Purchasing Power Parity as a long-term memory process: Evidence from some emerging countries

Mohamed Bilel TRIKI
Faculty of Economics and Management,
University of Tunis El Manar. Tunisia.
Samir MAKTOUF
Professor in
Faculty of Economics and Management,
University of Tunis El Manar. Tunisia.

Summery: this paper uses a fractional cointegration technique to test the Purchasing Power Parity (PPP) for thirteen emerging countries which knew financial crises during the floating exchange period from 1990:1 to 2009:12. The focus is on whether the deviations from the cointegrating relationship possess long memory and wether the fractional cointegration analyses may capture a wider range of mean-reversion behaviour than standard cointegration analyses. We found that PPP held, but very weakly, in the long run between the *Argentine, Brazil, Chile, Colombia, Indonesia, Korea, Mexico, Thailand and Venezuela* and *US* exchange rate during our floating exchange rate period but that the deviations from it did not follow a stationary process. Nevertheless, it is also found that the deviations from PPP exist and can be characterized by a fractionally integrated process in nine out of thirteen countries studied.

Keywords: Purchasing Power Parity, Fractional cointegration, GPH method, Modified R/S test.
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I. Introduction

The theory of purchasing power parity (PPP) has rather acquainted doubt on its empirical validity. PPP has been tested widely. Basically, two alternative strategies have been followed by recent research in order to overcome the power problem of the validity of the PPP. In a first approach, some researchers have applied unit root tests on the real exchange rates and tests for autoregressive unit root on the alternative fractionally integrated more than the standard

alternative of the series being I (0).[1] The second approach motivated the use of cointegration procedures in order to test the validity of the PPP condition in the long run, which should form a stationary linear (or nonlinear) combination if long-run PPP holds.

Applying cointegration procedures on the real exchange rate (RER) in order to test the validity of the PPP condition in the long run,[2] Darne and Hoarau (2008), failed to find any evidence in favour of a long-run PPP.

The recent progress of the panel data techniques has faced up to the traditional time series approach as a relative short time series. Hence, applying panel data is feasible to focus on this little number of observations with homogenous exchange rate regimes. In recent years the development of panel data variants by using different panel unit root and cointegration tests has been focused (e.g., Frankel and Rose (1996), Levin et al. (2002)). In addition, they have applied this approach on the free-float period of OECD countries by presenting some support for a long-run PPP relationship (Wu (1996)). Nonetheless, for fewer developed countries, the evidence for a stable PPP relationship is much weaker (Mark (1990)).

All these results found and considered below lead to empirical methodology used yet that has already been used. The advanced econometrics technique has been important to derive the results of empirical studies which are interested in times series that are characterized by the property of long memory[3]. Consequently, it is quite possible that a relevance of the usual cointegration test might find the PPP not valid, while the application of the fractional cointegration might find the validity of the PPP[4]. For that reason, using standard cointegration test holds simply tow two cases. First, where the deviation from equilibrium is short second, the

[1] In addition, numerous researchers estimated half-lives of deviations from PPP based on this approach. See, among others, Elliott and Pesavento (2006). In addition, other work has been to detect for nonlinearites in deviation from PPP movements. See, e.g., Kilian and Taylor (2003).

[2] Mark (1990) applied cointegration techniques that were developed by Engle and Granger on PPP tests. Cointegration which is interpreted as evidence in favour of PPP was rejected by Cheng and Lai. But these tests applied on PPP, can lead to problems of endogeneity and regressors and estimates of the cointegration vector. Besides, although theses methods provide non-proportional relationships, the issue of whether real exchange rates are mean-reverting or not is not taken into consideration.

3 The most popular technique for analysing long memory time series is through the fractional differencing. Cheung and Lai (1993) have found the long-term persistence property in aggregate output.

[4] More supportive evidence toward long-run purchasing power parity (PPP) hypothesis has been found recently by Cheung and Lai (1993) with the fractional integration or cointegration techniques. Baillie and Bollerslev (1994) also provide evidence for the existence of fractional cointegration among seven exchange rates by conducting fractional cointegration tests.

restoration of equilibrium is quick, while the fractional cointegration test can capture cases where the deviation from equilibrium is prolonged and the restoration of equilibrium is slow that which is more observed in the kind of time series envisaged in our context. In in addition, the fractional cointegration investigation captures a very slow mean-reverting process.

