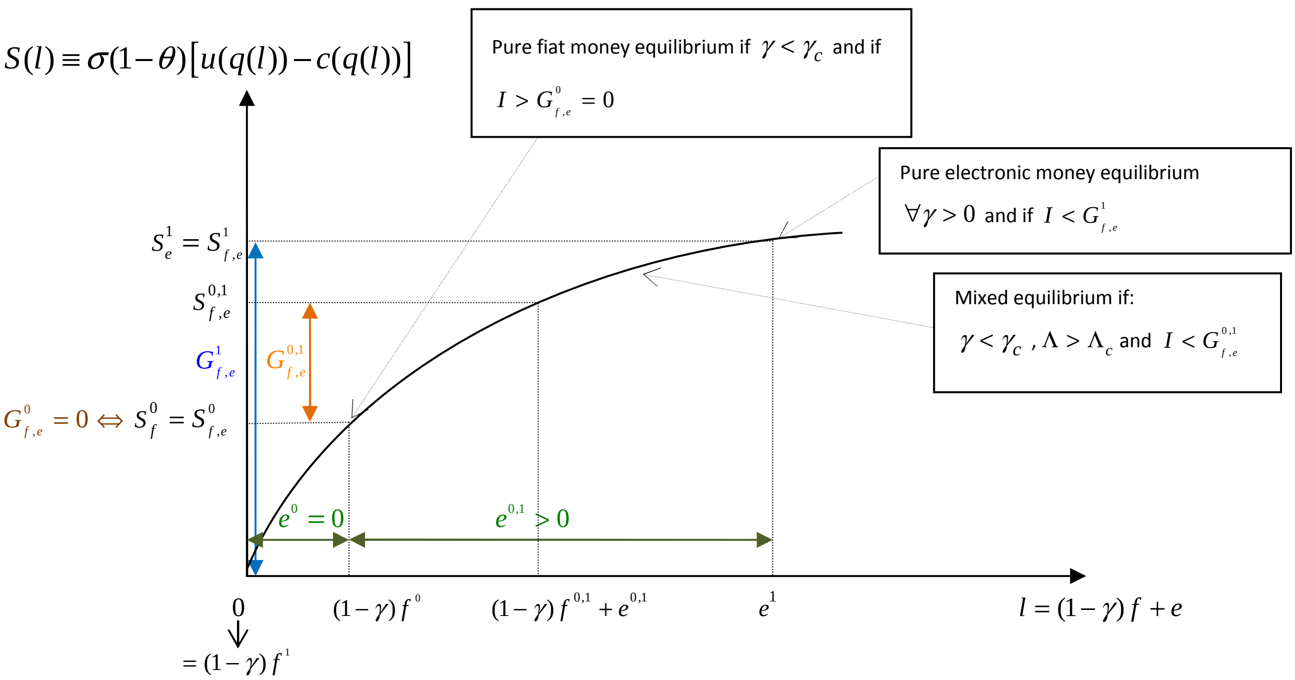
[17] For a presentation of concavity theorems, see Simon and Blume (2012), p510.

We consider now mixed equilibrium. From (57) and since $(1-\gamma)f^{0,1} < (1-\gamma)f^{0,1} + e^{0,1}$ we obtain:

$$G_{f,e}^{0,1} = \sigma(1-\theta)\left\{\left[u(q_{f,e}^{0,1}) - c(q_{f,e}^{0,1})\right] - \left[u(q_f^{0,1}) - c(q_f^{0,1})\right]\right\} \leq \sigma(1-\theta)S'\left((1-\gamma)f^{0,1}\right)(e^{0,1}) \qquad (59)$$

In figure 2, since $(1-\gamma)f^{0,1} < e^1$ and $e^{0,1} < e^1$ hence: $\sigma(1-\theta)S'\left((1-\gamma)f^{0,1}\right)(e^{0,1}) < \sigma(1-\theta)S'(e^1)(e^1)$, then conditions (58) and (59) imply that : $G_{f,e}^{0,1} < G_{f,e}^1$. Sellers may obtain a positive gain if they accept e-money as medium of exchange when some of them accept e-money since buyers will hold electronic money. Consequently, sellers will rationally decide to invest in the e-money technology if the investment cost doesn't exceed or is just equal to this gain: $I < G_{f,e}^{0,1}$.