To state the matter differently, in context of domestic and foreign prices in a different country, any shock for example in the foreign country (resulting in deviation from equilibrium) is quickly dissipated by arbitrage and the equilibrium is restored quickly but if the fractional cointegration in the tow markets is present, any shock to one of these country countries is not quickly dissipated hence, the deviation persists for a long time before the equilibrium is restored. Furthermore, testing the long-run validity of PPP theory has been very essential since it forms the foundation of much of exchange rate economics (e.g. flexible price monetary theory of exchange rate determination); Also, as a measure of long-run equilibrium exchange rate, its validity has important policy implications (Soofi, 1998).

On the other hand, from technical point of view, starting with the seminal contribution by granger (1981) and most of the work has considered the I(1)-I(0) type of cointegration in which linear combination, the deviation from equilibrium or the residual in the cointegrating equation is I(1) against an alternative I(0), . The the forgoing notion of cointegration is based on the Knife-edge dissimilarity between I(1) and I(0), thus a process is fractional cointegration tests I(1) against an alternative I(d) where d is less than unity. Those earnings in fractional cointegration are wider than the technique of the ADF test. If the coefficient of integration is less than unity, while the deviation from equilibrium or the residual may be large and may persist above a long phase. However in the long run the process is considered as a mean-reverting and the process can be returned over to its economic value. In that framework, in a fractional cointegration the deviation from intrinsic value or the residual is a long-memory process (unlike a deviation in a standard cointegration approach which is a short-memory process).

After further analyses (Diebold *et al*. (l991), Cheung and Lai (1993), Baillie and Bollerslev (1994), Chou and Shih (1997), Soofi (1998), Choudhry (1999), Alves et al. (2001), Nielen (2004), Masih and Masih (2004), Villeneuve et al. (2006)) it appears that the long-run cointegrating process between exchange rates and relative prices manifests over the time, they argue that this form of cointegration is associated with long memory and can reasonably be termed “fractional cointegration”.

The purpose of this study is to shed some light on the validity of PPP as a long-run condition and to display mean-reversion using a fractional cointegration approach that is suggested recently and applied by Cheung and Lai (1993).

Most long span studies on PPP have been undertaken for developed countries instead of developing countries, especially its validity with European and North American exchange rates.[5] As for the present analysis, we start with the relationship between nominal exchange rates, domestic and foreign prices, test step–by–step the necessary assumptions for long–run PPP, and focuses on the experience of the thirteen emerging countries exchange rate (Brazil, Argentine, Colombia, Chilly, Mexico, Uruguay, Venezuela, Malaysia, Singapore, Thailand, Korea, Indonesia and Philippines) over its recent floating regime, in which any formal analysis has been hardly conducted. While, little work has been done for emerging market economies.[6] In this paper, tests of mean-reversion in PPP under its least restrictive version using fractional cointegration analysis are conducted on USA's dollar using monthly data from 1983:4 to 2002:12, describing nominal bilateral exchange rates, domestic and foreign price levels. Later the paper examines the existence of a cointegrating relationship that illustrates PPP. Besides, that takes fractional cointegration analysis to examine if the deviations from the cointegrating relationship possesses long memory in the full period.

The remainder of this paper is organized as follows. Section II provides an overview of the PPP theory and its tests and discusses the empirical evidence from fractional cointegration analysis. Section III exposes a brief review of some econometric issues and comments the empirical results. Section IV reviews the main findings.

II. An overview of the PPP Hypothesis and its tests

The Purchasing Power Parity is the theory that is based on the law of one price, which declares that in competitive markets, identical goods will sell for identical prices when valued in the same currency. In the economics' literature, there are two versions of the theory of PPP. "The *Absolute PPP*: The price of internationally traded commodities should be the same in every country, that is, one unit of home currency should have the same purchasing power worldwide".

[5] Tests of PPP have been conducted on some countries outside the OECD, most notably high inflation countries such as those in Latin America (see for examples, Mahdavi and Zhou (1994)). This is due to the lack of long-span historical series for the latter group.

[6] Frankel and Rose (1996) and Lothian (1997) are two exceptions, but these studies use official nominal exchange rates and use fractional cointegration.

"The absolute version, popularly called the *law of one price*, is a very strong hypothesis and is usually considered by most economists as not relevant"[7]. In favor of the existence of features, like measurement errors in the variables, transportations costs and differently weighted price indices, we will study the weak version of long-run PPP, which relaxes the hypothesis of symmetry and proportionality that underlie the analysis of real exchange rates. Especially, we will gaze at expressions of the type:

$$s_t = c + \beta_1 p_t + \beta_2 p_t^* + v_t, \quad (1)$$

Where s_t is the logarithm of the nominal spot exchange rate (in the present case, the emerging countries devise value of one unit of the foreign currency), measured as domestic currency units per US dollar; p_t is the logarithms of the domestic price levels and p_t^* the logarithms of the foreign aggregate price level, and v_t is an error term incorporating deviations from parity. A finding of cointegration is a necessary condition for PPP to qualify as a long run constraint, so we will find that v_t should be stationary.[8] While basing on the condition of establishing the cointegration relation of these three variables, involving a stationary error term, deviations from parity will be mean reverting.

The parameters β_1 , β_2 are respectively domestic price and foreign price elasticities which allow for heterogeneous relations between exchange rate and prices. This kind of version does not impose the restrictions β_1=-1 and $\beta_2 = 1$ implicit in the strong version of PPP. This type of equations must be interpreted as long-run equilibrium relationships and, for this, it is required that there is cointegration among the variables. If cointegration is present, we will test for the strong version of PPP (i.e., whether β_1=-1 and $\beta_2 = 1$). In this respect, this paper investigates the proportionality condition on the relative price in equation (1) and the restriction commonly imposed is β_1=-1 and $\beta_2 = 1$, under this condition, we will investigate the strong version of PPP which can test weather the real exchange rate is stationary or not. The failure of a unit root indicates that the strong PPP holds in the long run.

[7] Definition is from Encyclopaedic Dictionary of International Finance and Banking.

[8] Following Junge (1984), this specification and its further restrictive version $\left[e_t = c + \beta_\tau \left(p_t + p_t^* \right) + \varepsilon_\tau \right]$ are related to as relative PPP as distinct from absolute PPP.

Empirical evidence from fractional cointegration analysis:

The empirical literature on the "Purchasing Power Parity" (PPP) hypothesis is as vast as its history (Froot and Rogoff, 1995).[9] In the present section, we are reviewing the empirical findings of several studies which resorted to the refinement of the cointegration results through fractional cointegration.

Many empirical studies tested the long run validity of PPP using the fractional cointegration analysis, more specifically the ARFIMA (p,d,q) model, we refer to Diebold, Husted and Rush (1991), Cheung and Lai (1993), Baillie and Bollesley (1994), Choundhy (1999), Lien and Tse (1999), Alves, Coti and Fovor (2001), Nielson (2004), Masih and Masih (2004) and finally Vileneuve and Handa (2006). Diebold et al (1991) discovered that purchasing Power Parity holds in the long run for each of the currencies studied and that the typical half of a shock to parity is approximately 3 years. Cheung and Lai (1993) implement the fractional cointegration procedure to six currencies for the period 1914-1989 (Canada, France, Italy, Japon, Uk, Us), which surround in different periods as well as great instability and turbulence including two world wars and two oil crises. Their empirical results display that PPP reversion exists and can be characterized by a fractionally integrated process in three out of five countries studied. The source of property of PPP deviations for many different countries can be accounted for by interactions between exchange rate and interest rates defined as economic fundamentals, such as, the levels of out put and money supply.

Baillie and Bollersley (1994) reported that the deviation from the cointegrating relationship suggests that it possesses long memory and may possibly be well described as fractionally integrated process. Soofi (1998) used cointegration and fractional cointegration methods in determining the mean-reverting properties of the parallel market exchange rates for several members of the OPPC (organization of Petroleum Producing countries). They applied the Geweke and Porter-Hudak (GPH) test and showed that PPP models for Algeria, Ecuador, Saudi Arabia and Venezuela are fractionally cointegrated. ChouDhry (1999) investigated the

[9] This brief review draws from some excellent new surveys on the PPP literature such as Froot and Rogoff (1995), and Sarno and Taylor (2002).

Purchasing Power Parity (PPP) between the United States and four high inflation Eastern European Countries (Poland, Romania, Russia and Solovenia). Using fractional and Harris-Inder cointegration test methods, they provided, when applying the relative version of PPP, an evidence for Russia and Solovenia but only very little evidence by using the absolute version of PPP.