Figure 2: Multiple equilibriums for different buyer's portfolio composition



As a result, an e-money equilibrium can coexist with a fiat money equilibrium when $\Lambda \in [0,1]$ and this result was quite intuitive. Indeed, the use of electronic money as medium of exchange is more advantageous than cash. The payment with e-money provides a greater quantity of goods produced and consumed since there is no risk premium to pay in addition with the selling price. If buyers expect that e-money may be accepted as means of payment, they will load electronic units on their support of storage and will enter the market with electronic funds. They will also hold cash in addition with e-money as long as electronic money will not be universally accepted by all sellers, as long as $\Lambda \neq 1$. However, the existence of this mixed equilibrium requires that fiat money be valued i.e. that $\gamma < \gamma_c$.

Moreover, it is interesting to determine the critical value Λ_c of $\Lambda \in [0,1]$ from which buyers decide to enter the market with e-money. Equation (36) gives the first order condition so that holding an additional unit of e-money increases the buyer's expected surplus. Hence, from (36) we can determine the threshold value from which an increase of one unit of e-money increases the buyer's marginal utility. This allows us to determine the role of the sellers' e-money adoption on the buyer's decision to hold e-money. From (36) the equilibrium output traded against fiat and e-money money is $q_{f,e}^{0,1}$ when $\Lambda \in [0,1]$ is solution to:

$$\frac{i}{\sigma\theta} = \Lambda\left[\frac{u'(q_{f,e}^{0,1}) - c'(q_{f,e}^{0,1})}{(1-\theta)u'(q_{f,e}^{0,1}) + \theta c'(q_{f,e}^{0,1})}\right] \tag{60}$$

After rearrangements, equation (60) implies:

$$\frac{u'(q_{f,e}^{0,1})}{c'(q_{f,e}^{0,1})} = \frac{[i+\sigma\Lambda]\theta}{[i+\sigma\Lambda]\theta - i} \tag{61}$$

Equation (61) gives the threshold value for Λ below which buyers don't hold e-money. Indeed, the right side of (61) is decreasing in Λ. An increase in the probability to meet a seller who accept e-money (Λ) increases the quantity of the search good produced and consumed in exchange for e-money. As a result, the buyer's choice of e-money balances is increasing in Λ. Hence, there is a critical value Λ_c for Λ below which buyers refuse to load their smart card with useless electronic money and decide to hold only cash[18]. This is the case when the denominator of equation (61) is equal to zero[19] or when:

$$\Lambda_c = \frac{i}{\sigma} \times \frac{(1-\theta)}{\theta} \tag{62}$$

Equation (62) gives the critical value Λ_c for the sellers' e-money adoption from which buyers decide to enter the market with e-money. The first term i/σ measures the cost of holding real balances as the product of the rate at which agents depreciate the future (i) and the average numbers of periods it takes to get matched (σ). The second term $(1-\theta)/\theta$ is the ratio of agent's bargaining power and it is increasing in the buyer's bargaining power (θ). The more the buyers' bargaining power is high, the more buyers require a high level of sellers' e-money adoption to enter the market with e-money in addition of cash.

Hence, buyers decide to hold e-money only if $\Lambda > \Lambda_c$, if not they continue to hold cash only. Moreover, $\Lambda_c > 0$, is associated with a positive seller's share of the total match surplus ($(1-\theta) > 0$): $\Lambda_c = i(1-\theta)/\sigma\theta > 0 \Rightarrow (1-\theta) > 0$. Indeed, if seller receives no share of the total

[18] The level of seller's e-money adoption is so low that the probability to meet a seller who accepts e-money is not sufficiently large in order buyers decide to choose the new means of payment. In this situation, agents prefer continue to use cash even if there is a risk of theft.