Alves et al (2001) also tested the validity of the Purchasing Power Parity (PPP) in Brazil vis-à-vis an international price index. Historical data for the period 1855-1996 are considered in order to compare the results with those obtained by Zini and Cati by using the standard cointegration models. The fractional cointegration is verified for relative version of PP but is rejected for absolute version. Neilson (2004) proposed a Lagrange multiplier test of the null hypothesis of cointegration relevance of the present test that was established by Monte Carlo experiments. By using this methodology to the analyzing of exchange rate dynamics for seven major currencies against the US dollar , Neilson tested the presence of Fractional cointegration however a mixed evidence of the existence of a cointegration relation.

Masih and Masih (2004) employed fractional cointegration to estimate the adjustment dynamics of PPP between Australia and Seven of its major trading partners using quarterly data over the period 1983-1994. They found that deviation from equilibrium followed a fractional cointegration process with mean-reversion, with the nominal exchange rate endogenously and with prices standing out as the econometrically relatively exogenous variable. Finally, Villeneuve and Handa (2006) used fractional cointegration techniques to test the PPP between the Canadian and the US currencies during the floating exchange period from 1974:1 to 2001:2. They found evidence that the deviation from PPP does not follow a fractionally cointegrated stationary process, so that at best holds only weakly even in the large run.

Few remarks can be stretched from this review of the result from PPP studies using integer cointegration and/or fractional cointegration analyses. First, all these studies do not examine certain emerging countries, so it is more important to test the PPP to these countries. Second, some version of PPP is less likely rejected for long data; the absolute version is more likely rejected to the relative version. Third, applying the ADF and KPSS test is critical so that it is more important to involve the fractional-ADF test. Fourth, the exchange rate follows a very long mean-reversion process, for that reason the fractional cointegration relationship for the PPP variables is more appropriate.

III. Econometric Issues

We envisage two series, x_t and y_t each of which being integrated of order 1. x_t and y_t are fractionally cointegrated[10] if there exists a cointegration relationship:

$$y_t = \alpha + \beta x_t + z_t \qquad (2)$$

Where z_t is a long-term memory process, such as an *ARFIMA* process[11]:

$$\Phi(L)(1-L)^d z_t = \Theta(L)\varepsilon_t \quad t = 1, 2, ..., \qquad (3)$$

Where $\Phi(L)$ and $\Theta(L)$ are autoregressive and moving average polynomials, respectively, ε_t is white noise, L is lag operator and:

$$(1-L)^d = 1 - dl - \frac{d(1-d)}{2!}L^2 - \frac{d(1-d)(2-d)}{3!}L^3 - .... \qquad (4)$$

The following fractional cointegration tests are based on the null hypothesis:

H_0 : x_t *and* y_t are not fractional cointegrated, i.e. z_t is *I* (1).

H_1 : x_t *and* y_t are fractional cointegrated, i.e. z_t is *I* (d), with $d \prec 1$

These tests are applied on the estimated residuals $\hat{z}_t$ of the long-term relationship (1). We adopt a test procedure which follows a methodology similar to that proposed by Engle and Granger (1987)[12] . Thus, we can initially test the order of integration of the original series, and if all of them have the same order of integration (say unit root), we can test the degree of integraton of the estimated residuals of the cointegrating structure. In this context, nonrejections of the null hypothesis that the order of integration of the estimated residuals is equal to that the original series will imply that the series are not cointegrated. On the other hand, rejections of the null in favour of alternatives with a smaller degree of integration represent evidence of fractional cointegration of a certain degree.

The result of this test is important from an economic point of view: if a variable is an *I*(d) process with $d \in [0.5, 1)$, it will be covariance nonstationary but mean-reverting inasmuch as an

[10] The concept of fractional cointegration has been recommended for the first time by Cheung and Lai (1993) in framework of Purchase Power Parity.

[11] For a presentation of Autoregressive Fractionally Integrated Moving Average processes, see Granger and Jouyeux (1980).

[12] for more details, see Gil-Alana (1997), we can also use the test of trust proposed by Johansen (1988).

innovation will have no permanent effect on its value. This is in contrast to an *I*(1) process which will be both covariance nonstationary and not mean-reverting, in which case the effect of an innovation will persist forever.

III-1 Fractional cointegration tests based on the estimation of ARFIMA processes (GPH):

We just remember the main lines of the technique employed here: the Geweke and Porter hudak (1983) method (GPH).