[19] By equating the denominator of (61) to zero, we determine the value of Λ below which a supplementary unit of electronic money involves no increase in the marginal consumption utility. When the probability of meeting a seller who accepts e-money $\Lambda \geq \Lambda_c$ is higher than its critical value buyers choose to hold e-money, below they continue to hold only cash.

match surplus he won't have incentive to support the cost of the e-money technology. This justifies our choice of the proportional bargaining solution to determine the terms of trade between agents.

Finally, we showed the existence of mixed equilibrium when $\Lambda \in [0,1]$ if both monies are valued by the buyers. This situation requires that two conditions are gathered. Fiat money has to be valued which requires a risk of theft lower than its threshold value ($\gamma < \gamma_c$) and e-money has also to be valued and that is the case when the level of the sellers' e-money adoption is sufficiently high, higher than its critical value ($\Lambda > \Lambda_c$).

6. Conclusion

In order to better understand why the launching of a new type of payment instrument may succeed or fail, it seems important to emphasize the crucial role of buyers and sellers decisions in terms of portfolio to hold and investment to incur. In our model, we investigate the policy of replacing fiat money with e-money by considering the risk of theft for using cash, absent with the use of e-money, which is nevertheless costly to manage. We determine multiple equilibria as a function of two main variables: the sellers' e-money adoption level (Λ), and the level of safety of the monetary instrument used (γ).

When all sellers possess the e-money technology ($\Lambda = 1$), fiat money is no longer valued and goods are only exchanged for e-money. In this situation, it is rational for sellers to invest in this technology if the investment cost is lower than the expected increase in the total match surplus involved by accepting the new means of payment. The only equilibrium is a *pure electronic money equilibrium*. At the opposite, when no sellers accept e-money ($\Lambda = 0$), e-money is not valued, and fiat money may or may not be valued, depending on the risk of theft of cash. In a safe economic environment, fiat money is valued and there is no need for another means of payment. However, when the risk of theft (γ) is higher than some critical value, fiat money becomes an unsafe medium of exchange, and is no longer valued by buyers who prefer autarky. The first case corresponds to a *pure fiat money equilibrium*, and the second one to a pure *autarky equilibrium*. As a result, for extreme values of the parameter Λ ($\Lambda = 0$ and $\Lambda = 1$) there is no possible coexistence of both monies; the buyers' portfolio will contain either fiat money or electronic money.

We also demonstrate the possibility of a mixed equilibrium where $\Lambda \in [0,1]$. We determine a threshold value below which buyers have no incentive to enter the market with e-money even if the measure of sellers who accept it is different than zero, and as long as this measure is below the agent's relative share of the holding cost of money: $\Lambda < \Lambda_c = i(1-\theta)/\sigma\theta$. If the probability to meet a seller who accepts e-money is higher that this cost, buyers will choose to enter the market with e-money. This result comes from the fact that there are strategic complementarities between the two sides of the market for the adoption of a new means of payment. Therefore, if the aim of the policymaker is to replace cash with e-money, it should intervene to give for free the technology to both sides of the market. Indeed, we have showed that starting from equilibrium where no one accepts e-money, we cannot obtain an equilibrium where e-money is used without giving for free the technology.

In conclusion, multiple equilibria arise from the strategic complementarities between the buyer's portfolio choice and the seller's decision to invest in the e-money technology. We can explain why the electronic money adoption is so different around the world, well accepted in Asia or Africa, less in Europe. Indeed, consider for example different economies with different levels of risk of theft. In a safe environment, agents may continue to use cash even though the government tries to replace it with e-money. In an unsafe economy, subsidizing the adoption of e-money could trigger the adoption of e-money by making its use better than cash.

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Appendix

In what follows we write q_f for $q(f)$ and $q_{f,e}$ for $q(f,e)$

Appendix 1

$$\left[u(q_f)-(1-\gamma)d_f\right]=\frac{\theta}{1-\theta}\left[-c(q_f)+(1-\gamma)d_f\right]$$
$$(1-\theta)u(q_f)+\theta c(q_f)=(1-\theta)(1-\gamma)d_f+\theta(1-\gamma)d_f$$
$$d_f=\frac{(1-\theta)u(q_f)+\theta c(q_f)}{(1-\gamma)}$$

Appendix 2

The expression of the total match surplus that the buyer maximizes when he enters the market with fiat money results from the following program:

$$(q,d)=\max_{d_f\leq f}\left\{u(q_f)-(1-\gamma)d_f\right\}$$

With : $d_f=\dfrac{(1-\theta)u(q_f)+\theta c(q_f)}{(1-\gamma)}$

$$(q,d)=\max_{d_f\leq f}\left\{u(q_f)-\frac{(1-\gamma)\left[(1-\theta)u(q_f)+\theta c(q_f)\right]}{(1-\gamma)}\right\}$$
$$(q,d)=\max_{d_f\leq f}\left\{u(q_f)-\left((1-\theta)u(q_f)+\theta c(q_f)\right)\right\}$$
$$(q,d)=\max_{d_f\leq f}\left\{\theta\left[u(q_f)-c(q_f)\right]\right\}$$

Appendix 3

$$d_f=\frac{(1-\theta)u(q_{f,e})+\theta c(q_{f,e})-d_e}{(1-\gamma)}$$
$$d_f+d_e=\frac{(1-\theta)u(q_{f,e})+\theta c(q_{f,e})-d_e}{(1-\gamma)}+d_e$$
$$d_f+d_e=\frac{(1-\theta)u(q_{f,e})+\theta c(q_{f,e})-d_e+(1-\gamma)d_e}{(1-\gamma)}$$
$$(1-\gamma)\left[d_f+d_e\right]+\gamma d_e=(1-\theta)u(q_{f,e})+\theta c(q_{f,e})$$
$$(1-\gamma)d_f+d_e=(1-\theta)u(q_{f,e})+\theta c(q_{f,e})$$

Appendix 4

The m**aximization program implies:**

$$(q,d)=\max_{d_f+d_e\leq f+e}\left\{u\left[(q_f,q_e)\right]-(1-\gamma)d_f-d_e\right\}$$

With : $(1-\gamma)d_f+d_e=(1-\theta)u(q_{f,e})+\theta c(q_{f,e})$

$$(q,d)=\max_{d_f+d_e\leq f+e}\left\{u(q_{f,e})-\left[(1-\theta)u(q_{f,e})+\theta c(q_{f,e})\right]\right\}$$

$$(q,d)=\max_{d_f+d_e\leq f+e}\left\{\theta\left[u(q_{f,e})-c(q_{f,e})\right]\right\}$$

Appendix 5

By moving $V^b(f,e)$ of period and by substituting its value in $W^b(f,e)$, we obtain the centralized market (CM) value function for a portfolio z whatever its composition ($z=f$ or $z=f+e$): $W^b(z)=z+\max_{z'\geq 0}\left[-z'+\beta V^b(z)\right]$

Then, by replacing the DM value function by its definition in (32), we can determine the quantity of monetary balances ($z=f+e$) to bring into the market given the probability Λ for the buyer to meet an "*e-money seller*". We obtain:

$$W^b(z)=(f+e)+\max_{z'\geq 0}\left\{-(f'+e')+\beta\begin{Bmatrix}\sigma(1-\Lambda)\theta\left(u\left[q(f)\right]-c\left[q(f)\right]\right)\\+\sigma\Lambda\theta\left(u\left[q(f,e)\right]-c\left[q(f,e)\right]\right)+(f'+e')+W(0,0)\end{Bmatrix}\right\}$$

$$W_t^b(z)=(f+e)+\max_{f',e'\geq 0}\begin{Bmatrix}-(f'+e')+\beta\left[(f'+e')+W(0,0)\right]\\+\beta\left[\sigma\theta(1-\Lambda)\left(u\left[q(f)\right]-c\left[q(f)\right]\right)+\sigma\theta\Lambda\left(u\left[q(f,e)\right]-c\left[q(f,e)\right]\right)\right]\end{Bmatrix}$$