The application of these procedures on residual series allows us to test the null hypothesis of a unit root (d=1) against the alternative of fractional integration $(d \prec 1)$. This is equivalent to a test of the null $d' = 0$ against $d' \prec 0$, with $d' = d - 1$ where d is the fractional difference parameter of the levels and d' the fractional difference parameter of the series in first differences.

Hosking (1981) explains that when $d \in (-0.5, 0.5)$ the process and any shock on z_t is stationary and invertible. When $d = 0$, the process z_t is stationary and any shock on z_t decays to zero at a quick rate. When $d \in (0, 0.5)$ the autocorrelations are positive and decay to zero with a very high hyperbolic rate, which implies that z_t is stationary and possesses a long memory effect. In the case where, an 'intermediate memory' characterizes the time series z_t as autocorrelations are negative. The process z_t is non stationary in the case where $d = 1$.

The aim of the Geweke and Porter hudak (GPH) method is to estimate the fractional integration parameter $\hat{d}'$ by the following regression:

$$\ln I(\lambda_j) = \hat{\alpha} - 2\hat{d}' \ln\left(2\sin\left(\frac{\lambda_j}{2}\right)\right) + \hat{e}_j \qquad (4)$$

Where λ_j is the Fourier frequency $\lambda_j = \frac{2\pi j}{T}$, $I(\lambda_j)$ is the periodogram of $\Delta z_t, t = 1, ..., T$, and j=1, 2,…, m where m corresponds to the number of periodogram ordinates. Traditionally the number of periodogram ordinates is chosen from the interval $\left[T^{0.45}, T^{0.55}\right]$. However, Hurvich et al. (1998) have recently showed that the optimal m is of order $\left(T^{0.8}\right)$.

Asymptotic normality of the estimated fractional difference parameter has been proved by Geweke and Porter hudak (1983) when $d \prec 0$ and by Robinson for $0 \prec d \prec 1/2$.

The GPH method is a two step estimating procedure. Indeed, one has to estimate the fractional difference parameter in a first step. In the second step, autoregressive and moving average parameters are estimated using traditional time series methods. There exist however one step and estimation procedures, like exact maximum likelihood (EML) one (see Sowell (1992) for details).

III.2 The modified *R/S* analysis:

Lo (1991) derived a test, called the modified *R/S* statistic, of the null hypothesis of short-range dependence which is invariant to a general class of short-term memory processes. The modified *R/S* statistic, denoted as Q_{mT}, is given by:

$$Q_{mT} = R / s_T(q) = \frac{1}{s_T(q)} \left[\underset{1 \le k \le T}{Max} \sum_{j=1}^{k} \left(\Delta \hat{z}_j - \overline{\Delta \hat{z}} \right) - \underset{1 \le k \le T}{Min} \sum_{j=1}^{k} \left(\Delta \hat{z}_j - \overline{\Delta \hat{z}} \right) \right] \tag{7}$$

Where

$$s^2{}_T(q) = \frac{1}{T} \sum_{j=1}^{k} \left(\Delta \hat{z}_j - \overline{\Delta \hat{z}} \right)^2 + \frac{2}{T} \sum_{j=1}^{q} w_j(q) \left[\sum_{j=1}^{k} \left(\Delta \hat{z}_j - \overline{\Delta \hat{z}} \right) \left(\Delta \hat{z}_{i-j} - \overline{\Delta \hat{z}} \right) \right] \tag{8}$$

With

$$w_j(q) = 1 - \frac{j}{q+1}, q \prec T \tag{9}$$

Where j = 1,…,q. We see that the autocovariance are weighted according to lags (see Andrews (1991) for the choice of q and Newey and West (1987) for the weights $w_j(q)$)[13]. The limiting distribution of the modified R/S statistic is known (see Lo, 1991) and it is thus possible to test the null hypothesis of short-term memory against the alternative of long-term memory (fractional integration) of the error term by comparing the statistic $V = Q_{mT} / \sqrt{T}$ [14] to critical values. At 5% significance level, the null hypothesis of the absence of long memory phenomenon

[13] Boutahar et Al. (2009) recommend to take the integer part of $[4(T/100)^{1/4}]$ as a value of q.
[14] this statistic converges to the range of a 'Brownian bridge' on the unit interval.

is rejected of the modified R/S statistic does not fall within the confidence interval [0.809 ; 1.862].

IV. Data and empirical Results

All data used for this empirical analysis are obtained from IFM. We consider monthly series of 230 observations. Both the domestic real exchange rate and the foreign domestic domestic and consumption price index foreign and domestic for thirteen emerging countries which are attacked by financial crises.These countries include Argentine, Brazil, Chile, Colombia, Indonesia, Korea, Malaysia, Mexico, Philippines, Singapore, Thailand, Uruguay and Venezuela on the M1:1990 to M2:2009 period.

In this section results are conducted from the GPH test for fractional cointegration and R/S modified.

IV.1 Testing for unit roots

The hypothesis tested in this paper is whether or not shocks had very long-term effects on the real exchange rate between thirteen emergent countries and USA. If so, there will be a high order of correlation in the time series, so that the Augmented Dickey Fuller (ADF) test for unit roots has been preferable to the Dickey-Fuller (DF) test, since the DF tests for an AR (1) process, while the ADF makes a parametric correction for higher-order correlation by assuming that the series tested follows an AR (p) process. Our three series were for the logarithmic values of the emergent's country–US dollar exchange rate and the two price indices for emergent countries and USA. Plotting them indicated that the ADF test should be applied to the series with both a constant and trend. The number of lags in the regression was determined by the statistics Q of Ljung-Box.

Results of usual unit root tests ADF are reported in Table 1 on each of the time series. At the 5% confidence level, the hypothesis of a unit root for each of the time series is not rejected but is rejected for their first differences only s_t for Chile, Colombia and Uruguay the hypothesis of a unit root is rejected for their first level. Therefore, p_t and p_t^* are all I(1).

***Table 1**: Unit root tests on s_t, p_t and p_t^**

		ADF
Argentine	s_t	-4.759* (3) (variation)
	p_t	-4.797* (3) (variation)
	p_t^*	-7.654* (3) (variation)
Brazil	s_t	-1.923** (1) (variation)
	p_t	-3.526* (3) (variation)
Chile	s_t	-5.343* (3) (level)
	p_t	-6.507* (3) (variation)
Colombia	s_t	-4.068* (3) (level)
	p_t	-4.383* (3) (variation)
Indonesia	s_t	-4.807** (3) (variation)
	p_t	-3.970* (3) (variation)
Korea	s_t	-7.670* (3) (variation)
	p_t	-5.659* (3) (variation)
Malaysia	s_t	-7.874* (3) (variation)
	p_t	-5.877* (3) (variation)
Mexico	s_t	-4.378* (3) (variation)
	p_t	-5.984* (3) (variation)
Philippines	s_t	-3.426* (3) (variation)
	p_t	-6.519* (3) (variation)
Singapore	s_t	-3.631* (3) (variation)
	p_t	-4.351* (3) (variation)
Thailand	s_t	-7.633* (3) (variation)
	p_t	-8.261* (3) (variation)
Uruguay	s_t	-4.099* (3) (level)
	p_t	-3.961* (3) (variation)
Venezuela	s_t	-3.995* (3) (variation)
	p_t	-7.688* (3) (variation)

(1): Model without neither constant nor deterministic trend.
(2): Model with constant and without trend. (3): Model with constant and trend.
***: stationary series at 1% significance level, **: stationary series at 5% significance level,
*: stationary series at 10% significance level. Variation means first difference of variable.

IV.2 Fractional cointegration and PPP

It seems thus interesting to test the existence of a stable long-term relationship, that led to the appearance of the real exchange rate when applying the strong version of Purchase Power Parity. Results of the application of traditional cointegration test (ADF) on residuals are reported in ***table 2***. These results show that the error term is non stationary at the 5% significance level, suggesting that the Fisher hypothesis does not hold.

However, this last result may be due to the fact that the usual concept of cointegration is too restrictive. We thus apply fractional cointegration tests. Since the tests should be applied on stationary series, they have been run on residuals in first differences. Results are reported in ***table 3*** for the modified *R/S* analysis, in ***table 4*** for the *GPH* test. Our test for fractional cointegration is divided into two steps. First, we use Equation (1) to estimate the value of residual derived from the strong relation of PPP. Second, using the GPH procedure, we estimate the parameter of integration *d*, and finally we test whether the residuals of the cointegrating relationship are *I (d)* with $0 \prec d \prec 1$ or not.