The term in brackets that the buyer maximizes is:

$$W_t^b(z)=\max_{f',e'\geq 0}\begin{Bmatrix}\dfrac{-(f'+e')(1-\beta)}{\beta}+\dfrac{\beta W(0,0)}{\beta}\\+\dfrac{\beta\sigma\theta\left[(1-\Lambda)\left(u\left[q(f)\right]-c\left[q(f)\right]\right)+\Lambda\left(u\left[q(f,e)\right]-c\left[q(f,e)\right]\right)\right]}{\beta}\end{Bmatrix}$$

After rearrangements we obtain the buyer's maximization problem which determines the quantity of monetary balances ($z'=f'+e'$) the buyer decides to hold in his portfolio before entering the following DM in order to consume the quantity $q(f)$ or $q(f,e)$ which maximizes his expected surplus net of the cost of money holdings.

$$W^b(f,e)=(f+e)+\max_{f',e'\geq 0}\begin{Bmatrix}-i(f'+e')\\+\sigma\theta\left[(1-\Lambda)\left(u\left[q(f)\right]-c\left[q(f)\right]\right)+\Lambda\left(u\left[q(f,e)\right]-c\left[q(f,e)\right]\right)\right]\end{Bmatrix}$$

The term in brackets is that the buyer maximizes:

$$\max_{f,e\geq 0}\left\{-i(f+e)+\sigma\theta\left[(1-\Lambda)\left(u[q(f)]-c[q(f)]\right)+\Lambda\left(u[q(f,e)]-c[q(f,e)]\right)\right]\right\}$$

Appendix 6

The calculation of $\frac{dq_f}{df}$ is conducted by differentiating the equation of terms of trade (17) that defines the terms of trade when buyer holds cash only we obtain the additional amount of good purchased with a marginal unit of fiat money i.e.: $\frac{dq_f}{df}=\frac{(1-\gamma)}{(1-\theta)u'(q_f)+\theta c'(q_f)}$

$$f=\frac{1}{(1-\gamma)}\left((1-\theta)u[q(f)]+\theta c[q(f)]\right) \qquad (17)$$

implies:

$$df=\frac{1}{(1-\gamma)}\left((1-\theta)\frac{\partial u[q(f)]}{\partial q}dq+\theta\frac{\partial c[q(f)]}{\partial q}dq\right)$$

$$df=\frac{1}{(1-\gamma)}\left[(1-\theta)u'[q(f)]dq+\theta c'[q(f)]dq\right]$$

We obtain finally:

$$\frac{df}{dq}=\frac{(1-\theta)u'[q(f)]+\theta c'[q(f)]}{(1-\gamma)} \text{ or } \frac{dq}{df}=\frac{(1-\gamma)}{(1-\theta)u'[q(f)]+\theta c'[q(f)]}$$

The calculation of $\frac{dq_{f,e}}{df}$ and $\frac{dq_{f,e}}{de}$ is conducted by differentiating the equation of the terms of trade (27). Indeed, by differentiating the expression $(1-\gamma)f+e=(1-\theta)u[q(f,e)]+\theta c[q(f,e)]$ we obtain:

$$(1-\gamma)df+de=(1-\theta)\frac{\partial u[q(f,e)]}{\partial q}dq_{f,e}+\theta\frac{\partial c[q(f,e)]}{\partial q}dq_{f,e}$$

and finally: $\frac{(1-\gamma)df+de}{dq_{f,e}}=(1-\theta)u'(q_{f,e})+\theta c'(q_{f,e})$ or $\frac{dq_{f,e}}{(1-\gamma)df+de}=\frac{1}{(1-\theta)u'(q_{f,e})+\theta c'(q_{f,e})}$

Given a fixed amount of electronic money $de=0$ we obtain the value of $\frac{dq_{f,e}}{df}$:

$$\frac{dq_{f,e}}{df}=\frac{(1-\gamma)}{(1-\theta)u'(q_{f,e})+\theta c'(q_{f,e})}$$