It is essential to note that the properties of these tests, such as the asymptotic distribution under the null hypothesis, are identified only if the true equilibrium errors v_t are observable. However this is not the case since the fractional cointegration tests are useful to estimated equilibrium errors. Therefore, the error correction term tends to be biased in favor of the stationarity hypothesis leading to too many rejections of the null hypothesis of no cointegration. One should thus use other critical values than those calculated on the basis of the true observed series.[15]

According to the modified *R/S analysis*, all residual series are fractionally integrated, since the statistic *V* does not range in its confidence interval given by Dittmann (2000) only for Argentine, Philippines and Venezuela. According to the *GPH* procedure, all real exchange rate series are fractionally integrated since the fractional difference parameter appears to be significantly different from zero for all country. We apply these two procedures in order to be sure that the *GPH* procedure has been reached.

[15] The reader is referred for example to Cheung and Lai (1993) and Diattmann (2000) for critical values of fractional cointegration tests.

The existence of a cointegration relationship between the variables studied requires that the equilibrium error, v_t, is mean reverting.[16] Further, the estimated value for certain country seems to be significantly different from 0.5 and we conclude that the presence of fractional cointegration between s_t, p_t and p_t^*, and the shocks induce non-stationary deviations from the long-run PPP relationship established from the cointegration vector.

The estimated values of d for all country countries are consistently smaller than one for the three values of μ for *Argentine, Brazil, Chile, Colombia, Indonesia, Korea, Mexico, Thailand and Venezuela*. Values of d when smaller than unity for three of our values of μ indicate that the error correction term v_t is a mean-reverting process, such results are confirmed confirming by Díaz, Lüders and Wagner (2003). Hence, our estimated *d* values are in the non-stationary but mean-reverting range (between 0.5 and 1.0) for statically significant values values significant. Recently, Taylor (2002) presents evidence for Argentina, Brazil and Mexico with more than 100 years of data, he finds that real exchange rates are stationary. These results support the PPP relationship in the long run, but very weakly.[17] With the exception of four countries, Malaysia, Philippines, Singapore and Uruguay, the value of *d* was greater than one; our results do not support evidence of fractional cointegration and so the PPP does not hold, however, such phenomenon seemed present in the series in the application of modified R/S method.

Our results do not provide evidence of fractional cointegration between the nominal exchange rates and relative price levels, but supports, though weakly, the PPP relationship for the long run.

***Table 2**: Unit root tests on real exchange rate*

	ADF
Argentine	-5.959* (3) (variation)
Brazil	-7.101* (3) (variation)
Chile	-6.838* (3) (variation)
Colombia	-7.794* (3) (variation)
Indonesia	-4.431* (3) (variation)
Korea	-4.228*(3) (variation)
Malaysia	-6.255*(3) (variation)
Mexico	-4.888*(3) (variation)

[16] The mean reversion behavior of the equilibrium error is of key interest, for unless the equilibrium error exhibits mean reversion, a shock to the system of variables studied will tend to permanently drive this system out of equilibrium.

[17] See table 5.

Philippines	-6.989*(3) (variation)
Singapore	-8.036*(3) (variation)
Thailand	-5.025*(3) (variation)
Uruguay	-3.934*(3) (variation)
Venezuela	-6.566* (3) (variation)

(1): Model without constant or deterministic trend.
(2): Model with constant and without trend. (3): Model with constant and trend.
***: stationary series at 1% significance level, **: stationary series at 5% significance level,
*: stationary series at 10% significance level.

***Table 3**: Modiied R/S analysis*

	d	*V*	*q=2*	*q=4*	*q=6*	*q=100*
Argentine	0.790		1.418	1.241	1.159	1.026
Brazil	0.734		1.536	1.490	1.475	1.944*
Chile	0.764		1.447	1.339	1.311	1.623*
Colombia	0.774		1.35	1.23	1.20	1.643*
Indonesia	0.713		1.373	1.509	1.538	2.103*
Korea	0.765		1.411	1.36	1.381	2.202*
Malaysia	0.723		1.486	1.378	1.345	1.711*
Mexico	0.725		1.129	1.075	1.117	1.981*
Philippines	0.825		1.493	1.342	1.282	1.313
Singapore	0.696		1.726*	1.66**	1.699	1.299
Thailand	0.746		1.402	1.339	1.322	1.695*
Uruguay	0.760		2.183*	1.936*	1.791*	1.317
Venezuela	0.692		0.956	1.025	0.986	1.585

**: stationary series at 5% significance level, critical value is 1.747
*: stationary series at 10% significance level, critical value is 1.620