Given a fixed amount of fiat money $df=0$ we obtain the value of $\frac{dq_{f,e}}{de}$:

$$\frac{dq_{f,e}}{de}=\frac{1}{(1-\theta)u'(q_{f,e})+\theta c'(q_{f,e})}$$

Appendix 7

Let f be a function of a real variable defined in an interval I in R. Then, f is concave on I if and only if:

$$f(y) - f(x) \leq f'(x)(y-x) \text{ for all } y, x \in I$$

This theorem means that f', the first derivative is a decreasing function that allows to write also that for each $x < y$:

$$f(x_0) - f(x_1) \leq f'(x_1)(x_0 - x_1) \Rightarrow f'(x_1)(x_0 - x_1) \leq f(x_1) - f(x_0)$$

$$f'(x_1)(x_0 - x_1) \leq f(x_1) - f(x_0) \leq f'(x_0)(x_1 - x_0)$$

$$\Rightarrow f'(x_1) \leq f'(x_0)$$

1) With: $\Lambda = 1 \Rightarrow f^1 = 0, q_f^1 = 0, q_{f,e}^1 = q_e^1$, from (43) and by applying the second concavity theorem given that $(1-\gamma)f^1 < (1-\gamma)f^1 + e^1$ we obtain:

$$G_{f,e}^1 \equiv \sigma(1-\theta)\left\{\left(u(q_{f,e}^1) - c(q_{f,e}^1)\right) - \left(u(q_f^1) - c(q_f^1)\right)\right\}$$

$$G_{f,e}^1 = \sigma(1-\theta)\left[S\left((1-\gamma)f^1 + e^1\right) - S\left((1-\gamma)f^1\right)\right]$$

$$\sigma(1-\theta)S'\left((1-\gamma)f^1 + e^1\right)\left((1-\gamma)f^1 + e^1 - (1-\gamma)f^1\right) \leq \sigma(1-\theta)\left[S\left((1-\gamma)f^1 + e^1\right) - S\left((1-\gamma)f^1\right)\right] = G_{f,e}^1$$

$$\sigma(1-\theta)S'(e^1)(e^1) \leq \sigma(1-\theta)\left\{\left(u(q_{f,e}^1) - c(q_{f,e}^1)\right) - \left(u(q_f^1) - c(q_f^1)\right)\right\} = G_{f,e}^1 \qquad (58)$$

2) With $\Lambda \in [0,1] \Rightarrow (1-\gamma)f^{0,1} \geq 0, e^{0,1} \geq 0, q_{f,e}^{0,1} > 0$

From (57) and by applying the second concavity theorem given that $(1-\gamma)f^{0,1} < (1-\gamma)f^{0,1} + e^{0,1}$ we obtain:

$$G_{f,e}^{0,1} \equiv \sigma(1-\theta)\left\{\left[u(q_{f,e}^{0,1}) - c(q_{f,e}^{0,1})\right] - \left[u(q_f^{0,1}) - c(q_f^{0,1})\right]\right\}$$

$$G_{f,e}^{0,1} = \sigma(1-\theta)\left[S\left((1-\gamma)f^{0,1} + e^{0,1}\right) - S\left((1-\gamma)f^{0,1}\right)\right]$$

$$G_{f,e}^{0,1} = \sigma(1-\theta)\left[S\left((1-\gamma)f^{0,1} + e^{0,1}\right) - S\left((1-\gamma)f^{0,1}\right)\right] \leq \sigma(1-\theta)S'\left((1-\gamma)f^{0,1}\right)\left((1-\gamma)f^{0,1} + e^{0,1} - (1-\gamma)f^{0,1}\right)$$

$$G_{f,e}^{0,1} = \sigma(1-\theta)\left\{\left[u(q_{f,e}^{0,1}) - c(q_{f,e}^{0,1})\right] - \left[u(q_f^{0,1}) - c(q_f^{0,1})\right]\right\} \leq \sigma(1-\theta)S'\left((1-\gamma)f^{0,1}\right)(e^{0,1}) \qquad (59)$$