***Table 4**: Results from the GPH test*

	GPH		*GPH*		*GPH*	
	d(μ=0.45)	*t-statistic*	*d(μ=0.50)*	*t-statistic*	*d(μ=0.55)*	*t-statistic*
Argentine	0.952**	3.502	0.952**	4.139	1.047**	5.598
Brazil	0.487	1.410	0.809**	2.959	0.895**	3.677
Chile	0.886**	3.257	0.995**	4.543	1.000**	5.347
Colombia	0.883**	3.246	1.018**	4.648	1.060**	5.668
Indonesia	0.729**	2.680	0.832**	3.744	1.018**	5.443
Korea	0.930**	3.419	0.818**	3.735	0.817**	4.368
Malaysia	1.050**	3.860	1.164**	5.315	1.046**	5.593
Mexico	0.828**	3.046	0.954**	4.356	0.967**	5.173
Philippines	1.118**	4.110	1.329**	6.068	1.244**	6.624
Singapore	1.271**	4.672	1.158**	5.287	1.171**	6.232
Thailand	0.860**	3.164	0.855**	3.904	0.913**	4.882
Uruguay	1.234**	4.528	1.190**	5.433	1.270 **	6.791
Venezuela	1.051**	3.857	0.877**	3.993	0.887**	4.637

**: stationary series at 5% significance level, critical value is 2.576

***Table 5**: Interpretation of the d value*

d value	PPP
>1.0	PPP is not valid
1.0	PPP is not valid
0.5 – 1.0	Deviation are mean-reverting but not Stationary; PPP holds, but very Weakly.
0.0 – O.5	Deviation are mean-reverting and Stationary in the long run; PPP hold Weakly.
d=0	Deviation follow a stationary and Mean-reverting process; PPP holds.

Source: Villeneuve and Handa (2006)

V. Conclusion

This paper has tested the relevance of PPP for the bilateral exchange rate between the thirteen emergent countries and the American dollar for the floating exchange rate period from 1990:M1 to 2009:M2. The fractional cointegration analyses were performed on this period for some countries.

The pattern of residuals using the cointegrating vector did not indicate mean reversion, so that fractional cointegration analysis was employed. The hypothesis of fractional cointegration over the period was rejected. The deviations followed a non-stationary, though mean-reverting, process. Hence, the process describing the deviations from the long-run PPP relationship was non-stationary and did not possess long memory. We conclude that PPP held, but very weakly, in the long run between the *Argentine, Brazil, Chile, Colombia, Indonesia, Korea, Mexico, Thailand and Venezuela* and US exchange rate during our floating exchange rate period but that the deviations from it did not follow a stationary process.

Considering the very close trading relationship between the *Argentine, Brazil, Chile, Colombia, Indonesia, Korea, Mexico, Thailand and Venezuela* and US economies and that our chosen period had a floating exchange rate regime, our findings indicate the relevance of PPP to be severely in doubt for explaining movements in the exchange rate under a floating exchange rate regime.

Our results lend support to the general conclusion in the empirical literature of the 'fragile' nature of the evidence in favor of the PPP hypothesis, and particularly the 'extra degree of fragility' for the *Argentine, Brazil, Chile, Colombia, Indonesia, Korea, Mexico, Thailand and Venezuela* –US dollar exchange rate.

Our findings are also consistent with a different slant on the PPP hypothesis provided by asking whether relative prices are affected by exchange rate fluctuations. There is mounting

evidence that relative prices are not much affected by exchange rate fluctuations in the short run. Therefore, given our finding that the short-run changes in the exchange rate do not respond, or respond very weakly, to the fluctuations in domestic and foreign prices and the general finding in the literature that relative prices are not very responsive in the short run to changes in the exchange rates, the conclusion seems inescapable that the short-run dynamic adjustments required for PPP between exchange rates and relative prices are very weak.

Our findings are consistent with the consensus of the empirical literature, reviewed earlier in this paper, on PPP between *Argentine, Brazil, Chile, Colombia, Indonesia, Korea, Mexico, Thailand and Venezuela* and the USA. Two possibilities that have been pursued in the empirical literature for other pairs of countries and could be investigated in future research for the *Argentine, Brazil, Chile, Colombia, Indonesia, Korea, Mexico, Thailand and Venezuela* –US real exchange rate would be to introduce non-linear dynamics, employing for example both Switching and fractional cointegration techniques (FI-STAR).

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